Interpreting Quantum Mechanics using Chaos Theory

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Abstract

The strange and often counter-intuitive aspects of quantum mechanics such as collapse and entanglement have led to a series of interpretations of the reality expressed by the discipline. The present work purports to the formulation and postulation of a new interpretation of quantum mechanics, one that stems from the principles of chaos theory. Specifically, the superposed state in a quantum system is viewed as an equivalent of a chaotic signal, whose collapse is decided procedurally by the initial conditions and timing-wise by the process of measurement. The basic postulates of quantum mechanics and a glimpse of entanglement is viewed in this perspective. Following this, the proposed chaotic interpretation is compared with other standard interpretations in light of various properties such as determinism, locality, realism, counterfactual definiteness, and hidden variables. The novel perspective of quantum mechanics fundamentals through chaos theory, where the latter has undergone significant progress in terms of studying the system behavior using standard tools and evolutionary patterns, offers a plethora of interesting and exciting options to study the hidden realities of nature.

Keywords: Quantum Mechanics, Chaos Theory, Superposition, Collapse, Entanglement

Introduction

At the heart of nearly all of science today is Quantum Physics. Quantum mechanics is a theory so counter-intuitive, yet repeatedly tested and validated for its accuracy and precision [1]. Of the many applications of this branch of science, quantum information and computation stands out [2]. This is essentially due to the fact that quantum information promises fanciful yet incredibly useful applications such as quantum teleportation and superdense coding, which could usher in the era of higher data security, capacity and ultrahigh processing speeds [3-6].

A theory as counterintuitive as quantum mechanics is bound to have its controversies. Most of the controversies surrounding this theory stem from the various interpretations given by Einstein, Bohr, Bohm and the like regarding its various ‘quirks’ such as wavefunction collapse, quantum entanglement, symmetry breaking and so on [7-9].

Chaos theory, as the flagship of Nonlinear Science, has found extensive applications in recent years in fields as diverse as biology, astrophysics, psychology and finance [10-14].

The present article takes advantage of the extensive knowledge accumulated in the domain of both chaos theory and quantum physics, proposes a new interpretation of quantum mechanics based on chaos theory. A fundamental postulate in the proposed interpretation is the equivalence between the superposed state and a chaotic signal. Though chaotic signals are deterministic, the impracticality of knowing completely and accurately all the initial conditions giving rise to chaos makes the predictability impossible, rendering the whole system ‘theoretically deterministic yet practically random’ [10,11]. In this light, the collapse of a quantum superposed state is viewed as the decrease in the chaotic nature of the system.

Following a detailed postulation of the chaotic interpretation, a glimpse into interpreting the phenomenon of entanglement is presented [9,15,16,17]. Following this, various properties of the chaotic interpretation such as determinism, reality, locality and counterfactual definiteness are discussed in comparison with other existing interpretations [7,8,9].

The implementation of a chaos based quantum system yields a number of advantages – firstly, quantum information applications such as quantum teleportation and superdense coding hitherto possible only in expensive superconducting systems or high magnetic field environments can hence be performed with much lesser cost using standard chaotic systems such as Chua circuits [18-22]. Secondly, it provides an experimental platform to verify most of the postulates of quantum mechanics including McHarris' chaos-quantum equivalency [23-26]. Thirdly, a lot of unified theories in particle physics such as the one proposed by Lloyd use quantum bits as the basis [27, 28]. A chaos enabled implementation of such 'computational universe' models could potentially answer a lot of unanswered questions in science such as dark matter, black hole singularities and so on [27].

The Schrodinger's Cat

The starting point for any quantum interpretation is a simplified discussion of one of the classical thought experiments that gave quantum physics to the world – the Schrodinger's Cat [29-32]. In essence, there is a closed box containing a cat. There is also poison in the box that may or may not activate. If the poison activates, the cat dies. If it doesn't activate, the cat lives. Simple probability dictates there is a 50% chance of survival; the cat is either dead or alive after the box is opened, but not both [29,31]. But, in quantum physics, one is interested in knowing if the cat is alive or dead before the box is opened. Of course, there is no definite way of knowing, which is why so many interpretations exist [7,8,9].

A direct real-life application of the Schrodinger cat concept is the qubit, an extension of the classical computing bit, with the unique additional property being defined as the "superposition of states". That is, a qubit can be 0 or 1 or a superposed state. This superposed state is seen as having both 0 and 1 [2]. But the superposed state is not permanent. The moment one "measures" it, the superposed state "collapses", changing to either 0 or 1.

These concepts are concisely represented in the Master Equation of Quantum Mechanics, termed the 'Schrodinger's Equation', described in standard Dirac Ket Notation [33-35]:

$$\boldsymbol{H}|\boldsymbol{\psi}> = \boldsymbol{E}.|\boldsymbol{\psi}> \qquad (\mathbf{1})$$

Here, '|ψ>' is the superposed state, representing both 0 and 1, or 'dead' and 'alive'. 'H', termed the "operator" is the process of measurement. What the equation says is that, the process of measurement H of a superposed state '|ψ>' leads to 'E', which is the measured value, or the collapsed state of the system.

It is well known that Einstein and Bohr had contradicting views about the principles encompassing the Scrodinger's cat [8-9]. The following is an extremely simplified summary of the individual contentions:

1. Einstein contended that the cat could never be both alive and dead. One doesn't know that it is alive or dead until the measurement, which only means one does not have enough information about the system, rather than the cat itself being in some kind of special state. So, in Einstein's views, Quantum Mechanics was "Correct, yet Incomplete" [8].
2. Bohr argued that the observer is an integral part of the entire system. Measurement is a physical process which affects both the measurer and the measured. So, in this perspective the observer is interacting with the system, changing its status upon measurement [9].

Though neither of these could be conclusively proved at that time, the debates lead to a series of interpretations, which is where the proposed chaotic interpretation is also placed [7].

An Overview of Chaos Theory

In physics, a Chaotic System is defined as a system whose behavior is highly fluctuating, almost always, depending on certain factors, called "initial conditions" [10-14,36,37]. This implies that, even if the initial condition changes very slightly, the system will show a drastic difference in the behavior and this property is aptly named "Sensitive Dependence on Initial Conditions", also explained more popularly as the "Butterfly Effect" [38-40]. Furthermore, the property that the initial events are carried over to subsequent events amplified implies that chaos contains "memory" [39].

Chaos is essentially deterministic. This means that if one knows the initial conditions, one can easily find out the output of a chaotic system at any point in time. But since the behavior is so fluctuating and it is almost always impossible to know all initial conditions, it appears like as if the chaos looks random, which is a clearly misleading appearance [10-14,36-40].

The Chaotic Interpretation

A fundamental principle to the chaotic interpretation of quantum mechanics proposed here is the equivalence between the qualities of a chaotic system and the "superposed state", outlined as follows:

a) A chaotic system is what could be called "rich". It has a lot of fluctuations, and hence a lot of options to offer. Consequently, it contains a lot of information [38].
b) A superposed state, such as a qubit also offers a lot of options. For example, in a qubit, there is an option of collapsing to 0 or 1 [2]. In a normal bit, no collapse, no options. So, visibly, a qubit contains more information than a regular bit.

This leads to the first point in the proposed interpretation:

I. The superposed state of a quantum system is a chaotic signal, and is seen as the wavefunction "ψ".

The next part is to interpret collapse. In collapse, the quantum state (qubit) transforms to a classical state (regular bit) [1,2]. It loses its information content, which according to Lloyd's formulation means that it loses its asymmetry [36]. Thus, the chaotic nature is destroyed in a collapse, which means, by extension,

II. Collapse is a stage in the evolution of "ψ" and is a reduction in its 'chaoticity' and asymmetry.

The next question is then one of determinism [41,42]. If one views the superposed state as a chaotic signal, it means that it is deterministic. This leads to the following point:

III. Given the exact initial conditions, one can predict the evolution of the chaotic system at any time. However the impracticality of knowing all the initial conditions make the system "practically random".

The next aspect would be measurement. For the sake of convenience, the initial configuration of the system is denoted by 'X', the system itself as 'S', the collapse as 'C' and measurement as 'M'.

Now, according to the property of determinism, S evolves in a chaotic fashion completely predictable. So, C is just a stage in the evolution of S. So, the question to be answered is, 'Is C connected to M?'

Suppose that C is not related to M. Thus C can happen before M, during M or after M. C happening before M is completely possible and plausible. But C happening after M is not a possibility. This is because it is known that measurement causes collapse and once a system is collapsed, it cannot collapse again. For example, if a qubit collapses to a 0, no matter how many times one measures it after that, it stays 0.

Thus, C is an irreversible operation. Also, collapse is defined as a decrease of asymmetry and increase in symmetry. As an example, if one takes a piece of clay, which is an asymmetrical shape and moulds it into a ball/sphere, a symmetrical shape, whichever way the ball is turned it looks the same. It isn't known anymore which is front, back, left or right. It is impossible to restore the sphere back to the original piece, since one doesn't know the direction. Asymmetry to symmetry is irreversible.

IV. Collapse is related to Measurement, and Collapse is an irreversible process.

But, it is also said that C is just a stage in S's life. So, if C depends on M, does S depend on M? The chaotic interpretation postulates that it does. S depends on M, as much as S depends on X.

V. Measurement and Initial Configurations together form the "initial conditions" of the system.

X is the initial configuration, which is part of the initial conditions. But as was said earlier, finding X is completely and accurately practically impossible and that makes chaos 'practically random'.

However, it is mentioned that M forms the initial conditions of S. But M occurs in a later stage of S's life. This might apparently be seen as leading to a notion of non-causality or retro-causality in the system, where a future event defines the present. This seems counter-intuitive and strange. But, it is postulated that the part of the initial condition that will determine S's behavior is already given by X. Thus according to X, S will evolve. All M does is to hasten the collapse of S.

VI. Measurement only determines "when System will collapse" and not "what System will collapse to".

The key postulates mentioned above are summarized in Fig. 1.

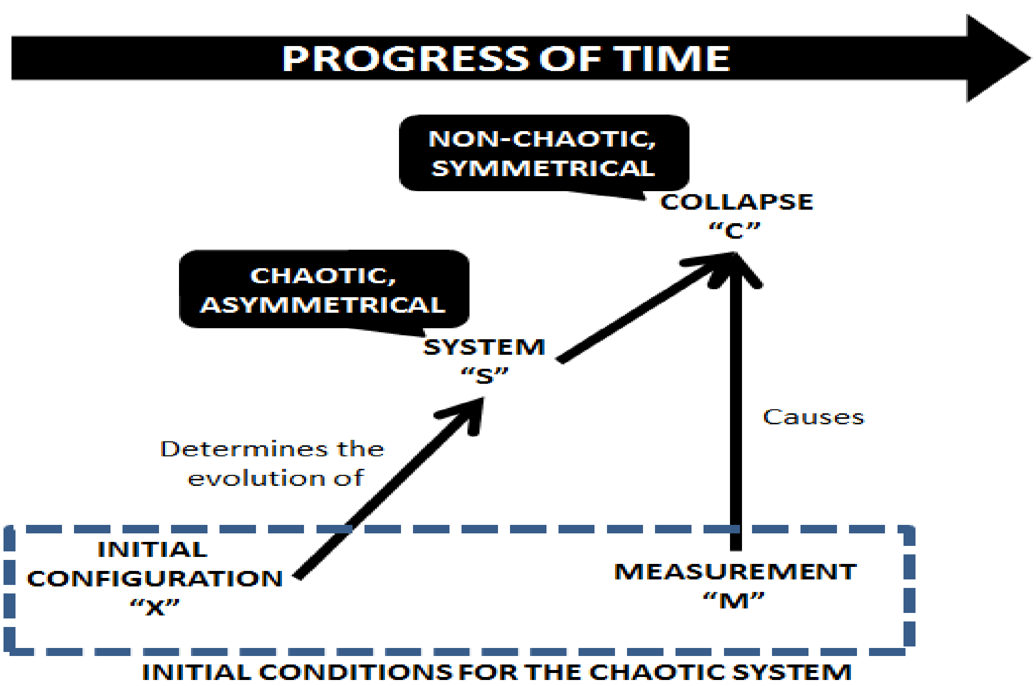

Figure 1 Illustration of the Chaotic Interpretation of Quantum Mechanics

The Basic Postulates of Quantum Mechanics

In this section, the 4 fundamental postulates of Quantum Mechanics are explored in the chaotic perspective [43-45]:

First Postulate:

The state of a quantum mechanical particle is represented by $|\psi>$ in a Hilbert space, a linear vector space consisting of an inner product space.

According to this postulate, $|\psi>$ is defined in a Hilbert space. This enables one to have a finite dimensional representation of $|\psi>$, as a linear combination of unit vectors chosen as the basis vectors. This postulate is unchanged in the chaotic interpretation, and the Hilbert space basis yields the space and time coordinates of the chaotic signal ψ.

Second Postulate:

The classical variables depicting the position 'x' and momentum 'p' are represented by the corresponding Hermitian operators 'X' and 'P' with the matrix elements in X basis given by

$$< x|X|x' > = x\,\delta(x - x')\ \textit{and} < x|P|x' > = -i\hbar \frac{d}{dx}\big(\delta(x - x')\big). \qquad (2,3)$$

This postulate concerns with the operators for various physical quanities such as position 'x', momentum 'p', and Hamiltonian 'H' leading to generation of basis vactors |x>, |p> and |E> respectively. In the chaotic interpretation, the various operators are viewed as functions acting on the chaotic signal, to give the output signals which become the corresponding basis vectors, or 'observables'.

Third Postulate:

When a quantum mechanical measurement is made by the action of a quantum mechanical operator Λ on a particle in state |ψ>, the state of the system changes from |ψ> to |λ>. The variable corresponding to the operator Λ will yield one of the Eigen values of λ of Λ with a probability P(λ) proportional to $|<\lambda|\psi>|^2$.

This postulate concisely explains the effect of measurement on the quantum state. Since the quantum state of a system is viewed as a chaotic signal, this postulate enunciates how a signal 'ψ' changes to another signal 'λ' upon effect of the measurement operator 'Λ'. The proportionality and probability must be seen as a purely 'practically probable' case, with the use of probability to compensate for the lack of information about initial configuration X.

Fourth Postulate:

The time dependent Schrödinger equation is given as follows, where 'H' is the quantum mechanical Hamiltonian, the energy operator.

$$i\hbar \frac{d|\psi(t)>}{dt} = H|\psi(t)> \qquad (4)$$

This postulate introduces the Energy basis with the operator H, and if |ψ> is used as the Energy Basis, the Eigen Value denoting the observable energy E is obtained according to Eq. 1. In the chaotic interpretation, 'H|ψ(t)>' is simply viewed as a function H[ψ(t)] of the chaotic signal ψ(t), and E is the resultant output of this operation. The same concept holds for other operators and their corresponding Eigen Values as well. Thus, in the chaotic interpretation, the equations 1 and 4 are rewritten together as $i\hbar \frac{d(\psi(t))}{dt} = H[\psi(t)] = E.\psi(t) \qquad (5)$

On a high-level perspective, the chaotic interpretation seems to have replaced the 'completely random' nature of conventional interpretations with the 'practically random' nature of chaos. This approach might open some new windows into understanding more about the elusive initial configuration 'X' by studying the patterns observed in the evolution of the system.

Entanglement – A Glimpse

One of the most attractive properties of quantum mechanics is entanglement, essentially a correlated activity of the states of a group of particles [15-17,46-49].

For example, two particles, each with a possible spin up or spin down with the net spin always zero are considered. If these particles are in superposed states and then are physically separated by a large distance, an attempt to measure the state of one of the particles, showing, say, a spin up, causes the other particle to instantaneously show a spin down.

The entanglement is apparently viewed as if the information somehow travelled instantaneously, faster than the speed of light from the first to the second particle, apparently violating the basic postulate of special theory of relativity that nothing travels faster than the speed of light, and this caused Einstein to term it “Spooky action at a distance” [46].

Mathematically, for two states ψ and ϕ, various combined states representing $|\phi\psi>$ can be written as (|00>+|01>)/2, (|00>+|10>)/2, (|00>-|01>)/2, (|00>-|10>)/2, (|01>+|10>)/2, (|00>+|11>)/2, (|01>-|10>)/2 and (|00>-|11>)/2 [2].

It is seen that, the states such as (|01>+|10>)/2 mathematically inseparable, i.e. one cannot factor these states into individual product states. These states are the “Entangled States” [2,46].

In the Chaotic Interpretation, Entanglement is seen as the special case of the interaction of the superposed states (chaotic signals) in such a way that the asymmetry and hence information content is maximized. Also since Chaos contains memory, the memory content of the two signals get mixed up and represent the “entanglement”. Thus, in the above experiment, as the two particles are being separated by physical space, the shared memory/information spreads through the space in between. Since the same “information field” acts on both particles on measurement, both collapse instantaneously. There is no travel involved here. However, this does mean that the information field is non-local.

Properties of the Chaotic Interpretation

In this section, various basic properties and principles dealt with in interpretations of quantum mechanics are outlined, and the similarities and differences between chaotic and other interpretations are tabulated.

Determinism and Reality: Determinism implies the fact that the outcomes of a system are completely predictable [41,42]. As outlined earlier, in the proposed interpretation, quantum mechanics is deterministic, provided exact initial conditions are known. Also, the wavefunction ψ is seen as a chaotic signal representing and defining the system, and the measurer’s knowledge/information, seen as measurement is only seen as an initial condition for ψ making ψ a real, rather than an informational entity.

Uniqueness of History: Some interpretations such as the many world and consistent history interpretations posit that each of the various probabilities of a superposition state correspond to multiple worlds or multiple histories [50-54]. However, that view is not held in chaotic interpretation, where history is seen as unique.

Hidden Variables: The Hidden world based interpretations such as the ones proposed by de Broglie and Bohm suggest that the wavefunction postulated by quantum mechanics does not give complete information about a system, and use this as an explanation for the apparent indeterministic nature of quantum mechanics [9,55-57]. However, Bell's Theorem that later settled the Einstein-Bohr debates suggests that local Hidden Variables are impossible and if there are any hidden variables, they would be non-local [58]. Bohm proposed that there is a hidden order that organizes a particle which in turn may be the result of a further order – the 'superimplicate order' [55].

However, since "the theoretically deterministic yet practically indeterministic" nature of quantum mechanics is succinctly described by chaos, one wouldn't need to resort to Hidden Variable based theories. Simply put, the indeterminism of System is transferred to the initial configuration 'X'.

Collapsing Wavefunctions: Collapse is essentially the reduction of a superposed state to a single state. It is the essence of "Measurement". As opposed to the deterministic evolution given by Schrodinger equations, collapse is often seen as a probabilistic, discontinuous, non-local change brought because of measurement [58-61].

An interesting point is that Collapse is generally seen as the representative of consciousness, when the system loses all unwanted information and gets a general awareness of what it is [62,63]. Also, in the many world interpretation, collapse is seen as just a selection of the multiple paths that the system can take, each with its own probability [50-52].

In the chaotic interpretation, collapse is seen as an event in the evolution of System, facilitated by Measurement, which is a fundamental component of the initial condition. This, by extension also implies that the observer has a role in determining atleast part of the system behavior (i.e., when it will collapse and not what it will collapse to).

Locality: Locality is the property that any system is affected only by other systems in its immediate surroundings [64,65]. Usually, one of the properties – realism or locality – is rejected by quantum mechanical interpretations to be consistent with experimentally observed results [64-67].

In the chaotic interpretation, a process such as measurement is seen as part of the initial condition of the system. Now, consistent with the Schrodinger equation, if the system had evolved in space, this means that the initial condition represented by measurement would affect the evolution of the system everywhere, making it clearly a non-local process.

Since, chaos is usually seen as a system involving memory, we can assume that as the system evolves in multiple locations in space, it carries the 'memorized' information with it, creating an informational field. The measurement process, being an initial condition, affects this information field, affecting the state of the chaotic waveform everywhere it is present.

Counterfactual Definiteness (CFD): CFD is the ability to assume the existence of objects and their properties even when they have not been measured [68-70]. In his famous discussion, Einstein argued that in each run of a measurement experiment, the system has some particular properties "#a" which determine the measurement outcome "a" [65].

Since in the chaotic interpretation, the system's collapsed outcome Collapse is viewed as dependent only on System (Measurement only determines when it will collapse, not the outcome of collapse), CFD holds.

Universal Wavefunction: This is a single, ultimate wavefunction, defined for the totality of existence, obeying at all times a deterministic wave equation [71,72]. Since the properties of determinism, realism and CFD hold in the chaotic interpretation, one may say that a Universal Wavefunction does exist.

Table 1 tabulates some of the main interpretations and their properties along with the proposed Chaotic Interpretation [50-56,73-85]:

Table 1 Comparison of Properties of Various Quantum Interpretations (Agn: Agnostic)

Interpretation	Determinism	Real	Unique History	Hidden Variable	Collapse	Observer Role	Local	CFD	Universal ψ
Ensemble	Agn	No	Yes	Agn	No	No	Agn	No	No
Hydrodynamic	Yes	Yes	Yes	Yes	No	No	No	Yes	Yes
Copenhagen	No	No	Yes	No	Yes	Causal	Agn	No	No
Broglie-Bohm	Yes	Yes	Yes	Yes	No	No	No	Yes	Yes
Von Neuman	No	Yes	Yes	No	Yes	Causal	No	No	Yes
Q – Logic	Agn	Agn	Yes	No	No	Interpret	Agn	No	No
T Symmetric	Yes	Yes	Yes	Yes	No	No	Yes	No	Yes
Many Worlds	Yes	Yes	No	No	No	No	Yes	No	Yes
Popper	No	Yes	Yes	Yes	No	No	Yes	Yes	No
Stochastic	No	No	Yes	Yes	No	No	No	Pos	No
Many Minds	Yes	Yes	No	No	No	Interpret	Yes	No	No
Consistent H	Agn	Agn	No	No	No	Interpret	Yes	No	No
Obj Collapse	No	Yes	Yes	No	Ye	No	No	No	No
Transactional	No	Yes	Yes	No	Yes	No	No	Yes	No
Relational	Agn	No	Agn	No	Yes	Intrinsic	Yes	No	No
Chaotic	**Yes**	**Yes**	**Yes**	**No**	**Yes**	**Timing**	**No**	**Yes**	**Yes**

Finally, for the sake of completeness, it is also noted that McHarris proposed a nonlinear and chaos based basis for quantum behavior. The key points are as follows [23-26]:

1. Nonlinear dynamics in its chaotic realm bridges the gap between the statistical nature of quantum mechanics and a more deterministic, more fundamental perspective without having to introduce hidden variables, adding completeness to a theory which was in Einstein's words 'correct yet incomplete'.
2. A one to one correspondence between cause and effect arising due to the determinism of chaos satisfies Einstein's perspective of Quantum Mechanics, whereas the extreme sensitivity nature of chaos drastically reducing the certainty of prediction and supplying a practical statistical interpretation satisfies Bohr's perspective of Quantum Mechanics, thus potentially resolving one of the most intense debates on Quantum Mechanics.
3. The classical derivation of Bell's inequality and subsequent violation of the same in the case of entanglement corresponds to an inaccuracy in the formulation. Specifically, the subtle assumptions of non-correlation statistics have greatly suppressed the correlations that could occur in real time and raise the level of the inequality.
4. The existence of attractors and basins in the dynamics of chaotic systems suggests a tendency of such systems to quantize themselves, without external influence.
5. A possibility of a nonlinear basis, motivated by instability, to explain spontaneous symmetry breaking and parity nonconservation.
6. The possible equivalence of quantum diffraction and intermittent periods of order in chaos.

Conclusion

An interpretation of quantum mechanics based on chaos theory is proposed. It is seen that the key principles for such an interpretation are the equivalence between the superposed state of a quantum system and a chaotic signal, the explanation of collapse as a stage in the evolution of the chaotic signal with the process determined by initial conditions and the timing determined by measurement and the 'theoretically deterministic yet practically random' nature introduced by chaos theory explaining the probabilistic aspect of quantum mechanics. Following glimpses of the fundamental postulates of quantum mechanics and the concept of entanglement through a chaotic perspective, various properties of the proposed chaotic interpretation, such as determinism, realism, locality, hidden variables and counterfactual definiteness are discussed and studied in comparison with existing interpretations. It is opined that the fresh perspective offered by chaos theory will help in using standardized chaos based tools and studies of evolution patterns effectively in understanding better, the hidden realities of nature.

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