Nonlinear Analysis and Chaotic Characterization in Annual GDP Growth Rates of National and Supranational Entities

Sai Venkatesh Balasubramanian

Sree Sai Vidhya Mandhir, Mallasandra, Bengaluru-560109, Karnataka, India.
saivenkateshbalasubramanian@gmail.com

Abstract

The Annual Growth Rate of Gross Domestic Product of an economy is often seen as a key indicator to the economy's growth and prosperity. In the present work, nonlinear analysis is performed on the GDP growth data of nations and supranational entities using two key tools, namely Averaged Lyapunov Exponent (ALE) and Distance Plot. The analyses ascertain the presence of chaos in the GDP growth rate data for both nations and supranational entities. Following this, some key inferences from the nonlinear analysis are presented, such as the high value of ALE obtained for fragile and conflict ridden economies, indicating the relationship between ALE and instability, the ALE analysis of BRICS nations as well as low ALE values of supranational entities such as OECD and EU. It is opined that the obtained results pave the way for unlocking a wealth of information regarding GDP growth rates ushering in a new era of 'Smart Economics'.

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Keywords

GDP Growth Rate, Nonlinear Analysis, Chaos, Lyapunov Exponent, Distance Plot

Introduction

Ever since the concept of Gross Domestic Product (GDP) was developed in the twentieth century, it has become the buzzword in the field of macroeconomics and a standard means of assessing a country's economy [1,2,3]. The GDP is essentially a measure of the gross value produced by all residents in an economy including product taxes but excluding subsidies that have not been included in the product values [3,4].

However, the concepts derived from the GDP such as the GDP growth rate have reached the general public more than the GDP itself [4,5]. It is well known that Public sentiment is a key factor affecting a nation's economy since, in most countries in the world, themselves democracies, it is the people that elect the government [6,7].

In the field of macroeconomics, the GDP growth rate is a topic of active research all around the world [1-7]. The key objective in this research is to assess the nature and identify any notion of determinism and predictability in the growth rate, or the absence of therein [8,9,10,11]. On these lines, it has been shown

that the GDP growth rate of certain economies exhibit chaotic nature, and this forms the key inspiration for the present work [10-17].

Chaos Theory, one of the newer branches of physics and a flagship of nonlinear science, deals with dynamical systems which are extremely sensitive to initial conditions, a phenomenon popularly known as the 'Butterfly Effect' [15,16,17]. In essence, very small deviations in the initial conditions, over evolution in time, drive the chaotic systems to drastic differences, making prediction near to impossible [16,17,18,19]. The key quantitative characterization of chaos is the Lyapunov Exponent, a measure of sensitivity, or how divergent trajectories closely separated can evolve [18,19].

The identification of Chaos in the GDP growth rates can be attributed to the sensitive dependence of a nation's economy on certain significant events in the country's timeline – elections, wars, terrorist attacks, global recession and so on [12,13,14,20].

Taking cue from the identification of chaos in GDP data, the present work focuses on a timescale of around fifty years from 1961 to 2014, and within this timescale, the GDP growth rate of various nations and supra-national entities are studied. The Averaged Lyapunov Exponent (ALE) is seen to indicate the persistence of sensitivity over the mentioned period, and it is seen that with the exception of a few, most GDP growth rates exhibit positive ALE. Also, among the supranational entities, it is seen that Fragile and conflict affected nations possess the highest ALE, indicating clearly that ALE is a measure of instability in an economy, politically as well as financially. In addition to the ALE which primarily studies the ling-term properties of GDP growth rates, a localized short-term study is also proposed using the Distance Plots. Some typical examples of Growth Rate analysis using Distance plots are presented as case studies. The ability to assess an economy's instability through the ALE and the extraction of associated short-term inferences from localized analysis using distance maps forms the novelties of the present work.

Methodology

The primary objective of the present work is to perform nonlinear analysis on the GDP growth rates of nations and supranational entities identifying the presence of chaos and obtaining long-term as well as short-term inferences from the analysis. For this purpose, two well-established tools of nonlinear analysis, namely Lyapunov Exponent and Distance plots are used. These tools, along with other key steps in the analysis are explained in this section.

Collection of Data:

The GDP Annual Growth Rate is calculated as the percentage increase or decrease in an economy's GDP over the period of a year [4].

The GDP growth rates of various countries and supranational entities (region-wise as well as income-wise) are obtained from the World Bank website (http://data.worldbank.org/indicator/NY.GDP.MKTP.KD.ZG/countries?display=default) [21]. According to the website, the aggregates in the GDP calculation are all based on constant 2005 US Dollars [21]. The GDP calculations do not make deductions for depreciation of fabricated assets or for depletion and degradation of natural resources [21].

The Distance Plot:

The main premise in the concept of distance plot is that most natural processes possess recurrent behavior in the form of periodicities and irregular cyclicities [22,23,24,25]. Here, a recurrence is defined as a condition where states in the system are arbitrarily close after some time of divergence [22,23,24,25].

On this concept, the distance plot (DP) is defined as follows:

For a discrete signal with N samples denoted by x(n), n<=N, the distance between the ith and jth point D(i,j) is given by

$$D(i,j) = \|x(i) - x(j)\|$$

The collection of all the distance points D(i,j) for all i,j<N form the Distance Matrix D, a plot of which is termed the Distance Plot (DP) [23].

It has been shown that certain dynamical invariants such as the Correlation Dimension, Kolmogorov Entropy and Mutual Information can be derived from distance plots [22,23,24,25]. The key advantage of distance plots is that these plots provide useful information about the chaotic nature even for short term and non-stationary data [22,23,24,25].

The Lyapunov Exponent:

The Lyapunov Exponent essentially quantifies the sensitive dependence of a system on initial conditions, a key aspect of chaotic behavior [18,19]. The maximal Lyapunov Exponent, denoted as the MLE refers to the largest Lyapunov Exponent value obtained and determines a notion of predictability of a dynamical system. A positive value of MLE asserts the presence of chaos [18,19].

Methods such as the Rosenstein's Algorithm exist to determine the Lyapunov Exponent from a time series [26]. Specifically, an evolution time Δt is defined and the ith sample of the divergence d for the jth trajectory is expressed as a function of λ in the following manner, where the λ*'s* denote the Lyapunov Exponents and C_j denote normalization constants:

$$d_j(i) = C_j e^{\lambda_i (i\Delta t)}$$

The MLE then is the largest of the obtained λ_i's, and is an indicator of the presence of chaos [26].

However, a more relevant parameter to the present work is the Averaged Lyapunov Exponent (ALE), computed by taking the average of the λ_i's, rather than the maximum. This quantity indicates the persistence of chaotic nature in the considered time period.

In the present work, the following algorithm is employed to calculate the ALE of GDP growth rate data of a given economy:

1. Consider an economy's GDP growth rate as a 1D signal x(n) with N samples.
2. Set a value for threshold E below which difference is to be analyzed. In the present work, E is set to 0.5%.

3. For each point i in 1 to N, and for each point j in i+1 to N, check if the difference between a(i) and a(j) is less than E. If the difference is less than E, then the ith and jth point are considered to be recurrences of each other.
4. For k in range of 1 to the remaining number of points, the logarithms of the differences between a(i+k) and a(j+k) are noted. These values represent the divergences.
5. The average of all the values of logarithmic differences noted above give L. L is the Average Lyapunov Exponent (ALE) of the System.

ALE Adjustment for Inadequacy – The Sudan Approach

Since the time scale chosen in the present work is 1961 to 2014, one significant issue arises in the calculation of ALE, namely the validity of the ALE for countries that have not existed in the entire timescale.

As an example, the ALE's of France and UK which have existed throughout in 1961-2014 are 0.1086 and 0.3711, whereas the ALE for Russia which did not exist as a nation till the 1990's is obtained as -1.5059 [27]. Similar negative values are observed for other new nations such as Timor-Leste, South Sudan, most of the Central Asian nations etc. [27].

It is also observed that, the lesser the number of growth rate data points available, the faster the ALE reduces, as shown in Fig. 1. Thus, a quadratic relation between ALE and Inadequacy is observed.

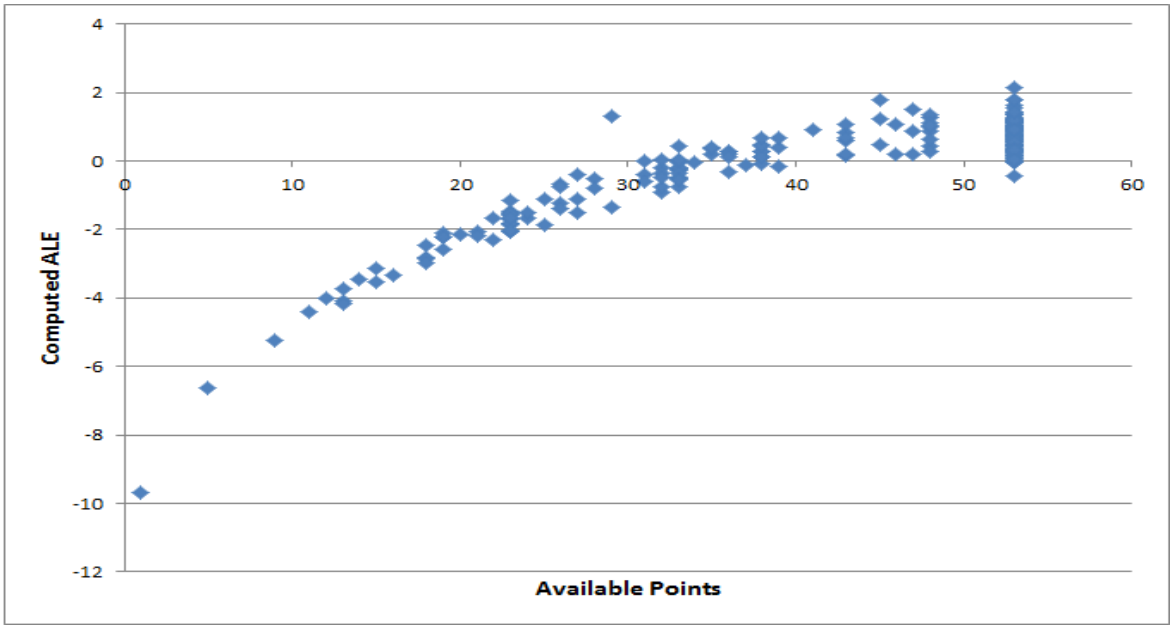

Figure 1 Computed ALE as a function of Available Data Points

Thus it is necessary to define some sort of correction/adjustment to the ALE's of nations whose growth rates are not fully obtained between 1961 and 2014.

In the present work, an ALE adjustment process, termed the 'Sudan Approach' is proposed as follows:

Among the nations listed by the World Bank, one of the latest to be formed is South Sudan, in 2011. This implies that for almost fifty years, Sudan and South Sudan shared the same economy and hence GDP.

Also, between 2011 and 2014, the GDP growth rate trends of Sudan and South Sudan do not show significant deviations from each other, implying that they almost follow the same trend.

Thus, if the GDP growth rates of Sudan and South Sudan are similar in the range 1961-2011 as well as 2011-2014, then the ALE of both the data sets ought to be similar. However, without any form of adjustment, South Sudan ALE is -6.6473, whereas Sudan ALE is 1.5322. The number of data points available in Sudan is 53 whereas for South Sudan it is 5.

The Sudan Approach to adjustment is described as follows:

1. Accounting for the quadratic relation of Fig. 1, the square of unavailable data points in South Sudan is computed. This is (53-5)^2=2304.
2. The adjustment factor is then defined by taking the difference between the ALE's of Sudan and South Sudan and dividing it by the value obtained in the previous step.

$$Adjustment\ Factor = \frac{ALE_{Sudan} - ALE_{SouthSudan}}{2304} = 0.00355$$

3. Finally, for any country with m number of unavailable GDP growth rate points, the adjusted ALE is defined as follows:

$$ALE_{Adjusted} = ALE_{Unadjusted} + 0.00355m$$

Results and Discussions:

Based on the DP and ALE techniques discussed earlier, the World Bank Data of GDP Growth Rates are analyzed and the ALE's have been computed. The ALE's of nations have been tabulated in the Appendix. Statistically, the calculated ALE's of nations have a mean of 1.1205, Standard Deviation of 0.5035 and Variance of 0.2536.

It is noteworthy that six of the fifteen highest ALE's, namely Vietnam, Iraq, Palestine, Ukraine, Afghanistan and Cambodia, have been victims of major wars/conflicts and political tensions between 1961 and 2014 [20,28].

Certain key results and inferences from these analyses are explored in this section as case studies.

Case Study 1 – The BRICS

The BRICS comprising of Brazil, Russia, India, China and South Africa are a group of developing and major national economies, and the five BRICS countries represent approximately 40% of the world's population [29,30,31]. The combined Purchasing Power Parity Gross Domestic Product (GDP-PPP) of the BRICS is nearly US$30 Trillion [31].

The Adjusted ALE's of the BRICS nations are obtained as 0.9325 (Brazil), 1.4797 (Russia), 0.7428 (India), 0.5519 (China) and 0.4311 (South Africa).

The DP's and Unadjusted ALE's (mentioned as 'Sensitivity Factor') of the BRICS nations are illustrated in Fig. 2 to Fig. 6.

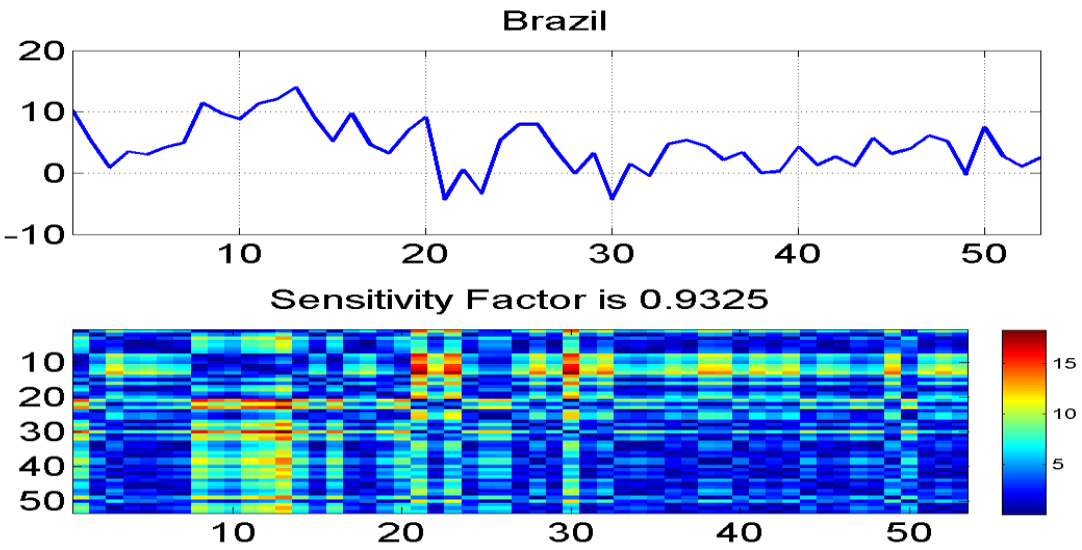

Figure 2 GDP Percentage Growth Rate Time Series (Top) and Distance Plot (Bottom) of Brazil. Horizontal Axis denotes Years since 1961.

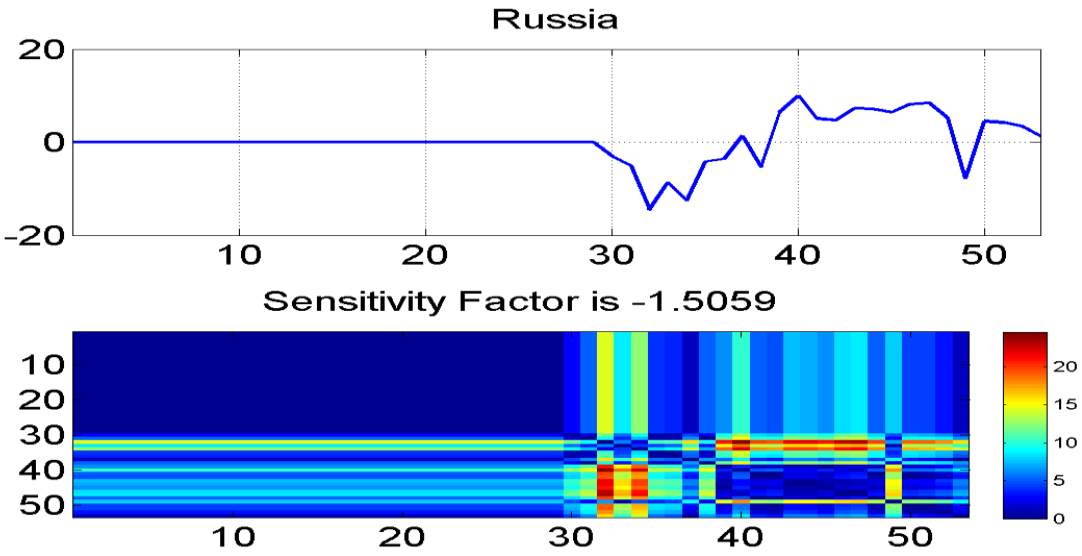

Figure 3 GDP Percentage Growth Rate Time Series (Top) and Distance Plot (Bottom) of Russia. Horizontal Axis denotes Years since 1961.

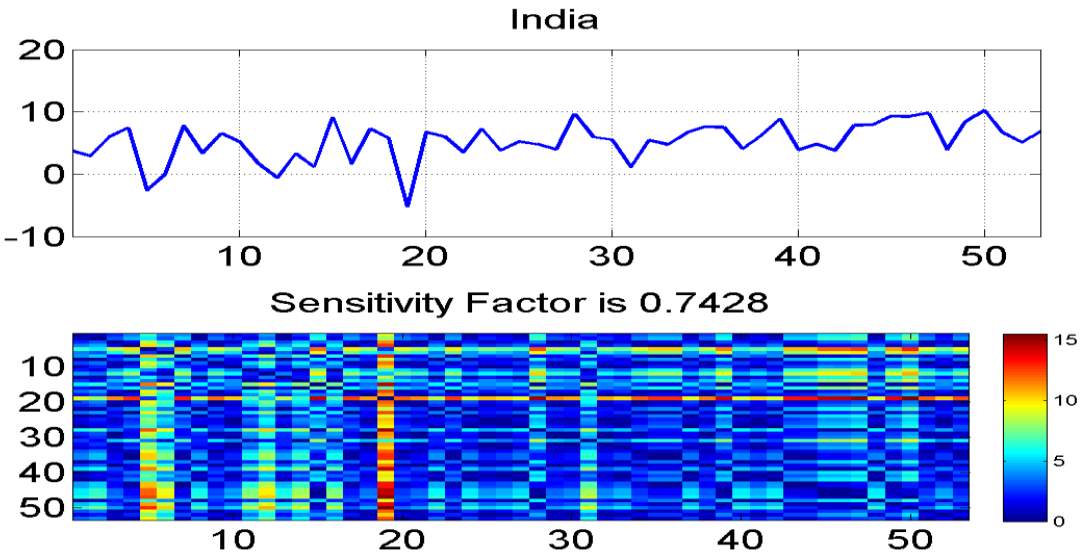

Figure 4 GDP Percentage Growth Rate Time Series (Top) and Distance Plot (Bottom) of India. Horizontal Axis denotes Years since 1961.

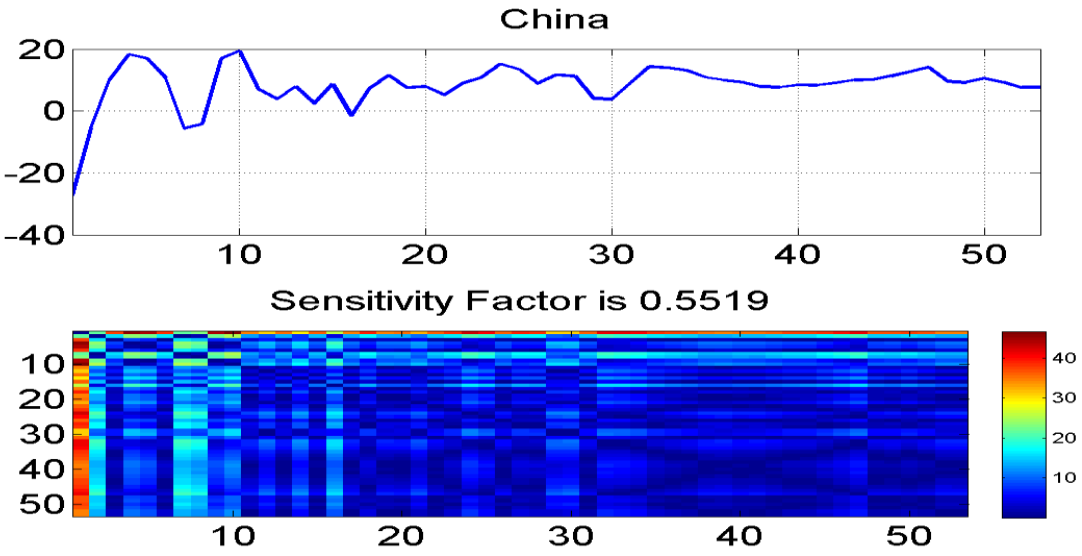

Figure 5 GDP Percentage Growth Rate Time Series (Top) and Distance Plot (Bottom) of China. Horizontal Axis denotes Years since 1961.

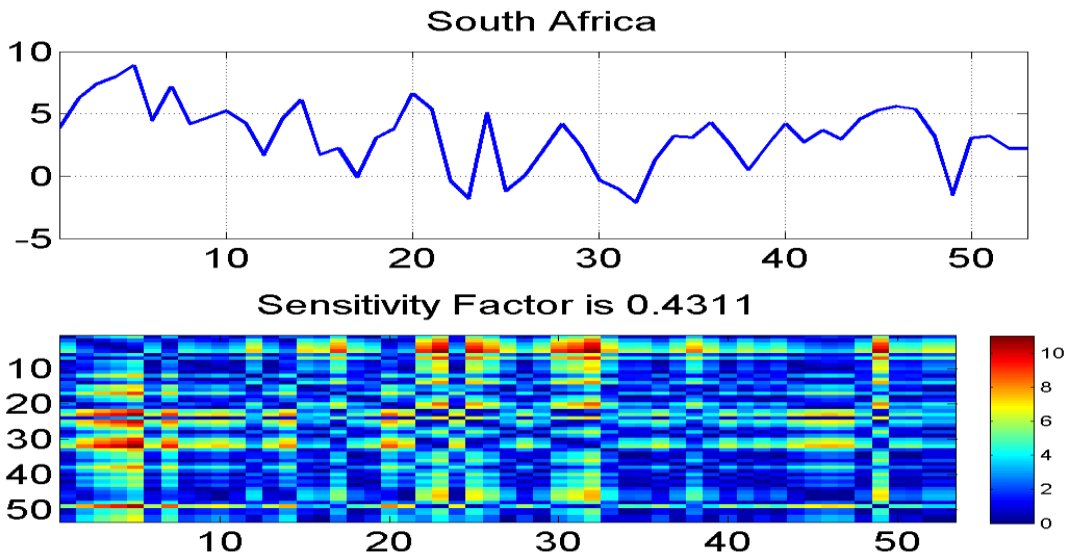

Figure 6 GDP Percentage Growth Rate Time Series (Top) and Distance Plot (Bottom) of South Africa. Horizontal Axis denotes Years since 1961.

From the DP and ALE's the following inferences can be obtained:

1. In terms of Growth Rates in percentages, China has recorded some of the highest values, close to 20%.
2. China also shows the most streamlined and consistent of the growth rates, seen by the dark blue patch in the DP between timescales of 35 to 45. Also, the number of bright patches seen in China DP is relatively low, indicating that the sensitivity induced deviations are low in China. This has a direct consequence in China's ALE of 0.5519 being among the lowest in the BRICS nations, higher only than South Africa.
3. From the DP's, it is evident that India and South Africa exhibit some of the most vibrant trends, with a lot of variations seen as bright patches in their respective DP's. However, the maximum difference value in South Africa is restricted to around 10%, whereas in India, the differences approach nearly 15%. This difference is seen in the ALE of India (0.7428) being higher than that of South Africa (0.4311).
4. The highest ALE among the BRICS nations is seen for Russia at 1.4797, more than a whole unit higher than the lowest ALE, South Africa. From the data available for Russia (1990 onwards), the DP shows that the GDP growth of Russia has seen violent swings from nearly -15% to around 10%.
5. A visual inspection of the density of bright patches along the DP's of all the five nations reveals that, the overall slope of Growth Rates in India is positive, Brazil and South Africa are slightly negative, China is almost constant and Russia is slightly positive.
6. Among the five BRICS nations, only Brazil and India have registered major growth rate deviations in recent times (2010 onwards). This might be a heralding factor of sharp transitions and accelerated/decelerated growth to come in these nations.

Case Study 2 – ALE's of Supra National Entities

In addition to nations, the World Bank data also lists the GDP growth rates of Supranational entities. These include regions and continents (such as 'Middle East and North Africa'), Income Status ('Low Income Countries'), Special Groups ('OECD Countries'), Political Groups ('European Union Countries') and Others ('Fragile and Conflict Affected Nations'). The ALE's of all these entities are computed and tabulated in Table 1.

Supra National Entity	ALE	Supra National Entity	ALE
Arab World	0.92375	Low and Middle Income	0.2913
Caribbean Islands	1.06115	Low Income	0.38925
Central Europe and Baltic	1.115	Lower Middle Income	0.338
East Asia and Pacific - All	0.0944	Middle East and North Africa - All	0.7135
East Asia and Pacific - Developing	0.4293	Middle East and North Africa - Developing	0.96715
Euro Nations	0.2184	Middle Income	0.2788
Europe and Central Asia - All	0.233	North America	0.3557
Europe and Central Asia - Developing	1.35985	OECD Countries	-0.0402
European Union	0.1669	Pacific Islands	0.82725
Fragile and Conflict Affected Nations	1.645	North Asia (Russia)	1.47965
Heavily Indebted Poor Countries	0.2783	Small States	0.6517
High Income	1.32705	South Asia	0.5541
OECD High Income	-0.0152	Sub Saharan Countries – All	0.4202
Latin America and Caribbean - All	0.5982	Sub Saharan Countries - Developing	0.4563
Latin America and Caribbean - Developing	0.65	Upper Middle Income	0.3295
Least Developed	0.67135	World	0.0493

Table 1 GDP Growth Rate ALE's of SupraNational Entities

From Table 1, the following can be inferred:

1. The Highest ALE's are seen in Fragile and Conflict affected nations. Thus, the ALE can be seen as a window to the economic instability and unpredictability in these nations [20,28]. This also means that due to repeated wars and conflicts, the GDP growth rates have fluctuated wildly, and that in order to achieve economic stability, a lot of effort may be required to 'desensitize' the GDP growth rate to political instabilities.
2. The next highest values of ALE are seen in Russia and the Developing nations of Central Asia and Europe. A common factor linking most of the countries in these groups is that they were members of the Soviet Union (USSR) until the 1990's [27]. In the first few years after 1990, the stabilization of economies of most former soviet states displayed some fluctuations, seen in the high ALE values.
3. The negative ALE's are seen only in the OECD and OECD: High Income Groups. It is noteworthy that most of the OECD nations are well developed nations, ranking consistently high in the Human Development Index [32,33,34]. The low value of ALE shows that the growth rates of these nations are more or less streamlined, without growth surges as seen in nations such as India and China.

4. The ALE of the World is a low positive value at 0.0493 and the DP is shown in Fig. 7. The low ALE signifies an almost desensitized behavior of GDP growth rates, leading it to almost appear random.
5. The mean of the ALE's listed in Table 1 is 0.588, and the closest group to this mean is South Asia at 0.5541. The ALE and South Asia DP of Fig. 8 indicate that South Asian GDP growth shows optimal Chaotic Behavior with moderate ALE. This factor, if analyzed for its consequences, might prove to be an attractive feature for large scale investments in South Asian Nations [35,36].

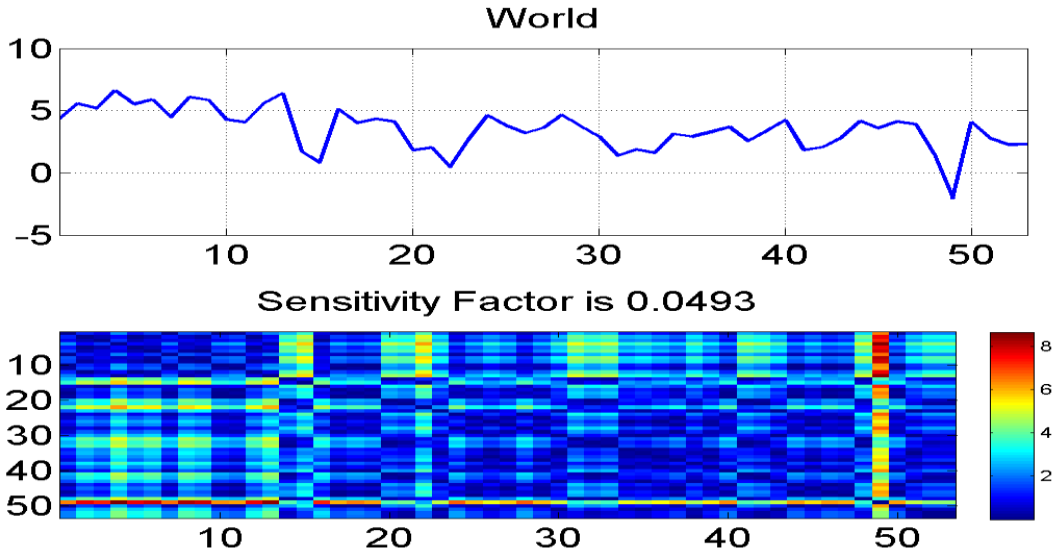

Figure 7 World GDP Percentage Growth Rate Time Series (Top) and Distance Plot (Bottom). Horizontal Axis denotes Years since 1961.

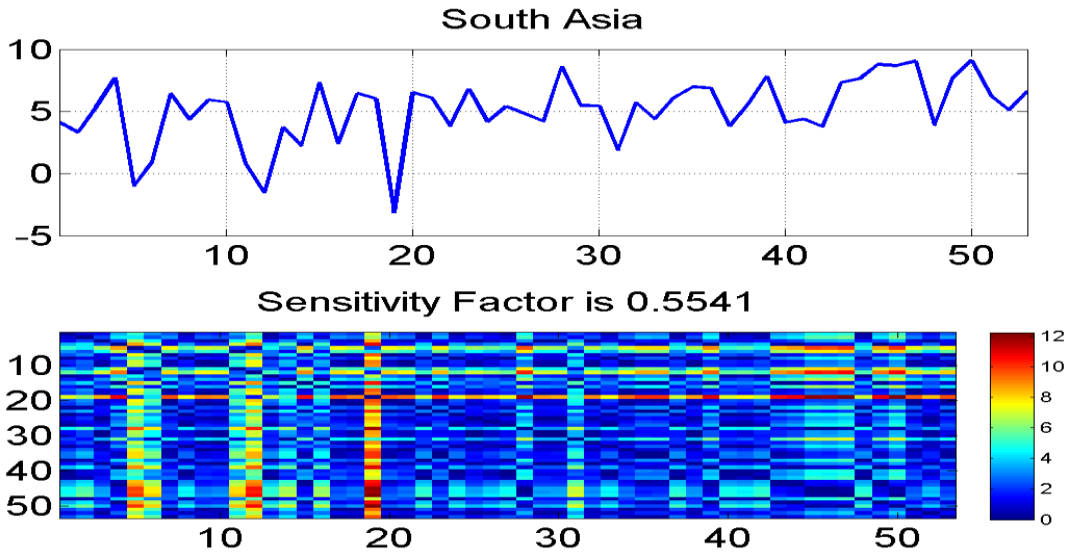

Figure 8 GDP Percentage Growth Rate Time Series (Top) and Distance Plot (Bottom) of South Asia. Horizontal Axis denotes Years since 1961.

The Standard Deviation and Variance for the Supranational Entity ALE's of Table 1 are obtained as 0.4404 and 0.1939 respectively. The lower statistical values of supranational entities compared with national entities is due to the fact that multiple nations taken as one group serves to desensitize the chaotic variations that are typically seen in single nations. The European Union with an ALE of 0.1669 may thus be a lesson in context.

Conclusion

The Annual GDP Growth Rate data from the World Bank is taken and nonlinear analyses are performed for nations and supranational entities. Two key tools, the Distance plot and Averaged Lyapunov Exponent are considered. In accordance with the existing literature, the presence of chaos is confirmed in the GDP Growth Rate of countries. Specific Case Studies are presented focusing on the GDP Growth of BRICS Nations and Supranational Entities. The main inferences obtained from such case studies include the nature of growth in BRICS, High ALE of Fragile and Conflict Nations indicating the relationship between ALE and Instability, and the lowering of ALE when countries operate as a group, as in the European Union. The nonlinear analyses provided in the present work enable the potential unlocking of a wealth of information regarding economic growth and stability at the national and supra national levels, ushering in a new era of 'Smart Economics'.

Appendices

The GDP Growth Rate Analysis for Sudan and South Sudan (Unadjusted)

According to the 'Sudan Approach' mentioned in the Methodology Section, the GDP Growth trends of Sudan and South Sudan are shared as a single entity between 1961 and 2011, and are similar between 2011 and 2014. The following shows the GDP growth rate of the two nations along with the distance plot and unadjusted ALE values.

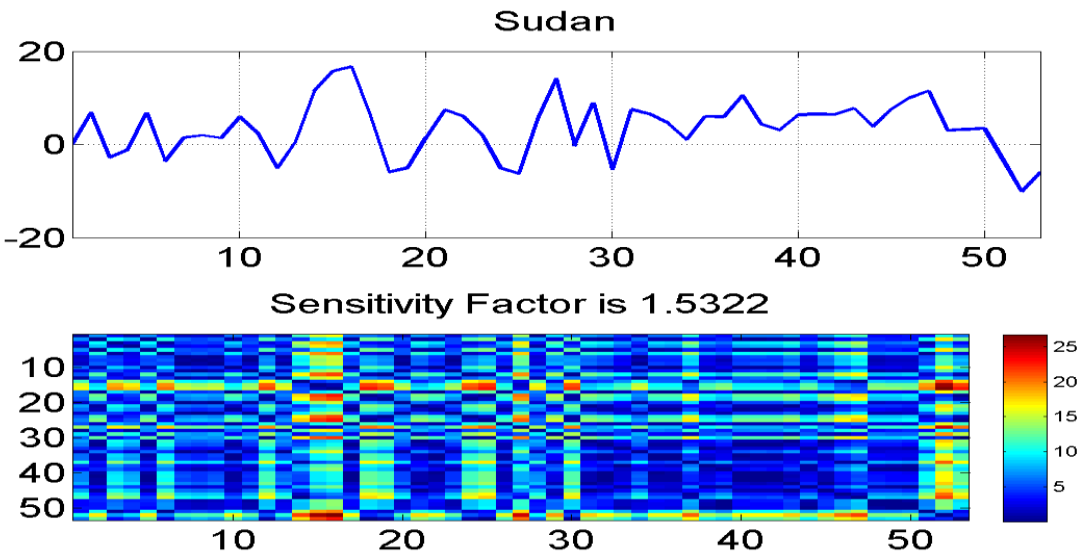

Figure 9 GDP Percentage Growth Rate Time Series (Top) and Distance Plot (Bottom) of Sudan. Horizontal Axis denotes Years since 1961.

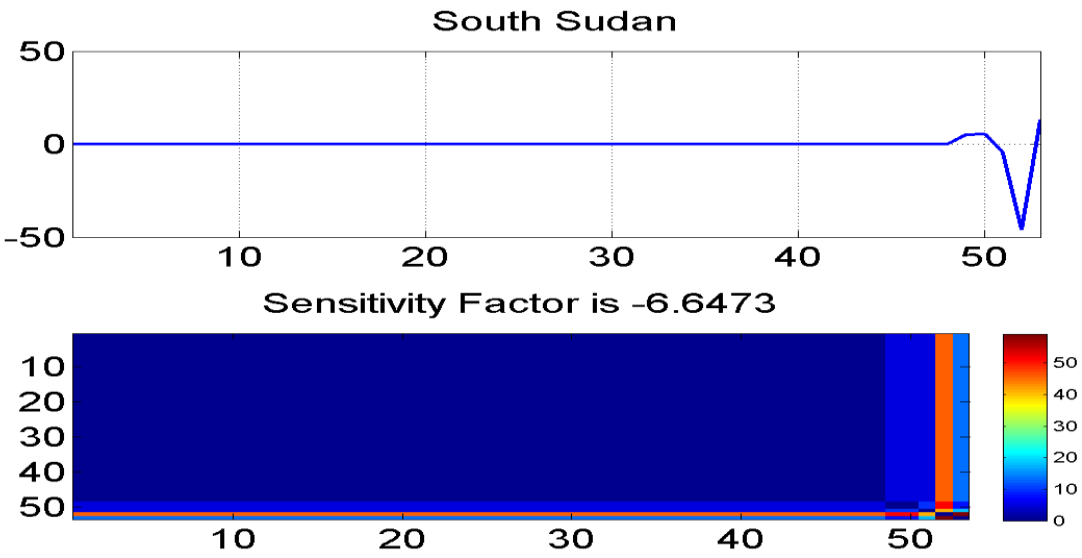

Figure 10 GDP Percentage Growth Rate Time Series (Top) and Distance Plot (Bottom) of South Sudan. Horizontal Axis denotes Years since 1961.

ALE's Computed for GDP Growth Rates for various Nations

NAME	UNADJUSTED ALE	ADJUSTED ALE	2013 GROWTH	NAME	UNADJUSTED ALE	ADJUSTED ALE	2013 GROWTH	NAME	UNADJUSTED ALE	ADJUSTED ALE	2013 GROWTH	NAME	UNADJUSTED ALE	ADJUSTED ALE	2013 GROWTH
afghanistan	-4.419	1.844	1.934	el salvador	0.268	0.357	1.676	lesotho	0.767	0.767	5.489	russia	-1.506	1.480	1.319
albania	-0.225	1.195	1.417	equatorial guinea	0.424	1.844	-4.840	liberia	2.136	2.136	11.307	rwanda	1.215	1.215	4.685
algeria	0.482	0.482	2.800	eritrea	-2.052	1.584	1.332	libya	-3.447	1.952	-10.879	samoa	-0.393	1.325	-1.139
angola	-0.516	1.703	6.800	estonia	-2.452	1.897	1.630	lithuania	-1.569	1.626	3.251	sao tome and principe	-4.148	1.532	4.000
antigua barbuda	0.284	1.310	-0.071	ethiopia	-0.178	1.388	10.490	low and middle income	0.291	0.291	4.961	saudi arabia	1.241	1.468	3.950
arab world	0.125	0.924	3.352	euro	0.218	0.218	-0.465	low income	0.215	0.389	5.645	senegal	0.692	0.692	2.804
argentina	1.352	1.352	2.925	europe central asia all	0.233	0.233	0.420	lower middle income	0.338	0.338	5.819	serbia	-2.847	1.502	2.600
armenia	-1.461	1.734	3.500	europe central asia developing	-1.228	1.360	3.741	luxembourg	0.909	0.909	1.990	seychelles	1.434	1.434	5.276
australia	-0.045	-0.045	2.512	european union	0.167	0.167	0.060	macau	-0.008	1.710	11.888	sierra leone	1.022	1.022	5.519
austria	0.300	0.300	0.228	fiji	1.027	1.027	3.470	macedonia	-1.840	1.355	3.100	singapore	1.195	1.195	3.851
azerbaijan	-1.169	2.027	5.797	finland	0.850	0.850	-1.206	madagascar	0.925	0.925	2.414	slovakia	-2.162	1.473	1.425
bahamas	0.897	0.897	0.667	france	0.109	0.109	0.285	malawi	1.122	1.122	4.969	slovenia	-2.980	1.369	-0.998
bahrain	-0.223	1.197	5.338	fringe and conflict affected	-1.550	1.645	0.770	malaysia	0.718	0.718	4.734	small states	-0.768	0.652	2.676
bangladesh	0.069	0.069	6.014	gabon	1.171	1.171	5.887	maldives	-3.991	1.977	3.714	solomon islands	-1.538	1.657	2.951
belarus	-1.500	1.695	0.895	gambia	0.859	0.986	4.802	mali	1.065	1.238	2.149	south africa	0.431	0.431	2.212
belgium	0.305	0.305	0.275	georgia	1.289	1.377	3.320	malta	0.668	1.023	2.900	south asia	0.554	0.554	6.592
belize	0.971	0.971	1.526	germany	0.173	0.528	0.106	marshall islands	0.030	1.596	2.989	south sudan	-6.647	1.532	13.131
benin	0.529	0.529	5.640	ghana	0.781	0.781	7.585	mauritania	1.131	1.131	6.715	spain	0.628	0.628	-1.230
bhutan	-0.189	1.231	2.045	greece	1.196	1.196	-3.316	mauritius	-0.130	0.779	3.200	sri lanka	0.432	0.432	7.251
bolivia	0.427	0.427	6.776	grenada	0.219	1.245	2.423	mexico	0.853	0.853	1.070	st kitts and nevis	0.110	1.136	4.211
bosnia	-2.237	1.867	2.480	guatemala	0.179	0.179	3.690	micronesia	-1.093	1.306	-4.000	st lucia	0.028	1.448	-0.432
botswana	1.156	1.156	5.827	guinea	-1.490	0.910	2.300	middle east and north africa all	0.486	0.714	2.245	st vincent grenadine	1.139	1.139	1.664
brazil	0.933	0.933	2.492	guinea bissau	0.822	1.177	0.329	middle east and north africa developing	0.878	0.967	-0.500	sub saharan all	0.420	0.420	4.030
brunei	0.410	1.106	-1.752	guyana	1.186	1.186	5.222	middle income	0.279	0.279	4.944	sub saharan developing	0.456	0.456	4.125
bulgaria	-0.292	1.129	1.066	haiti	-3.514	1.612	4.297	moldova	-0.054	1.366	8.899	sudan	1.532	1.532	-6.004
burkina faso	0.929	0.929	6.647	heavily indebted poor	0.278	0.278	4.562	mongolia	-0.375	1.191	11.743	suriname	0.450	1.249	2.880
burundi	0.747	0.747	4.594	high income	-1.659	1.327	1.414	montenegro	-3.330	1.530	3.338	swaziland	0.213	0.568	2.785
cambodia	-2.126	1.740	7.406	high income oecd	-0.015	-0.015	1.314	morocco	0.888	0.888	4.381	sweden	0.565	0.565	1.498
cameroon	0.436	0.436	5.562	hondureas	0.651	0.651	2.564	mozambique	-0.481	0.939	7.444	switzerland	-0.545	0.875	1.916
canada	0.328	0.328	2.022	hong kong	0.994	1.083	2.932	namibia	-0.369	1.051	5.125	tajikistan	-0.793	1.425	7.400
caribbean	0.262	1.061	1.503	hungary	-2.281	1.131	1.525	nepal	0.235	0.235	3.782	tanzania	-1.874	0.909	7.282
central african republic	1.632	1.632	-36.000	iceland	0.978	0.978	3.459	netherlands	0.450	0.450	-0.726	thailand	1.040	1.129	1.766
central europe and baltic	-2.080	1.115	1.344	india	0.743	0.743	6.899	new zealand	-0.329	0.697	2.497	togo	1.004	1.004	5.118
chad	1.792	1.792	3.966	indonesia	0.392	0.392	5.781	nicaragua	0.759	0.759	4.606	tonga	-0.473	1.093	0.500
chile	0.769	0.769	4.075	iran	1.351	1.440	-5.800	niger	1.354	1.354	4.105	trinidad and tobago	1.182	1.182	1.600
china	0.552	0.552	7.671	iraq	1.774	2.002	4.212	nigeria	1.393	1.393	5.394	tunisia	0.641	0.729	2.521
colombia	0.443	0.443	4.678	ireland	0.594	0.949	0.174	north america	0.356	0.356	2.205	turkey	1.145	1.145	4.124
comoros	-0.495	0.925	3.500	israel	0.701	0.701	3.248	norway	0.419	0.419	0.647	turkmenistan	-0.685	1.903	10.199
congo dr	1.252	1.252	8.482	italy	0.479	0.479	-1.925	oecd	-0.040	-0.040	1.352	tuvalu	-1.677	1.518	1.298
congo R	1.029	1.029	3.441	jamaica	0.191	0.319	1.275	other small	-0.513	0.907	3.320	uae	0.658	1.456	5.200
costa rica	0.794	0.794	3.497	japan	0.476	1.274	1.613	pacific islands	-0.738	0.827	2.266	uganda	-0.590	1.128	3.271
cote d'ivoire	1.218	1.218	8.701	jordan	0.169	0.169	2.829	pakistan	0.475	0.475	4.409	uk	0.371	0.371	1.729
croatia	-2.817	1.532	-0.939	kazakhstan	-1.682	1.513	6.000	palau	-1.674	1.737	-0.331	ukraine	-0.737	1.851	1.876
cyprus	0.272	1.071	-5.400	kenya	0.723	0.723	5.743	palestine	-2.120	1.984	-4.429	upper middle income	0.330	1.205	4.685
czech	-1.849	1.346	-0.702	kiribati	1.083	1.438	2.973	panama	1.027	1.027	8.352	uruguay	1.205	1.560	4.397
denmark	0.427	0.427	-0.486	korea R	1.081	1.081	2.971	papua new guinea	1.183	1.183	5.537	usa	0.410	0.330	2.221
djibouti	-1.816	1.379	5.000	kosovo	-4.081	1.600	3.000	paraguay	1.279	1.279	14.225	uzbekistan	-1.393	1.195	8.000
dominica	0.278	1.304	-0.913	kyrgyzstan	-0.406	1.994	10.534	peru	0.971	0.971	5.785	vanuatu	-0.023	1.259	1.969
dominican R	0.999	0.999	4.584	laos	-1.336	0.709	8.518	philippines	0.602	0.602	7.181	venezuela	1.438	1.438	1.343
east asia and pacific all	0.094	0.094	4.205	latin america caribbean all	0.598	0.598	2.490	poland	-2.067	1.128	1.668	vietnam	1.324	3.369	5.421
east asia and pacific developing	0.429	0.429	7.134	latin america caribbean developing	0.650	0.650	2.473	portugal	0.944	0.944	-1.355	world	0.049	0.049	2.295
ecuador	0.668	0.668	4.642	latvia	1.112	1.201	4.110	puerto rico	0.877	0.877	-0.577	yemen	-2.039	1.156	4.156
egypt	0.435	0.524	2.101	least developed	-0.894	0.671	4.643	qatar	-2.239	1.865	6.322	zambia	0.900	0.900	6.713
				lebanon	-1.103	1.681	0.900	romania	0.018	1.438	3.500	zimbabwe	1.771	1.771	4.484

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