Chaos Theory and Nonlinear Analysis for the Business Strategist

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Abstract:

The complex nature of the growing organization has in recent years necessitated a fundamental change towards management strategy and formation of policies. In this light, the present article focuses on the use of various tools and techniques of nonlinear analysis, including Time Series, Phase Portrait, Multiscale Analysis, Polar Plots, Largest Lyapunov Exponents, Fractal Dimension and Kolmogorov Entropy to analyze important data such as the share revenue and price, and draw conclusions to reveal key information about public sentiment and market trends. The management could potentially use this information towards effective formation of various policies such as compensation, dividend declarations and layoff operations.

I. Introduction

Technology explosion, globalization of workforce and international trade development have all improved the quality of lives of employers and employees alike - but it also comes with a price of its own [1,2]. The management policies that have to be framed for such worldwide enterprises are extremely complex [1, 2].

The present work attempts to serve as a starting step towards the effective framing of policies in such complex organizations by using radically new approaches such as systems theory and chaos theory. The motivations of the present work are as follows:

1. This is the era of information technology. Terms like Big Data, Information Explosion and Internet of Things are becoming buzzwords. Hence it is no surprise that Big Data and Cloud Computing have become the new focal points of most Fortune 500 companies such as IBM [4,6].
2. Chaos involves complexity. Complexity involves patterns. Patterns are extremely rich in information. By studying few quantifiable parameters of an organization like IBM, the patterns in which the organization evolves can be ascertained. The wealth of information that is uncovered can be used purposefully and constructively to frame effective policies corresponding to various issues such as investment portfolios, internal budget and remuneration policies [7].
3. Thus, in essence, the work combines the best features of both of the above mentioned points – information and chaos theory, to come up with an innovative, feasible, simplistic solution to the problem of complex management.
4. Fortunately, the last few years of research in chaos theory by eminent scientists and mathematicians worldwide have resulted in a lot of parameters and metrics that can effectively describe the nature and behavior of the patterns that the system displays while evolving.

II. An Organization as a Chaotic System

One of the key features of the systems theory of management is that the organization is viewed as a system exhibiting chaos. In this section, this concept will be explored, which will form the basis for the analysis methodology proposed.

Input, the first component in a system, is the external driving force behind the organization. For typical multinational companies, inputs could be resources, assets etc. Output denotes the processed result of the system. Most usually, the output variables are profits and returns, shares purchased etc. The system, which is the organization itself, is a complex unit consisting of the proverbial 6 M's – Men, Money, Machines, Materials, Methods and Management.

Chaotic systems can be roughly defined as systems which are clearly distinguished by sensitive dependence on initial conditions and having deterministic evolution even though the phase space appears to be quite random [13].

The Chaotic system is a nonlinear, memory system which displays a very unique property - Sensitive Dependence on initial Conditions

Since chaotic systems are nonlinear, they tend to amplify and exaggerate certain inputs. Also, since they have memory, this causes further spilling over of the results into other areas, and in due course, the trend of the whole system changes drastically [14].

The following assumptions are made in the nonlinear analysis and strategy-data mapping of a typical organization. Special references and allusions are made to a Fortune 500 Company as a case study.

1. A Fortune 500 Company can be viewed as a large, growing and evolving organization, which can be suitably classified as a chaotic system, hence enabling us to apply the principles of contemporary theories of management such as systems theory and chaos theory.
2. The principle investment of such companies is remuneration and thus this is the perfect indicator of the input signal.
3. The share price and revenue data of a company reflects the market's opinion on its progress, and is one of the principal sources of revenue for the company. Thus it represents the output signal.
4. Various policy revelations and changes within an organization such as Declaration of dividends and earnings per shares, annual hike announcements, layoffs and retrenchments, and acquisitions clearly reflect in the share revenue data, usually with effects such as the butterfly effect dominant in such trends.
5. Thus, the analysis methodology proposed in this work serves to observe the trends and patterns in the share revenue data, try to correlate it with the policy changes and observe the effect that these changes have on the employees and the outside market.
6. There are two signals considered for the analysis – daily share price (closing) and share revenue (closing price*volume sold).

The metrics and their respective analyses detailed in this work may be grouped broadly into three categories:

1. Temporal Analysis: 'Temporal' is the adjective of time, and temporal analysis refers to the study of how a given signal, such as the share revenue varies with respect to time. Thus, this kind of analysis, apart from giving a glimpse of the 'ups and downs' of a signal, also give other useful results such as the evolution patterns and short term patterns of interest.
2. Spectral Analysis: This pertains to the frequency aspect of a signal. The frequency refers to the rate of change of share variables (i.e. how often it changes values). A signal may have multiple frequency components at the same time. For example, there may be weekly as well as monthly patterns in the variables. Also, the periodicity of the signal can be ascertained using spectral analysis.
3. Nonlinear Analysis: Since the system considered here is nonlinear (chaotic system), nonlinear analysis is required to bring out the true characteristics of the system. These include crucial aspects such as sensitivity, information capacity of the signal, and the self-repeating nature of the signal.

1. Time Series:

The time series is a representation, a graph of how the share variable varies with respect to time [8-12]. From this data, the general trend as well as the 'ups and downs' of the signal can be ascertained. For example, the time series of IBM share revenue for a period of two years from 4th February 2012 to 4th February 2014 is plotted as follows:

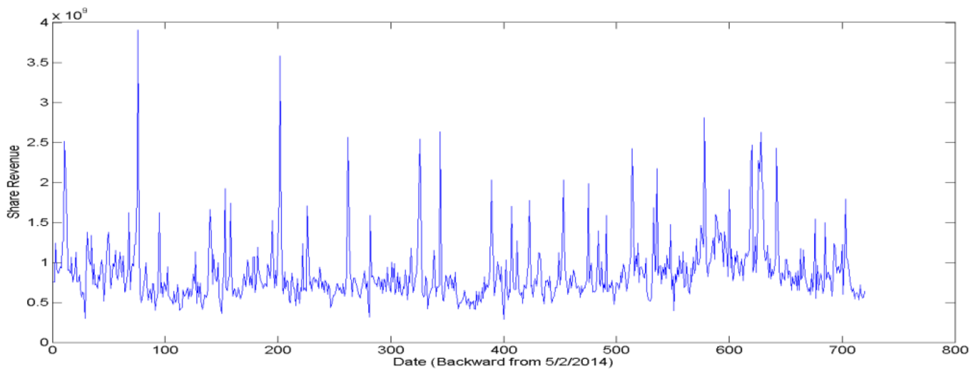

Figure 1 Time series of Share Revenue of IBM

The horizontal (x) axis indicates the number of days going backwards from 04/02/2014 to 04/02/2012. The vertical axis (y) indicates the revenue in dollars (US$).

From the time series, a few inferences become clear:

1. The share revenue is sporadic. While on most days the daily revenue remains in the vicinity of 1 billion dollars, it occasionally shoots up to as much as 4 billion dollars.
2. A certain minimum level is maintained. Very rarely has the revenue fallen below 0.5 billion dollars.
3. By looking at the 'baseline' one sees a dip near 365 days. And on either side of the dip, the baseline slopes up to a gradual 'hillock'. Roughly, this indicates a yearly periodicity in the share revenue.

2. Phase Portrait:

Phase portrait is one of the most valuable analysis results of a chaotic system. It is a qualitative analysis of the evolution of the system. In other words, it shows the path or 'trajectory' that the system follows [15].

A typical example of a chaotic signal and its phase portrait is shown as follows. This signal is obtained by the multiplication of two sine waves of different speeds.

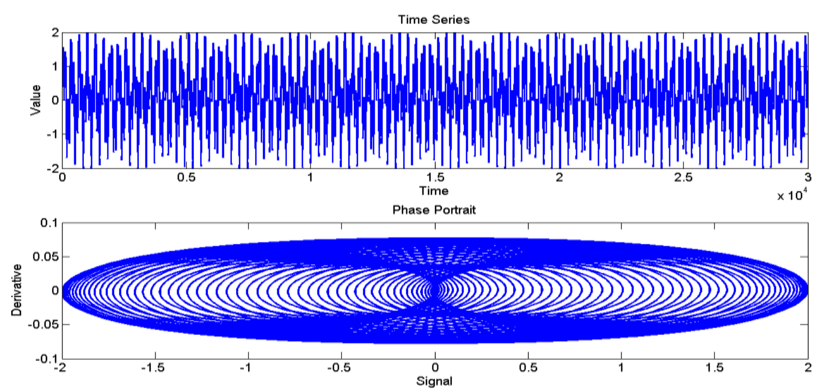

Figure 2 Chaotic Signal Phase Portrait

As can be clearly seen, the signal gives a noisier and more complicated appearance, whereas the phase portrait appears more ornamental. In general, the overall pattern of a phase portrait is called an ‘attractor’. Hence, circles, limit cycles, torus etc. are all attractors [5, 13, 15, 16].

From the phase portrait, one can understand the following:

1. The degree of complexity in the signal, and hence the system from where it originates.
2. The pattern, and the approximate trend it will take in the future, given its present position in the phase plane. This corresponds to the famous quote on chaos: "When the present determines the future, but the approximate present does not approximately determine the future.”

Thus, for a chaotic system, the phase portrait is one of the most valuable tools for a qualitative ascertaining of complexity.

3. Multiscale Analysis:

Most financial variables such as share data vary over different time scales simultaneously. For example, yearly events such as budget announcements and monthly events such as salary pay-days are both influencing factors in the purchasing of shares [17]. In such cases, multiscale analysis becomes a useful tool. A typical multiscale analysis technique is the wavelet analysis [18]. Here, a wavelet is defined as an aperiodic oscillatory signal.

The multiscale analysis technique involves expanding or contracting the wavelet to different scales and trying to match it with the share data. A strong match indicates that the corresponding scale influence on the share data is present [18].

As an example of the multiscale analysis, the wavelet analysis of S&P 500 index is shown as follows:

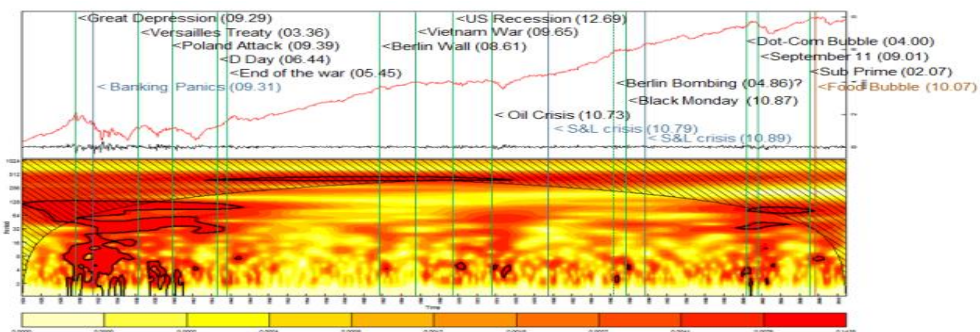

Figure 3 Wavelet Analysis of S&P 500 index

The figure shows the time series of S&P 500 index since 1923, marking important historical events. The colored image is the contour result of the wavelet analysis. Here, the vertical axis indicates the timescale, in days. Darker colours indicate more influence of that time scale on that signal. As can be seen, an event such as the dot com bubble shows a prominent effect on the 30 day time scale (monthly). More serious events such as the D-day of World war 2 signaled larger time scale effects of nearly 500 days.

The wavelet analysis is most effective when focused in shorter time scales to understand the impact of an event. From the wavelet analysis, both the timescale of influence of that event as well as the duration for which the influence actually existed can be obtained.

4. Magnitude Spectrum

Spectrum is a compilation of frequencies. Frequency indicates repeatability, periodicity. Frequency is thus the periodicity of a signal over a particular time scales. Thus, spectrum is the distribution of the periodicity of the share data over different time scales [19]. The magnitude spectrum is numerically calculated by taking a Fourier transform of the signal [19]. The Fourier transform converts time changing signals to frequency distributions.

5. Polar Plot

While the magnitude spectrum indicates the periodicity of a signal, it does not indicate the locations of such periodicity. For example, the magnitude spectrum indicates yearly periodicity, but does not indicate which time of the year, the highest rates occur. In such cases, the polar plot comes in handy.

The polar plot is derived from the polar coordinates [20]. Mathematically, a pair of Cartesian coordinates such as time and value can be converted into polar coordinates – those are magnitude and phase. Phase answers the question which part in the whole (time of the year), and magnitude answers the question how much in that phase.

While the temporal and spectral analyses provide an effective qualitative representation of the patterns and trends underlying the IBM share price and revenue data, it is the nonlinear analysis that gives quantitatively assertive information about the nature of the patterns and trends.

6. Largest Lyapunov Exponent (LLE)

The Lyapunov Exponent quantifies "Sensitive dependence on Initial Conditions" [21]. In other words, the Lyapunov exponent quantifies the Butterfly effect [14].

In a typical attractor such as the Lorenz attractor, the initial time is considered, where a group of very closely separated points are taken. In due course, the same points are noted after letting the system evolve for some time. As can be seen, what were closely separated points in the left pane are scattered wildly in the right pane. The left pane indicates the 'initial condition' and the scattering of the points indicates the sensitivity [21]. The largest Lyapunov exponent is the final measure of sensitivity obtained. Though the calculation of Lyapunov exponent requires the actual system to evolve over a period, modern techniques such as the Rosenstein algorithm can be effectively used to obtain the Largest Lyapunov exponent from the time series directly [22]. If the LLE is positive, chaotic nature dominates (that is, the complexity of the system is high).

7. Fractal Dimension (D2)

A fractal is a system in which, 'any part of the system resembles the whole'. Most chaotic signals are fractal [23]. Typically, after four or five self-similar iterations, we end up with a complex and ornamental looking structure. It is called a fractal, since in each iteration, we developed on a fraction (1/3) of the original. The significance of the final pattern is that each part of the final pattern is a miniature version of the whole. This property is known as 'self-similarity' [23].

The fractal dimension of a space denotes the degrees of freedom of a particle to move around that space [24]. For instance, a particle in a square can move in two directions: up/down, and left/right. Thus the dimension is two. For fractal structures, since a fraction of the whole is being chewed off every time, the dimension ends up as a fraction such as 0.7 or 0.4. Thus D2 is a measure of the dimensionality of the signal. A high fractal dimension such as 0.9 or 0.8 indicates more instability than lower dimensions such as 0.6 or 0.4. Various methods and techniques have been proposed to calculate the fractal dimension of a signal. One of the famous methods is the box counting algorithm whose result is called the 'Minkowski Bouligand Dimension' [25, 26].

8. Kolmogorov Entropy (K2):

The Kolmogorov entropy is an indicator of the information content of the signal. The concept originated from information theory [54, 55]. The number of times that the time series of a share data crosses a particular value is seen as the empirical probability of that value occurring in the sample space. In other words, the time series of the share data is mapped onto a probability space. Then, the Kolmogorov entropy, denoted as K2 is defined as the uncertainty associated with the probability space. 'Information' in the managerial sense can be interpreted as the value that the market sees in a particular share data sample at any instant of time. Hence, more information content can be viewed as potential revelations to the market. The revelation could be related to the internal working of the system that gave rise to the signal (i.e. about the company).

III. CASE STUDY - IBM

This section considers three particularly significant events in the recent history of IBM:

1. Acquisition of Business Analytics Software Firm Cognos in 2007.
2. Announcement of hikes of the likes of 20-25% for IBM India employees in 2011.
3. Layoff and retrenchment in Q1 2014.

In each case, the policy implications of the events are qualitatively assessed. Then, suitable time durations are defined before and after the event. For both time durations, the temporal, spectral and nonlinear analyses are performed, first using share revenue data, and then using share price data.

1. Acquisition of Cognos

Cognos Incorporated was a company in Ontario, Canada, dealing with business intelligence (BI) software [29]. Business Intelligence essentially refers to the effective harnessing of useful information from raw data [30]. On November 9th, 2007, IBM announced that it would be acquiring Cognos. The acquisition of BI giant Cognos put IBM in the elite league of a BI 'megavendor' with competitors such as Oracle, Microsoft and SAP. Soon, IBM came up with radically new concepts such as "Smarter Planet", "Big Data" and "Cloud Computing" [4, 6, 31], each bringing immense value to the future. With the decision to sell the microelectronic fabrication unit [32], IBM expects to focus big time on data and BI, and capture a huge share of the market.

To understand the implications of the acquisition of Cognos, two time durations are selected.

1. Before Acquisition announcement: September 18th 2006 to November 9th 2007.
2. After Acquisition announcement: November 10th 2007 to 2nd January 2009.

		BEFORE	AFTER
SHARE PRICE	LLE	1.112	4.699
	D2	0.661	0.661
	K2	5.658	5.661
SHARE REVENUE	LLE	0.723	4.022
	D2	0.661	0.661
	K2	5.61	5.55

The analysis reveal key trends in the share market performance of IBM caused due to the acquisition of Cognos BI. An increased sensitivity, as witnessed by the LLE increase is seen. Thus the slight decrease in K2, signifying lesser information content, meant that the market did not have a clear understanding of which way the Cognos acquisition would turn IBM's fortunes. Hence there were differences of opinion, and while optimistic buyers went bullish, pessimistic buyers went bearish. Hence, a sharp contrast in the share revenue data is seen, as witnessed by the multiscale analysis. This also affected the share price data, as the variation in the prices as well as the maximum price went down.

2. Announcement of hikes 2011

2010-2011 was one of the happier years for IBM. Just a year away from celebrating its centenary, this was the year IBM reached and relished several milestones. The SmartCloud [33], the smarter planet initiative, a lot of business awards at the global level were the hallmarks of the year. In India too, similar trends were seen. While the software departments were recruiting from premier institutes all over India such as IITs, IIITs and NITs by tens and hundreds, the systems and research departments were aggressively hiring interns. Productivity reached a new high, and the annual hike announcement during July 2011 was a happy occasion. Average employees were promised atleast 15% hike, whereas excellent performers would be earning hikes as much as 25%. While this may seem to be a totally internal affair, the announcement of hikes clearly affected the share market performance of IBM.

To understand these implications, two time durations are selected.

1. Before hike announcement: July 20th 2010 to July 20th 2011.
2. After hike announcement: July 20th 2011 to July 20th 2012.

		BEFORE	AFTER
SHARE PRICE	LLE	-4.82	8.064
	D2	0.98	0.98
	K2	5.528	5.535
SHARE REVENUE	LLE	-1.68	3.31
	D2	0.661	0.661
	K2	5.475	5.463

The initial couple of months after the hike announcement saw a marked increase in the share revenue baseline. This could indicate a generally bullish trend. This also helped to even out the contrasts caused before the hike announcement. But, the purchasing patterns of shares varied greatly. Rather than a systematic approach towards buying shares every week or every ten days, the polar plot indicates a less coordinated approach. This drove the system from a periodic one to a chaotic one, as witnessed by the Lyapunov exponent. All these results point towards an improving yet chaotic trend in the share performance of IBM. AS the company turned 100 in 2011, mixed opinions across India as well as the US prevailed, and more people joined in the share purchasing fray. Though synchronization took a hit, the overall results turned favorable for the company, encouraging it to set sail for yet another 100 years.

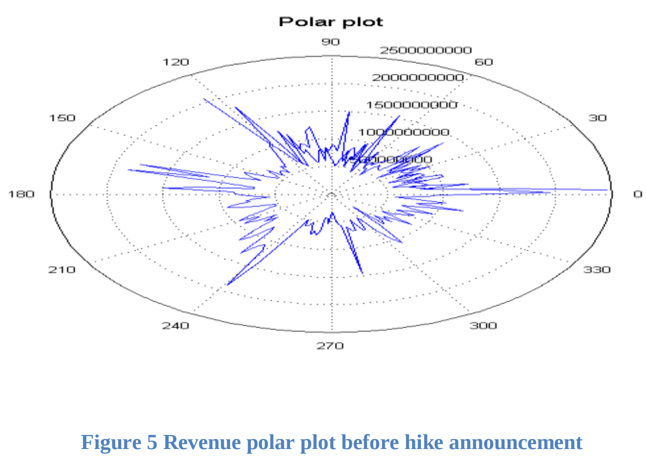

Figure 5 Revenue polar plot before hike announcement

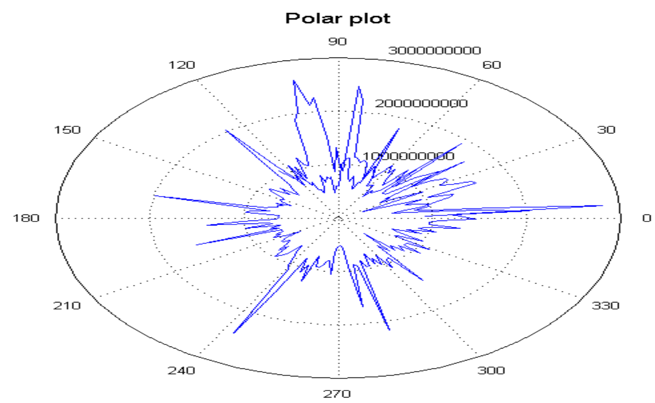

Figure 6 Revenue polar plot after hike announcement

3. Layoffs and Retrenchments of Q1 2014

The decline in the sales of servers and storage systems in 2013 had lead IBM to initiate a "Global Restructuring" program [34], whereby the top executives would forego their bonuses and nearly 15000 job cuts would be seen. This is in part, a measure to pioneer new, high value segments of the IT industry, with the company shifting focus from hardware based areas semiconductor research, mainframes and power processors to information based areas such as cloud computing and big data. While the job cuts started in India, Brazil and European Union soon followed suit. Following that, US based offices such as the Research Triangle Park (RTP) also witnessed job cuts [3]. Following this, on mid-April, IBM announced the Q1 2014 results, with profits and earnings from the systems and technology group (STG) significantly down by almost 11%.

While many long timers knew that this was only a restructuring and shift in focus in IBM, the market did not seem to welcome this change. The layoffs were perceived by many potential shareholders in both India as well as abroad as a decrease in the performance of the company.

To analyse the implications, the following time durations are defined:

1. During the Layoff (Q1): January 2nd 2014 to April 16th 2014
2. After the Layoff (Q2): April 17th 2014 to July 31st 2014

		DURING	AFTER
SHARE PRICE	LLE	31.67	16.77
	D2	0.68	0.68
	K2	4.28	4.29
SHARE REVENUE	LLE	500	707
	D2	0.68	0.68
	K2	4.23	4.21

The layoffs in 2014 Q1 have invariably been perceived adversely by the share market. This reflects in the clear drop in the share revenue during mid-February as witnessed by the multiscale analysis. While people followed the general trend of month end purchase of shares, the announcement of Q1 results changed the mindset. The average market value of the share went down in Q2, since people were not ready to take extravagant risks by quoting out of the money (OTM) prices. This is reflected by the decrease in LLE and increase in K2 for the price data. On the contrary, mixed opinions prevailed in the volume of shares purchased. This showed as significant rise in LLE and a drop in K2. In other words, lesser information about the revenue was available, and people were in a slightly confused state.

Overall, the moderate value of D2 and the extremely high values of LLE clearly indicated periods of extreme sensitivity.

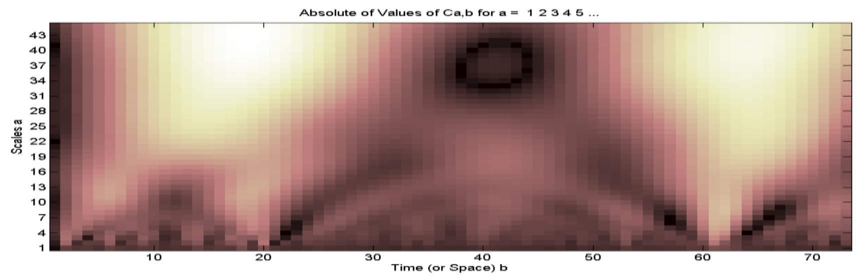

Figure 7 Revenue multiscale analysis before

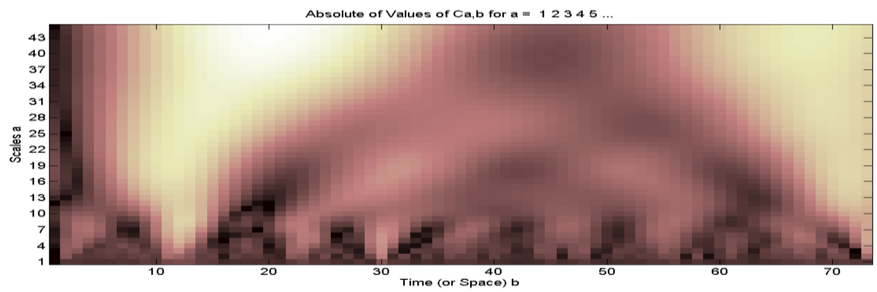

Figure 8 Revenue multiscale analysis after

CONCLUSION

This section details the implications of the nonlinear analyses elaborated in detail above.

1. System based revelations: All the case studies and the nonlinear analyses of each clearly point towards the fact that IBM is a huge, evolving, chaotic organization. This is in accordance with the assumption made earlier. Since chaotic systems contain memory, the output signal clearly depends on the system and past values. Thus, the output signal contains a wealth of information of the system it arises from. In these case studies, the IBM management would view the share revenue data as a direct effect, a market response to their internal policies, decisions and declarations.
2. Quantification: While it may be intuitively evident that a high hike would clearly boost the company's morale and the market response would certainly be bullish, the magnitude to which such reactions occur depend heavily of what existing and potential shareholders perceive of the company. People's opinion of a company is a totally abstract quantity – a feeling. But the nonlinear analyses and parameters such as sensitivity have done an effective job quantifying them.
3. Short-term trends: Multiscale analysis such as the wavelet analysis clearly highlights the short term trends immediately before or after an event. The contrasting dark and white spots on the wavelet analysis in certain timescales clearly illustrate the magnitude of influence of such decisions on the share market. Since the equity is one of the major sources of income for IBM, proper assessment of the company decisions and policies and their influence on the share market is crucial.
4. Planning ahead: The analyses techniques proposed in the present work serve to know well in advance, how to 'award the good news' or 'break the bad news', since most declarations of dividends, hikes and layoffs affect the share market performance. For example, the 2014 Q2 results for IBM STG have been announced in July 2014 at a loss of nearly 12%. This, and given that Q1 results were also at 11% loss, prediction of the share market performance for the next three months is not so difficult. It will continue along the same

trend. Also, the effect of the hike announcement in 2011, quantified in this work will help to assess what effect similar hike announcements will have in the forthcoming years.

5. Finer details: The polar plot reveals the share purchase pattern over the days of a month. The multiscale analysis reveals short term ‘spots’ and ‘patches’ arising due to decisions and announcements. Though the detailed analysis and interpretation of such finer details are out of scope of the present work, such analysis can be done by the company’s internal management who have access to most of the employee’s details about performance, customer satisfaction etc. to come up with a clearer understanding.
6. Marketing management: Once the company understands how the share market generally responds to the announcements and decisions, and is able to assess the magnitude of change to such declarations, the company can use these to play its cards safe. It can use this as a platform to release a new product, service or technique and see how the share market responds towards promoting the long term sustenance of the same. This could be a supplement to conventional marketing management strategies like customer surveys, Delphi technique etc.
7. Competitive advantage: By skimming through rival firms and how their decisions and announcements made an impact on the share market, IBM can have a competitive advantage in delaying/advancing these announcements if properly planned.
8. Innovation and Invention: Since IBM is one of the major players in the IP market alongside filing the largest number of patents, the company can direct its employees towards fruitful research by looking at what fields of research have been able to get the most confidence of shareholders. This is a win-win situation as it benefits the company to increase its revenue, benefits the employees to advance their career and benefits the society in general, leading to a truly “smarter planet”.

The application of the basic concepts and techniques of chaos theory and systems theory to the formulation of an effective analysis methodology opens up new avenues in the field of business strategy. The methods and techniques proposed in the present work can be adopted with slight modifications for the analysis of any company. It can also be used by Governments for effective framing of nationwide policies. One of the advantages of the presented analysis is that it requires minimum resources. Few news releases and the historical stock quotes are the only requirements for such an analysis. Moreover the metrics proposed in this work have all been used in various departments of science and technology since a long time and have been thoroughly time tested. Also, using certain algorithms, the computations of such metrics are extremely simple and can be done with basic mathematical software.

Future steps in this direction could involve refining the analysis methodologies, inclusion of more relevant metrics, and crafting a universal analysis methodology that could lead to a whole new way of looking at management. Part of such enhancements would include proper methodologies to gather interpretations and inferences from such analyses. Also, the analyses and interpretations could give rise to a new way of arriving at close to accurate predictions of future share prices. While this may be useful to the company, it will immensely benefit investors in the share market.

The effective harnessing of publicly available share market information and using it in a managerial capacity to understand and plan effectively for the growth of the organization, giving it an innovative and strategic competitive advantage forms the crux of the present work.

REFERENCES

[1] M.Lissack, The Interaction of Complexity and Management, Greenwood, 2002.
[2] A.Adekola and B.S.Sergi, Global Business Management, Ashgate, 2012.
[3] 13000 IBM Layoffs expected: http://www.businessinsider.com/ibm-layoffs-expected-2014-2.
[4] F.J.Ohlhorst, Big Data Analytics: Turning Big data into Big money, Wiley, 2012.
[5] E.E.Peters, Fractal Market Analysis: Applying Chaos Theory to Investment and Economics, Wiley, 1994.
[6] K.Jamsa, Cloud Computing, Jones and Bartlett, 2013.
[7] G.V.S.Sekhar, Business Policy and Strategic Management, I.K.International, 2009.
[8] A.V.Oppenheim and A.S.Willsky, Signals and Systems, Prentice-Hall, 1997.
[9] A.V.Oppenheim, Discrete time signal processing, Pearson, 2006.
[10] R.J.Elliot, W.C.Hunter and B.M.Jamieson, Financial Signal Processing, Federal Reserve Bank, 2000.
[11] H.K.Khalil, Nonlinear Systems, Pearson, 2013.
[12] A.C.Scott, The Nonlinear Universe: Chaos, Emergence and Life, Springer, 2007.
[13] C.H.Skiadas and I.Dimotikalis, Chaotic Systems: Theory and Applications, World Scientific, 2010.
[14] J.Gleick, Chaos: Making a new science, Open Road Media, 2011.
[15] L.S.Packer: Graphical Phase plane methods in dynamics and vibrations, Cornell, 1956.
[16] S.H.Strogatz, Nonlinear Dynamics and Chaos, Westview, 2008.
[17] F.In and S.Kim, An introduction to wavelet theory in finance, World Scientific, 2012.
[18] I.Daubechies, Ten Lectures on Wavelets, SIAM, 1992.
[19] P.Bloomfield, Fourier Analysis of Time Series, Wiley, 2004.
[20] W.S.Levine, Control System Fundamentals, CRC Press, 1999.
[21] J.Perry, Chaos in Real Data, Springer, 2000.
[22] M.T.Rosenstein, J.J.Collins and C.J.DeLuca, A practical method for calculating largest Lyapunov exponents from small data sets, Physica D, 65, 1993.
[23] K.Falconer, Fractal Geometry, Wiley, 2013.
[24] C.Tricot, Curves and Fractal Dimension, Springer, 1995.
[25] P.Abry, P.Goncalves and L.Vehel, Scaling, Fractals and Wavelets, Wiley, 2013.
[26] C.Skiadas, Topics on Chaotic Systems, World Scientific, 2009.
[27] R.M.Gray, Entropy and Information Theory, Springer, 2011.
[28] Kolmogorov in perspective, American Mathematical Society, 2000.
[29] Cognos Inc. Information: http://www.answers.com/topic/cognos-incorporated-usa.
[30] C.Vercellis, Business Intelligence, Wiley, 2011.
[31] S.Nunes, Smarter planet?, Island Press, 2013.
[32] IBM looking to sell chip manufacturing operations: http://online.wsj.com/news/articles/SB10001424052702303496804579367710245657016.
[33] E.Schouten, IBM SmartCloud Essentials, Packt, 2013.
[34] IBM Global Restructuring: http://www.bloomberg.com/news/2014-02-27/ibm-begins-cutting-u-s-jobs-in-1-billion-restructuring-plan.html.