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Neutrino Oscillation – a Natural Effect of Band Theory

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The No-Boundary Wave Function (NBWF) instanton formulation provides a particle theory basis due to its consistency with causality and cosmology. Terms of the NBWF instanton action are identified with intrinsic metrics of finite-dimensional particle theory. This approach yields a new type of finitary particle theory, in which the string (or finite graviton) is replaced with "band" representation geometry. A band is simply the well-known string with an intrinsic stiffness. Representation geometric metrics of area and curvature are thus intrinsic descriptors of particle mass and energy. The ground states of a band are inherently tripartite (having three portions), which motivates assignment of QCD color local gauge fields to portions of the band. For completeness, band-theoretic representation requires a quantum state algebra. A quantum state algebra that is one-to-one with standard model particle theory was sought – and found. The band theory quantum state algebra is a subgroup of index 12 of a cross product of two wreath products. A subgroup of this unique non-commutative algebra has proven to be 1-1 with standard model QCD particles, via its fundamental tie to preons. Further, band theory produces _exactly_ three particle generations (or families or flavors). Three flavors of neutrinos are thus predicted by a well-founded particle formulation, which is qualitatively consistent with observations of massive oscillating neutrinos.

Band theory is the particulate portion of a comprehensive theory, which includes cyclic cosmology. Cyclic cosmology has been demonstrated to have no cosmological coincidence problem.

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1. INTRODUCTION

The no-boundary wave function [1, 2], NBWF, expressed as the leading terms in its expansion, has the form:

$$\Psi(b,\chi) \approx \exp\{[-I_R(b,\chi) + iS(b,\chi)]/\hbar\} \quad (1)$$

The NBWF is a *causal* formulation owing to its development to describe cosmology. Although it includes an "instanton", it cannot describe QC/ED because it lacks quantum algebraic state representations. This paper presents a well-founded formulation which has been proposed [3, 4] to resolve this fundamental deficiency.

Causality requires that a particle's time coordinate be *uninvolved* with its (representative) non-commutative matrix algebra [5]. The Standard Model is thus known to be fundamentally *a-causal*, although it has a causal interpretation, its formulation is not consistent with cosmology or black hole theory.

Consistent mathematical formulations [6] describe finite geometries algebraically, and thus suggest a finite representation geometry and associated algebra be defined as a foundation for particle physics. The finite particle representation geometry and its Langland-dual quantum algebra [3, 4] which underlies QC/ED is shown in Figure 1.

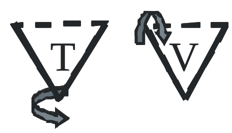

Figure 1: Two fundamental geometric quanta and their associated algebraic basis notation.

String theory [7] and some quantum gravity theories are also finite-geometric formulations. Note that the well-known dynamically triangulated quantum gravity [8] metric is a four-tetrahedron, although related to that under study, has not been found to be a QC/ED basis representation. In fact, neither has any form of string theory. Observe that strings are 'limp', that is, they have no intrinsic stiffness. By introducing stiffness to such finite particles the long-sought QC/ED particle representation geometry is determined. The notation of Rishon theory [9] is retained for convenience, but the quantum foundation created here is not a composite or parton theory – which is consistent with observations [10].

2. REPRESENTATION GEOMETRY – BAND THEORY

A closed Band (defined as a string with intrinsic stiffness) of *any* scale has a natural ground state, called a trecoil. {In this geometric representation, a trefoil knot would model a sterile neutrino.} A coiled Band has exactly two higher-order states with topologically distinct world sheets. These quantum states can easily self-interact – that is, they naturally *oscillate* between quantum states, as indicated in Figure 2.

Figure 2: Three intrinsic states of a finite band's world sheets; also the topological basis for three generations of fermions.

Neutrino band-state self-interactions incorporate zero to two gluons, and can only occur through the emission or absorption of a single gluon. As such, an oscillatory, vs classical, mixing angle is expected to be time/range-dependent. Because the Tau flavor can only be achieved from the Electron flavor via the Muon flavor, $\theta_{1,3}$ is expected to be very low. This is experimentally supported by Daya Bay observation of $\theta_{1,3}$ mixing angle. A range-dependent mixing angle is qualitatively supported by additional evidence from T2K [11].

A trecoil, in any state, exists with both right- or left-handed chirality (coiling). As such it is not its own anti-particle, and thus not Majorana, which property has not yet been concluded by experimental evidence. However, the representation geometry of π^0 and *Z* bosons have an extra symmetry as discussed in Section 5.

Partitions of the Band are inherent to this geometric representation. Finite-particle partitions, as designated for the band by Rishon notation in Figure 1, are attributed as the origin of color-local gauge fields. How a partitioned band replicates QC/ED states 1-1 is most evident through its quantum algebra.

3. QC/ED REPRESENTATION ALGEBRA

Review of band (nee tripartite string) noncommutative matrix algebra; recall that temporal (and mass) metrics must be uninvolved, and thus will be factored in.

Hofstadter [12] suggested that a non-commutative matrix algebra, now known to be a *subgroup of index 12 of a cross product of two wreath products* [13], could replicate QCD fermions. The fundamental representation algebra was quickly found and subsequently published in 1992 [3, 4] and developed in later presentations [14, 15].

The explicit QC/ED group is symbolized by Я (Ya), as such it not extensible – unlike the Standard Model of particles. Those familiar with Quantum Gravity may note that the symbol Я is similar to a generic symbol historically used for hypothetical QG particles that replicate QCD. By use of Я notation, particle wave functions $|H\rangle$ are expanded to explicitly notate the color states of quarks, even though they interact too quickly to be observed. Leptons are also fundamentally partitioned, with an extra quark-lepton symmetry just as in the eight-fold way. Neutrinos and electrons are thus composed of VVV and TTT 'Rishon-like' partitions, and their anti-particles states are $\overline{V}\overline{V}\overline{V}$ and $\overline{T}\overline{T}\overline{T}$. The Я algebraic notation for e^- and ν is thus also Dirac-like. A neutrino and free electron are represented in the non-commutative algebra Я in Figure 3.†

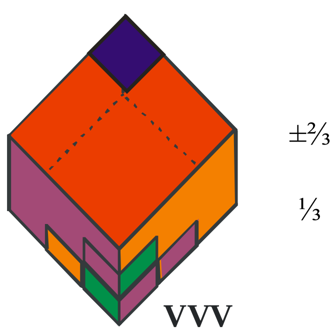

Figure 3: A Muon neutrino's algebraic state representation. Note- a gluon is represented quite naturally in this algebra.

The explicit chromo-dynamic states of partitioned band-theoretic particles are associated with QCD color-local gauge fields, simply by labeling the partitions as R, Y & B. Each partition thus has two extra space-like intrinsic dimensions: a local radius, r_R, r_Y, r_B, and Kähler angle $Ø_R$, $Ø_Y$, $Ø_B$. This notation serves to map band theoretic states explicitly one-to-one

† The familiar Rubik's algebra is restricted such that the corners are fixed in place, with ±twist. Cube faces have been re-coloured to match the requirements of QCD eight-fold way symmetry. Thus $Я_{RYB}${R,Y,B,G,P,O}≡Singmaster{R,D,B,L,U,F}.

with QCD states when quark color is defined to be determined by the orientation of fermion spin with respect to its quantized, colored partitions. Spin is restored as a geometric property, not merely a quantum number. In this way, leptons are colorless because their spin, if any, cannot be oriented to partition color. In other words the electron's color local gauge fields are superimposed and neutralized.

The band's area and curvature are two intrinsic properties that allow a complete description of all fundamental particles. By attributing particle mass to world-sheet *cross-sectional* area [16], and energy to intrinsic curvature, we obtain a theory that is one-to-one *and onto* the standard model of particles, *including* massive, oscillating neutrinos.

These properties appear in the NBWF formulation (Eq. 1) in the exact same form! The Planck scale, α', is a natural quantum metric of mass which is consistent with gravitation [16]. Defining a quantum metric for energy as *i*ç, a unit of intrinsic curvature, now write the explicit neutrino state function:

$$|H_{\nu\mu}\rangle=(\alpha'+i\text{ç})|e^{-\lambda(\nu\mu+\text{gluon})}B^2(\bar{P}O P\bar{O})^2(\bar{O}GO\bar{G})^2B^2\rangle\ , \tag{2}$$

where λ is a Randall-Sundrum scaling exponent defined as a functional ב (Beth) having *six extra intrinsic dimensions*:

$$\lambda=\beth(r_R,\ r_Y,\ r_B,\ \text{Ø}_R,\ \text{Ø}_Y,\ \text{Ø}_B). \tag{3}$$

This formulation meets the requirements of causality [5], is the explicit QC/ED ground state of band theory, and thus resolves coupled problems of fundamental theory. In particular, this geometric-algebraic representation theory models neutrino oscillations in a fundamental way, which is more consistent with observations [11] than an extension of standard a-causal theory. It also supports the normal ordering of neutrino masses [17].

The band theoretic geometric representation with its dual algebraic notation has the form of NBWF equation 1, but is an explicit (not fuzzy) 'instanton' that is 1-1 and onto QC/ED. This fact allows one to select a Friedman-Walker-Robertson cosmological formulation which is consistent with particle theory.

4. CONSISTENT PARTICLE AND COSMOLOGY FORMULAE

Recognize that standard model particle theory is essentially a space-time average of allowable particle interactions over scales accessible by high-energy colliders. That weak scale has been reduced in equation 2 to a Planck-like scale to construct a fundamental particle formula. The theory allows string-like computations of particle interactions, which can also be computed over measurable space-time scales. One can equally well consider the theoretic formulation at a cosmological scale, in which modelling particle states is rendered moot, but the underlying causal formulation remains.

A cosmological space-scale sum of equation 2 particle states yields the unifying formulation:

$$e^{(i\theta_{mt}-\lambda)}|Я\rangle, \tag{4}$$

where temporal evolution is included via the *original* cyclic cosmology [18] imaginary time variable, θ_{mt}, and Я is retained to allow the unifying formula to be evaluated at the usual weak interaction scale.

First observe that the universe is cyclic in *mass* and time. Formula 4 can be evaluated at a fixed (current) θ_{mt} 'time' to yield the well-known equation of General Relativity, but with an *imaginary* temporal term. The generalized QC/ED state algebra, Я, averages to unity at macroscopic scales, as usual. The result is set equal to **one** "universal size" to yield:

$$\frac{G^{C}_{\mu\nu}}{\Lambda g_{\mu\nu}} + \frac{8\pi i}{\Lambda g_{\mu\nu}} T_{\mu\nu} = 1 \quad (5)$$

where the Randall-Sundrum scale factor, λ, has been evaluated to reveal $G_{\mu\nu}$ and $T_{\mu\nu}$. Here the cosmological constant simply appears in the denominator, although it can be multiplied through and terms re-arranged to easily recognize General Relativity. Temporal curvature is naturally an imaginary quantity, like any curvature out of the coordinate plane in physics.

Cyclic cosmology thus requires a flat Riemannian geometric space-time. It also eliminates the cosmological coincidence problem, since the legacy GR is orthogonal to the cyclic formulation when evaluated at the present time [19].

5. MAJORANA PARTICLES

Band theoretic representation geometry and its associated algebra require an "extra" internal symmetry. The Я algebra requires a Majorana operator, which is akin to a super-symmetric operator, to model the π^o meson since it is comprised of $q\bar{q}$ pairs:

$$J_r = R\bar{G}O^2R^2G^2O\bar{B}P^2Y^2O\bar{B}R^2G^2B^2G^2R^2. \quad (6)$$

The geometric dual to this algebraic operator is shown in Figure 4. An open string, which is also a finite representation geometry, is included for comparison. The Lund open string has been postulated to require a width, b, to model its mass. However, open string theory attempts to encode chromodynamics state algebra by unfounded assignment of quanta, and doesn't model energy.

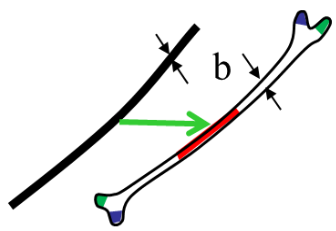

Figure 4: A π^o meson geometric representation reveals (a) its Majorana/extra internal symmetry and (b) the large scale difference between quark and weak scales, to which is attributed the relative success of Lund open string theory.

Majorana properties of particles have been experimentally determined for both π^o and *Z*. The representations of *Z* are similar to that for π^o, although two operators like (or a simplified version of) that in equation 6 are required. It is likely that Higgs is also Majorana, based on observations of its decay into $p\bar{p}$ pairs. Existence of several Majorana particles should motivate re-consideration of grand unification, since it is an intrinsic super-symmetry.

6. THREE QCD FLAVORS/GENERATIONS

The gluon subgroup is mapped in figure 5 for the neutrino in Я, since it is essential to the algebraic group [13]. The two gluon operators each form a 3-cycle. Either Hamiltonian operator representing a gluon results in a muon neutrino, while both will yield a tau neutrino when combined correctly.

The gluon algebraic operators, フ and レ, are defined by both the direction that the flipped edge-subcube moves and the color of the face it crosses. The direction of motion is indicated by the character: フ flips a subcube to the left, while レ

flips a subcube to the right as diagrammed in Figure 5. Three orientations of the operators are indicated by the numbers 1-3. The two basic gluon operators, oriented with respect to the purple face are:

$$\text{フ}_{P=2}=P^2B\bar{R}|\bar{Y}PG^2Y\bar{P}O^2|R\bar{B}P^2 \tag{7}$$

and

$$\text{レ}_{P=2}=P^2B\bar{R}|\bar{P}YO^2P\bar{Y}G^2|R\bar{B}P^2\ . \tag{8}$$

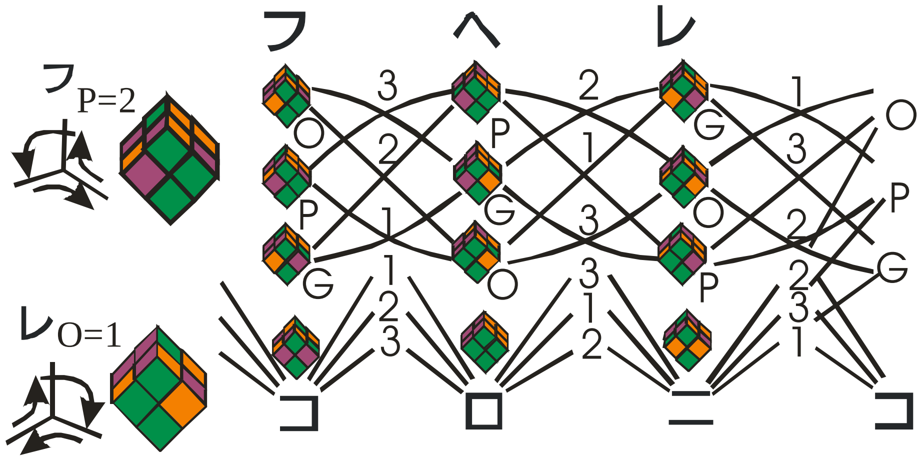

Figure 5: Map of gluon operator in Я allows exactly three flavors (or quark generations).

The orientation of resulting states is annotated by O,G,P here since each tau neutrino state can be oriented with respect to each colored face. The tau neutrino state is only accessible via the muon neutrino state, as in the band representation. The algebraic representation thus yield exactly three flavors, or quark generations.

7. CONCLUSIONS

Band theory is a well-founded geometric representation and noncommutative, causal, algebraic formulation of particles in a manner consistent with physics at observable and macroscopic scales.

The finite geometric construction yields quantum states of a string-like theory, in which Rishon-like quantum gravitational partitions are used to ensure that the Standard Model of particles is realized. A formulation consistent with these theories ensures that the usual computations of particle theory are enabled. As such, the theory establishes the connection between a specific type of string & quantum gravity theories and QC/ED.

The following important, albeit qualitative, results were established:

- The band theoretic representation of neutrinos is inherently massive and includes oscillations in a fundamental way.
- Band theory explains the observation of three particle generations/families/ flavors, as well as Majorana properties in terms similar to super-symmetry.
- Band theory is fundamentally consistent with a flat FRW cyclic cosmological which includes a cosmological constant. This cyclic formulation has no cosmological coincidence problem.

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