Demonstration that the equation $\mathbf{a}^{\mathbf{x}} + \mathbf{b}^{\mathbf{y}} = \mathbf{c}^{\mathbf{z}}$ has no solution in positive integers if a, b, c not have a common prime factor.

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Abstract

Beal's conjecture is studied, which arises from investigations Andrew Beal, on Fermat's Last Theorem in 1993.

Beal's conjecture, proposed to the equation $a^x + b^y = c^z$ where A, B, C, x, y, z positive integers x, y, z> 2 so that the equation has a solution A, B, and C must have a common prime factor.

Given the vain attempts to find a counterexample to the conjecture (which has been proven by the help of modular arithmetic, for all values of the six variables to a value of 1000) values for the six variables. To advance the theory of numbers.

Given the relationship that has Beal's conjecture with Fermat's last theorem; it is considered important to number theory, demonstration of this conjecture.

To solve Beal's conjecture, using Fermat's last theorem and the remainder theorem is proposed.

Keywords

Beal´s conjecture, Fermat

Introduction

Beal's conjecture is a conjecture in theory proposed by Andrew Beal on 1993 numbers; A similar conjecture was suggested independently by Andrew Granville those dates.

While researching generalizations of Fermat's Last Theorem in 1993, Beal made the following conjecture.

Beal's conjecture states that if $A^x + B^y = C^z$ where x, y and z greater than 2; solution is then positive integers A, B and C must have a common prime factor.

The equation $A^x + B^y = C^z$ has infinite solutions if A, B and C have a common prime factor, using a computer and modular arithmetic, the conjecture has been verified for all values of 6 variables until 1000.

Fermat's last theorem says that the equation $A^x + B^y = C^z$ do not have solutions integer other than 0 for x = y = z where x> 2. Then there will be no solutions in positive integers, so that Beal's Conjecture is true, for this particular case.

For demonstration it requires the following theorems.

Theorem 1 Fermat's last theorem

If n is a positive integer such that $n \geq 3$ then the equation $x^n + y^n = z^n$ has no solution in positive integers.

Proof.

Demonstrated by Andrew Wallis and Richard Taylor. [1]

Theorem 2

In a full and rational polynomial in x which is divisible by a binomial of the form x-a, to replace the x by a in the polynomial, the polynomial is zero.

Proof.

Be the $Ax^m + Bx^{m-1} + Cx^{m-2} + \cdots + Mx + N$ a polynomial which is divisible by x-a. Dividing the polynomial by x-a, as it is divisible, its residue is 0. Let Q the quotient obtained of this division.

As in any exact division the dividend is equal to the product of the divisor by the quotient.

$$Ax^m + Bx^{m-1} + Cx^{m-2} + \cdots + Mx + N = Q(x-a)$$

This equality is true for all values of x. substituting x by a, is obtained

$$Aa^m + Ba^{m-1} + Ca^{m-2} + \cdots + Ma + N = Q(a-a)$$

$$Aa^m + Ba^{m-1} + Ca^{m-2} + \cdots + Ma + N = Q(0)$$

$$Aa^{m} + Ba^{m-1} + Ca^{m-2} + \cdots + Ma + N = 0$$

Q. A. D.

Proof of the conjecture

Theorem 3 (Beal's Conjecture)

If the equation $\mathrm{A}^{\mathrm{x}} + \mathrm{B}^{\mathrm{y}} = \mathrm{C}^{\mathrm{z}}$ with x, y, z> solution 2 has positive integers A, B, and C must have a common prime factor.

Proof

The equation $\mathrm{A}^{\mathrm{x}} + \mathrm{B}^{\mathrm{y}} = \mathrm{C}^{\mathrm{z}}$ has infinite solutions if A, B and C have a common prime factor, so to prove the theorem is sufficient to prove that the equation has no solution if A, B and C not have a common prime factor.

First it is proved that the equation $\mathrm{A}^{\mathrm{x}} + \mathrm{B}^{\mathrm{y}} = \mathrm{C}^{\mathrm{z}}$ x, y, z are equal to 4 or an odd prime number.

If an exponent x is not divisible by any odd prime, then it is a power of 2 , and x>2 , then it is equal to 4 or divisible by 4, is $x = 4x'$ by substituting in equation $A^{x} = (A^{x'})^{4}$ and being $(A^{x'})$ another positive integer, the equation is satisfied for x = 4.

If an exponent x is divisible by any odd prime, then is $x = px'$ by substituting in equation $A^{x} = (A^{x'})^{p}$ and being $(A^{x'})$ another positive integer, the equation is satisfied for x = p.

The proof will be by cases.

1) Case 1 $x = y = z = 4$ $\quad a^4 + b^4 = c^4$

By Theorem 1 it has no solution in positive integers.

2) Case 2 $x = y = z$ $\quad a^x + b^x = c^x$

By Theorem 1 it has no solution in positive integers.

$$a^x = c^z - b^x$$

3) Case 3 $x = y \neq z$ $\quad a^x + b^x = c^z$

Solving for a^x

$$a^x = c^z - b^x$$

Assuming a, b and c numbers co-prime positive integers; a^x is a positive integer, then there is a positive integer such that a^x is divisible by e. then there is a integer d such that e = c-d.

Then $c^z - b^x$ is divisible by c-d. By theorem 2.

$$d^z - b^x = 0$$

Solving d

$$d^z = b^x$$

$$d = b^{\frac{x}{z}}$$

x, z are co-prime $b^{\frac{x}{z}}$ is an irrational number, so that d is not integer. And then e can not be a positive integer. Then a^x has no divisors positive integers, which is a contradiction; therefore the equation $a^x + b^x = c^z$ has no solution in positive integers if a, b and c are co-prime.

4) Case 4 $x = z \neq y$ $a^x + b^y = c^x$

Solving for a^x

$$a^x = c^x - b^y$$

Assuming a, b and c numbers co-prime positive integers; a^x is a positive integer, then there is a positive integer such that a^x is divisible by e. then there is a integer d such that e = c-d.

Then $c^x - b^y$ is divisible by c-d. By theorem 2.

$$d^x - b^y = 0$$

Solving d

$$d^x = b^y$$

$$d = b^{\frac{y}{x}}$$

x, y are co-prime $b^{\frac{y}{x}}$ is an irrational number, so that d is not integer. And then e can not be a positive integer. Then a^x has no divisors positive integers, which is a contradiction; therefore the equation $a^x + b^y = c^x$ has no solution in positive integers if a, b and c are co-prime.

5) Case 5 $x \neq y \neq z$ $a^x + b^y = c^z$

Solving for a^x

$$a^x = c^z - b^y$$

Assuming a, b and c numbers co-prime positive integers; a^x is a positive integer, then there is a positive integer such that a^x is divisible by e. then there is a integer d such that e = c-d.

Then $c^z - b^y$ is divisible by c-d. by theorem 2

$$d^z - b^y = 0$$

Solving d

$$d^z = b^y$$

$$d = b^{\frac{y}{z}}$$

y, z are co-prime $b^{\frac{y}{z}}$ is an irrational number, so that d is not integer. And then e can not be a positive integer. Then a^x has no divisors positive integers, which is a contradiction; therefore the equation $a^x + b^y = c^z$ has no solution in positive integers if a, b and c are co-prime.

6) Case 6 $x = 4$ $\qquad a^4 + b^y = c^z$

Solving for b^y

$$b^y = c^z - a^4$$

Assuming a, b and c numbers co-prime positive integers; b^y is a positive integer, then there is a positive integer such that b^y is divisible by e. then there is a integer d such that e = c-d.

Then $c^z - a^4$ is divisible by c-d. by theorem 2

$$d^z - a^4 = 0$$

Solving d

$$d^z = a^4$$

$$d = a^{\frac{4}{z}}$$

4, z are co-prime $a^{\frac{4}{z}}$ is an irrational number, so that d is not integer. And then e can not be a positive integer. Then b^y has no divisors positive integers, which is a contradiction; therefore the equation $a^4 + b^y = c^z$ has no solution in positive integers if a, b and c are co-prime.

7) Case 7 $z = 4$ $\qquad a^x + b^y = c^4$

Solving for b^y

$$b^y = c^4 - a^x$$

Assuming a, b and c numbers co-prime positive integers; b^y is a positive integer, then there is a positive integer such that b^y is divisible by e. then there is a integer d such that e = c-d.

Then $c^4 - a^x$ is divisible by c-d. by theorem 2

$$d^4 - a^x = 0$$

Solving d

$$d^4 = a^x$$

$$d = a^{\frac{x}{4}}$$

4, x are co-prime $a^{\frac{x}{4}}$ is an irrational number, so that d is not integer. And then e can not be a positive integer. Then b^y has no divisors positive integers, which is a contradiction; therefore the equation $a^x + b^y = c^4$ has no solution in positive integers if a, b and c are co-prime.

8) Case 8 $x = y = 4$ $a^4 + b^4 = c^z$

Solving for a^4

$$a^4 = c^z - b^4$$

Assuming a, b and c numbers co-prime positive integers; a^4 is a positive integer, then there is a positive integer such that a^4 is divisible by e. then there is a integer d such that e = c-d.

Then $c^z - b^4$ is divisible by c-d. By theorem 2

$$d^z - b^4 = 0$$

Solving d

$$d^z = b^4$$

$$d = b^{\frac{4}{z}}$$

4, z are co-prime $a^{\frac{4}{z}}$ is an irrational number, so that d is not integer. And then e can not be a positive integer. Then a^4 has no divisors positive integers, which is a contradiction; therefore the equation $a^4 + b^4 = c^z$ has no solution in positive integers if a, b and c are co-prime.

By the cases the equation $a^x + b^y = c^z$ has no solution in positive enteors if a, b and c have no common prime factor.

Q.A.D.

Conclusions

Beal's conjecture is a generalization of Fermat's last theorem, so the proff is important to the development of the theory of numbers. It establishes a condition to see if the equation $a^x + b^y = c^z$ has a solution in positive integers.

To the proff Beal's conjecture stating that for the equation $a^x + b^y = c^z$ with x, y, z> 2 has a solution in positive integers if A, B, and C have a common prime factor.

Bibliography

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[2] W. J. LeVeque. Elementary Theory of Number, (1962). Addison-Wesley Publishing Company.

[3] http://www.bealconjecture.com/