# The Dark Matter entropic force and Newtons energetic force as a complete first law of thermodynamics set of gravitational forces

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## Abstract

In this paper I derive an emergent Dark Matter force using the virial theorem in the context of the Dark Matter halo model. This emergent force is then used to inductively derive a Dark Matter entropy S and a Dark Matter number of microstates W. I then show that this emergent force can be interpreted as an entropic force. Using the first law of thermodynamics a set of two forces can be derived from my model's potential function, with the Newtonian force of gravity derived from the energy as the first one and the emergent Dark Matter force derived from entropy as the second one.

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## I. THE ELEMENTARY PARTICLE DARK MATTER HALO MODEL

In this paper I derive an entropic force regarding galactic Dark Matter phenomena, in the context of the model I am working on, as presented in [1], [2], [3] and in [4]. This will be a brief presentation, assuming that the interested reader will look through the previous parts of this work in progress.

Given a rest mass $m_0$ at $r = 0$, it will have an additional spherical Dark Matter halo containing an extra mass, with Dark Matter properties only, in the sphere with radius r as

$$m_{\rm DM} = \frac{r}{r_{\rm DM}} m_0. \tag{1}$$

In our model the total gravitational source mass $m_g$ of an elementary particle $m_0$ contained within a sphere with radius $r$ will then be given by

$$m_g = m_0 + m_{\rm DM} = m_0 + \frac{r}{r_{\rm DM}} m_0 = m_0 \left(1 + \frac{r}{r_{\rm DM}}\right). \tag{2}$$

We accordingly define a Dark Matter halo mass density as

$$\rho_{\rm DM} = \frac{m_0}{4\pi r^2 r_{\rm DM}} \tag{3}$$

and then the spherically symmetric gravitational source mass $m_g$ inside a sphere of radius $r$ is given by

$$m_g = \int_V \rho_{\rm DM} dV = \int_r \rho_{\rm DM} 4\pi r^2 dr = \int_r \frac{m_0}{r_{\rm DM}} dr = \frac{m_0}{r_{\rm DM}} r + m_0 \tag{4}$$

with the last factor as the obvious constant of integration, given the starting point of our model that we have $m_g = m_0$ at $r = 0$.

Given the definition of the gravitational potential as

$$\phi = -\frac{GM}{r} \tag{5}$$

with gravitatinal source mass $M$ as

$$M = M_0 + \frac{rM_0}{r_{\rm DM}} \tag{6}$$

we get a gravitational potential at $r$ as

$$\phi = -\frac{GM_0}{r} - \frac{GM_0}{r_{\rm DM}} = \phi_0 + \phi_{\rm DM} \tag{7}$$

For the resulting force of gravity on a classical mass $m$ we get the Newtonian result

$$\mathbf{F} = -m\nabla\phi = -m\nabla\phi_0 + -m\nabla\phi_{\text{DM}} = -m\nabla\phi_0 = -\frac{GM_0 m}{r^2}\hat{r}. \tag{8}$$

This is due to the fact that the new mass factor varies linear over $r$ and thus results in a additional potential term that is constant. Our Dark Matter halo acts as a gauge term in the source that produces a constant term $\phi_{\text{DM}}$ in the potential and thus has zero effect on the Newtonian force. This of course presents a problem because MOND phenomenology clearly indicates that a deviation of Newtonian forces on the galactic scale matches the experimental findings, see [8] and [9]. This gap between model and phenomena is what the entropic force will close. Put in a simple equation, we have

$$F_{MOND} - F_{Newton} = F_{Entropic}, \tag{9}$$

at least as far as our model is used. In our perspective, MOND is a phenomenological theory only, crucial for a better understanding of total galactic entropy.

Although the extra term in $m_g$ doesn't effect Newtons force law of gravity, it is effecting the gravitational energy of a satellite mass $m$ in the field of a source mass $M$. This gravitational energy is given by

$$U_g = m\phi = m\phi_0 + m\phi_{\text{DM}} = -\frac{GM_0 m}{r} - \frac{GM_0 m}{r_{\text{DM}}}. \tag{10}$$

Now we assume that the virial theorem is still valid, that it is more fundamental than the force analysis from which it was originally derived. Using $2U_k = -U_g$ we get $v^2 = -\phi$ for orbiting satellites and

$$v^2 = -\phi = \frac{GM_0}{r} + \frac{GM_0}{r_{\text{DM}}}. \tag{11}$$

If we let $r \to \infty$ then

$$v_f^2 = \frac{GM_0}{r_{\text{DM}}}, \tag{12}$$

which is a constant, the galaxy rotational velocity curves' final constant value. In Fig.(1) the result is compared to the Newtonian virial expectation for $v$.

## II. THE EMERGENT FORCE, THE VIRIAL THEOREM AND BARYONIC TULLY-FISHER

At all times in the galactic disk, the centripetal force $F_c$ must match with the virial theorem, so $F_g = F_c$. The difference between the needed $F_c$ and the Newtonian force of

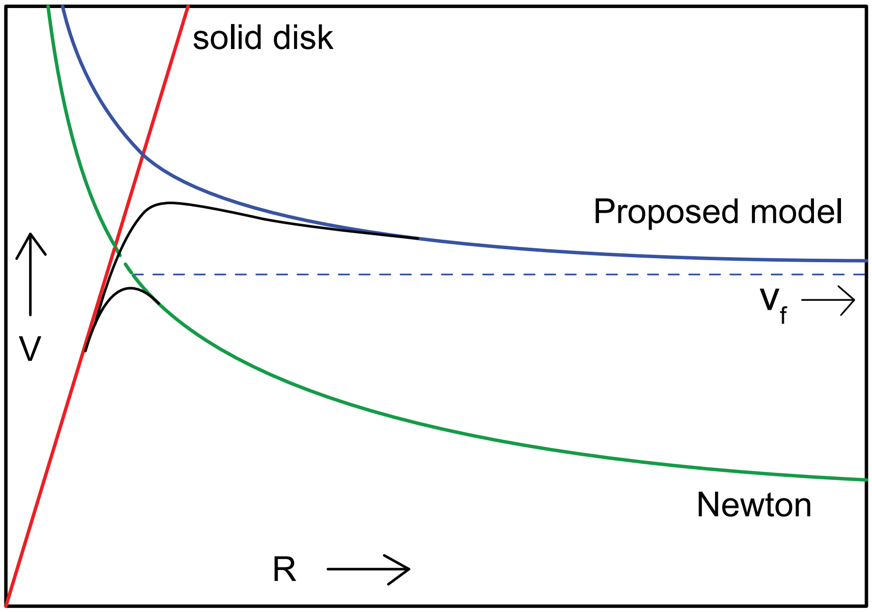


FIG. 1. Rotation velocity versus radius curves with the Newtonian and our new model expectation.

gravity $F_N$ must be delivered by the emergent Dark Matter force $F_{\text{DM}}$. Assuming the orbits to be circular, we can insert Eqn.(11) in the formula for $F_c$ to get

$$F_c = \frac{m_0 v^2}{r} = \frac{GM_0 m_0}{r^2} + \frac{GM_0 m_0}{r\ r_{\text{DM}}} = -F_N - F_{\text{DM}} \tag{13}$$

and so the entropic Dark Matter force must result in

$$F_{\text{DM}} = -\frac{GM_0 m_0}{r\ r_{\text{DM}}} \tag{14}$$

For $r \gg r_{\text{DM}}$, $v = v_f$ and $F_c \approx -F_{\text{DM}}$ so

$$F_c = \frac{m_0 v_f^2}{r} \approx \frac{GM_0 m_0}{r\ r_{\text{DM}}} = -F_{\text{DM}} \tag{15}$$

leading again to

$$v_f^2 = \frac{GM_0}{r_{\text{DM}}} \tag{16}$$

We can define a special Dark Matter centripetal acceleration given by

$$a_{\text{DM}} \equiv \frac{v_f^2}{r_{\text{DM}}} \tag{17}$$

so $r_{\text{DM}}$ can be given by

$$r_{\text{DM}} \equiv \frac{v_f^2}{a_{\text{DM}}} \tag{18}$$

which, inserted into Eqn.(16) gives

$$v_f^2 = \frac{Ga_{\rm DM} M_0}{v_f^2} \tag{19}$$

and to

$$v_f^4 = Ga_{\rm DM} M_0, \tag{20}$$

a relation that we recognize as Milgrom's form of the Baryonic Tully-Fisher relation.

The Tully-Fisher relation is a relation between the luminosity of a spiral galaxy and its, maximum, rotation velocity [6]. The physical basis of the Tully-Fisher relation is the relation between a galaxy's total baryonic mass and the velocity at the flat end of the rotation curve, the final velocity. According to McGaugh both stellar and gas mass of galaxies have to be taken into account in the relation that is referred to as the Baryonic Tully-Fisher (BTF) relation [7]. In 2005 McGaugh determined the baryonic version of the LT relation as

$$M_d = 50 v_f^4, \tag{21}$$

see [7] and Fig(2). In this form, $M_d$ is expressed in solar mass $M_\odot = 1,99 \cdot 10^{30}\ kg$ units

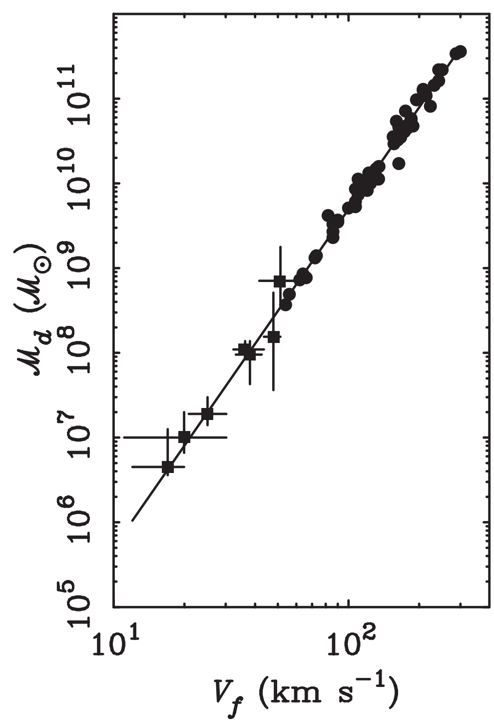


FIG. 2. The double log graph for the Baryonic Tully-Fisher relation with vertical the total baryonic mass, from McGaug 2005.

and the final velocity of the galactic rotation velocity curve $v_f$ is expressed in $km/s$. If we express the galactic mass in $kg$ and the velocity in $m/s$ we get the total baryonic mass, final velocity relations in SI unit values as $M_b = 1,0 \cdot 10^{20} v_f^4$. In the double log form with SI values we get $\log M_b = 4 \log v_f + 20$.

In 1983, Milgrom interpreted the BTF relation as an indication of a deviation from Newtonian gravity, making a modification of Newtonian dynamics or MOND necessary [8]. Using McGaug's 2005 values in SI units, Milgrom presented the BTF relation in the form

$$v_f^4 = 1,0 \cdot 10^{-20} M_b = G a_0 M_b, \tag{22}$$

resulting in an acceleration $a_0 = 1,5 \cdot 10^{-10}\ m/s^2$ in McGaug's values. According to Milgrom, this relation should hold exactly, thus interpreting it as an inductive law of nature instead of looking at it as just an empirical relation [9]. The resulting acceleration can be written as $5 \cdot a_0 \approx cH_0$, with the velocity of light $c$ and the Hubble constant $H_0$. According to Milgrom, the deeper significance of this relation between the galactic critical acceleration and the Hubble acceleration should be revealed by future cosmological insights [8]. This allows us to identify our $a_{\scriptscriptstyle DM}$ with Milgrom's $a_0$. And we can conclude that in our model, $a_{\scriptscriptstyle DM} = v_f^2 / r_{\scriptscriptstyle DM}$ is the galactic Dark Matter constant. In MOND terminology, our entropic DM force is given by

$$F_{\scriptscriptstyle DM} = -\frac{G a_0 M_0 m_0}{r\ v_f^2}, \tag{23}$$

so with $F_{\scriptscriptstyle DM} \propto r^{-1}$.

## III. FROM THE EMERGENT FORCE TO THE ENTROPY

Our Dark Matter force emerged from virial requirements. But we can also connect the DM force to emergent gravity, using Boltzmann's entropy. According to Verlinde, see Eqn. 3.7 in [5], we have for the entropic force of gravity the general requirement

$$F \Delta r = T \Delta S \tag{24}$$

in which $T$ stands for the absolute temperature connected to $m_0$ and $S$ for the entropy of $m_0$ in the DM field of $M_0$. We can write it in integral form as

$$\int_{S_i}^{S_f} T dS = \int_{r_i}^{r_f} F dr. \tag{25}$$

If we focus on galactic gas clouds of neutral hydrogen H 1 with a mass $m_0$ and an internal temperature $T$ of 100 $K$ that is constant over vast distances in $r$, so isothermal, this combines with our entropic DM force to

$$\int_{S_i}^{S_f} dS = \int_{r_i}^{r_f} -\frac{GM_0 m_0}{r\ r_{\rm DM} T} dr = -\frac{GM_0 m_0}{r_{\rm DM} T} \int_{r_i}^{r_f} \frac{1}{r} dr \tag{26}$$

and to

$$S_f - S_i = -\frac{GM_0 m_0}{r_{\rm DM} T} \ln \frac{r_f}{r_i}, \tag{27}$$

which can also be written as

$$S_f - S_i = \frac{U_{\rm DM}}{T} \ln \frac{r_f}{r_i}. \tag{28}$$

We have the Boltzmann definition of entropy $S$ as

$$S = k_B \ln W \tag{29}$$

with for W the number of microstates. We assume that

$$\ln W \propto \ln \frac{r}{r_m} \tag{30}$$

with $r_m$ as the radius of the particle with mass $m_0$ that is at distance $r$ in the field of $M_0$. We further assume that for the macroscopic temperature of particles on the galactic disk, all moving with $v_f$ with one single degree of freedom perpendicular to $r$, we can assume $k_B T = 2U_{kf} = mv_f^2 = -m\phi_{\rm DM} = -U_{\rm DM}$.

This can be combined with the above

$$S_f - S_i = \frac{U_{\rm DM}}{T} \ln \frac{r_f}{r_m} - \frac{U_{\rm DM}}{T} \ln \frac{r_i}{r_m} \tag{31}$$

to a Dark Matter entropy as

$$S = k_B \ln \left(\frac{r}{r_m}\right)^{\frac{U_{\rm DM}}{k_B T}} \tag{32}$$

so with

$$W = \left(\frac{r}{r_m}\right)^{\frac{U_{\rm DM}}{k_B T}} \tag{33}$$

with $T$ related to the average total kinetic energy of the particle relative to the Dark Matter field in question. This implies that in the flat rotation curve domain, the Dark Matter entropy is given by

$$S = k_B \ln \left(\frac{r}{r_m}\right)^{-1} = -k_B \ln \left(\frac{r}{r_m}\right) \tag{34}$$

producing a decreasing entropy as we go further away from the baryonic matter $M_0$. A decrease of entropy outwards produces an entropic force inwards.

## IV. FROM POTENTIAL TO ENTROPIC FORCE AND THE FIRST LAW OF THERMODYNAMICS

Now let's inverse the derivation, starting with a gravitational charge $m_0$ in the Dark Matter field of the gravitational source $M_g$. This gives us the DM potential

$$\phi_{\text{DM}} = -\frac{GM_0}{r_{\text{DM}}}, \tag{35}$$

the DM energy as the constant

$$U_{\text{DM}} = -\frac{GM_0 m_0}{r_{\text{DM}}}, \tag{36}$$

and the Dark Matter entropy

$$S = k_B \ln \left( \frac{r}{r_m} \right)^{\frac{U_{\text{DM}}}{k_B T}} \tag{37}$$

From the entropy we can derive the entropic DM force using

$$F_{\text{DM}} = T \left( \frac{dS}{dr} \right)_{U_{\text{DM}}} = T \frac{d}{dr} k_B \ln \left( \frac{r}{r_m} \right)^{\frac{U_{\text{DM}}}{k_B T}} = U_{\text{DM}} \frac{d}{dr} \ln \left( \frac{r}{r_m} \right) = \frac{U_{\text{DM}}}{r} = -\frac{GM_0 m_0}{r\ r_{\text{DM}}}. \tag{38}$$

The crucial thing here is that we didn't, we couldn't derive the force directly from the potential, the last being a constant. But the DM potential energy of $m_0$ in the Dark Matter halo of $M_0$ was connected to an entropy and from this entropy we could derive an entropic force, giving the lacking part of the centripetal force.

Newtonian gravity then is given by

$$F_N = -\left( \frac{dU_N}{dr} \right)_{S_{\text{DM}}} \tag{39}$$

and we have used the first law of thermodynamics as

$$dU = TdS - Fdr \tag{40}$$

or as

$$F_g = -\left( \frac{dU}{dr} \right)_{S_{\text{DM}}} + T \left( \frac{dS}{dr} \right)_{U_{\text{DM}}} = F_N + F_{\text{DM}} \tag{41}$$

Thus, the first law of thermodynamics is the foundation for our proposal for an expansion of Newton's law of gravity.

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