

Four applications of a Navier-Stokes Cosmology

Victor Christianto^{*}

ABSTRACT

In a recent paper I presented a possible route from an exact analytical solution of the Navier-Stokes equations to Navier-Stokes Cosmology on Cantor Sets. But that was just an outline or a view that there is coherent hydrodynamics picture for everything in this Universe, from smallest scales to galaxies and beyond. Therefore in the present paper I will review four applications of a Navier-Stokes-Cosmology: a. Active Galactic Nuclei, b. Oceanography, c. Relativistic Hydrodynamics, and d. Vortex model of elementary particles. While I borrow most of these applications from some existing literatures, but to my present knowledge there is no attempt so far to view these applications under a unified theme of Navier-Stokes cosmology.

Key Words: Navier-Stokes equations, Navier-Stokes cosmology, Active Galactic Nuclei, relativistic hydrodynamics, elementary particles, oceanography, Mathematica.

Introduction

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In my previous paper [1], I conclude it with this note: “The next task is how to find observational cosmology and astrophysical implications. This will be the subject of future research.” Therefore in the present paper I will review four applications of the proposed Navier-Stokes-Cosmology:

- a. Active Galactic Nuclei,
- b. Oceanography,
- c. Relativistic Hydrodynamics, and
- d. Vortex model of elementary particles.

While I borrow most of these applications from some existing literatures, but to my present knowledge there is no attempt so far to view these applications under a unified theme of Navier-Stokes cosmology. It is interesting to note here that some features of this framework may have similarities with the fractal vortex universe model of Patrick Driessen, who also mentioned similarity between galaxies and particles [16].

^{*} Correspondence: Victor Christianto, Independent Researcher and administrator of www.sciprint.org, Phone: (62) 878-59937095, URL: http://researchgate.net/profile/Victor_Christianto. Email: victorchristianto@gmail.com

a. Dipole toroidal vortex model in Active Galactic Nuclei

The 2D Navier-Stokes equation for a steady viscous flow can be written as follows [6]:

$$\rho(\vec{v} \cdot \nabla)\vec{v} = -\nabla p + \rho\vec{f} + \mu\Delta\vec{v} \quad (1)$$

Argentini obtained a general exact solution of ODE version of 2D Navier-Stokes equation in Riccati form as follows [1][2]:

$$\dot{u}_1 - \alpha u_1^2 + \beta = 0, \quad (2)$$

where:

$$\alpha = \frac{1}{2\nu},$$

and

$$\beta = -\frac{1}{\nu} \left(\frac{\dot{q}}{\rho} - f_1 \right) s - \frac{c}{\nu}.$$

The solution of Riccati equation is notoriously difficult to find, so this author decides to use Mathematica software in order to get an exact analytical solution [4][5]. The result has been presented in a recent paper [3].

One possible solution of Navier-Stokes equations comes in the form of vortex, for example it is known that Serrin's swirling vortex is a solution of Navier-Stokes equations. In this regard, it is interesting to remark here that Bannikova and Kontorovich have proposed a dipole toroidal vortex model for Active Galactic Nuclei.[12][13]

They began their paper as follows: "Starting with the Antonucci and Miller's outstanding work, tori have been considered as a necessary element of the AGN-structures forming the basis of the AGN unified model. A brilliant achievement was the first direct observation of the obscuring tori described by Jaffe, Meisenheimer, Röttgering *et al.* (2004). Existence of tori was confirmed by observation with VLT optical interferometer equipped with MIDI IR-camera." [12] They also suggested that "Since the preliminary observational data Jaffe, Meisenheimer, Röttgering *et al.* (2004) point at significantly larger torus sizes, it should be natural to suggest *the "matrjoshka" scheme: there are tori of smaller radii within the outer big torus.* In the case of Eddington luminosity, the mass of torus that replenishes the accretion disk is proportional to its big radius." [12] They also concluded that "A dipole-toroidal vortex can be an essential element of AGN-structure, which replenishes the accretion disk." [12]

The figure below shows their concept [13]:

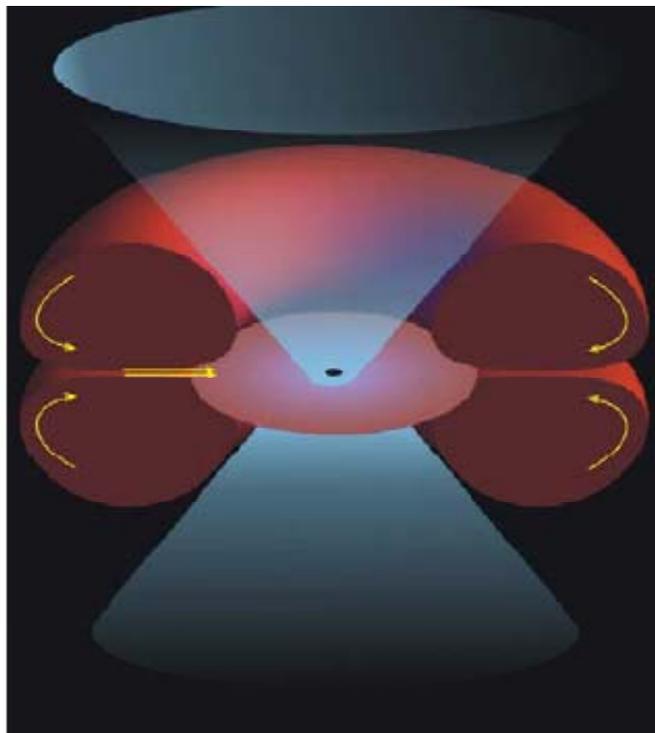


Fig. 1b. Dipole toroidal vortex in the AGN center: 3D picture.
Cones sketch out the wind and radiation.

The momentum is related with circulation and mass, and it is given by [13]:

$$\Gamma = \oint v \cdot dr = 2\pi r \cdot v_{\phi} \quad (3)$$

According to Patrick Driessen, Bohr's quantization condition also holds at galaxies scale where the cosmic Bohr radius can be estimated and it yields value: $A_0=41.8$ Mpc [16].

b. Atmospheric flow and Oceanography

The use of dipole vortex in atmospheric flow has been discussed by Snyder, Plougonven and Muraki [14]. Their paper investigates the generation mechanisms for the stationary inertia-gravity waves embedded within a larger-scale dipole vortex. Similarly, various applications of vortical flows can be observed in oceanography field. [15] Atmospheric and oceanographic fields are few of applications of Navier-Stokes equations.

c. Relativistic Hydrodynamics

Now we can extend further the Navier-Stokes equations to relativistic hydrodynamics (RHD). In essence, the relativistic hydrodynamics will yield Generalized Relativistic Magneto-Hydrodynamics (GRMHD). Although there are a number of approaches to RHD, here we will follow the presentation of Jose A. Font [17]. For other treatment, see for instance Andreev [18].

Relativistic hydrodynamics is the part of physics devoted to the study of both, flows in which the velocities attained by individual particles or by the fluid as a whole approach the speed of light in vacuum, or those for which the strength of the gravitational field is important enough to render mandatory a description in terms of Einstein's theory of gravity [17]. In particular, flow velocities as large as 99% of the speed of light are required to explain the apparent superluminal motion inferred in many of the observed jets in extragalactic radio sources associated with active galactic nuclei.[17]

In essence, general relativistic MHD is concerned with the dynamics of relativistic, electrically conducting fluids (plasma) in the presence of magnetic fields. In terms of the (Faraday) electromagnetic tensor $F^{\mu\nu}$, defined as [17]

$$F^{\mu\nu} = U^\mu E^\nu - U^\nu E^\mu - \eta^{\mu\nu\lambda\delta} U_\lambda B_\delta, \quad (4)$$

Maxwell equations read [17]

$$\nabla_\nu * F^{\mu\nu} = 0, \quad \nabla_\nu F^{\mu\nu} = J^\mu, \quad (5)$$

Maxwell's equations can be simplified if the fluid is a perfect conductor. This case corresponds to the so-called ideal MHD condition. Under this assumption the electric field measured by the Eulerian observer has components: [17]

$$E^0 = 0, \quad E^i = -\alpha \eta^{0ijk} \nu_j B_k, \quad (6)$$

And Maxwell's equations $\nabla_\nu * F^{\mu\nu} = 0$ reduce to the divergence-free condition of the magnetic field plus the induction equation for the evolution of the magnetic field [17]:

$$\frac{\partial(\sqrt{\gamma} B^i)}{\partial x^i} = 0, \quad (7)$$

$$\frac{1}{\sqrt{\gamma}} \frac{\partial}{\partial x^0} (\sqrt{\gamma} B^i) = \frac{1}{\sqrt{\gamma}} \frac{\partial}{\partial x^j} (\sqrt{\gamma} [\alpha \nu^i B^j - \alpha \nu^j B^i]), \quad (8)$$

d. Vortex model of elementary particles

Now we extend it further to vortex model of elementary particles as proposed by Rockenbauer [19].

According to Rockenbauer, the Dirac equation describes the motion of electrons in electromagnetic field, but it considers spin as intrinsic property without any real motion. Despite the fact that customarily applied point charge models avoid clarifying whether spin is related to any physical motion, there were efforts to relate certain motion to spin.[19]

He argues that spin kinetic energy can be written as [19]:

$$E_{spin} = \frac{3}{2} I \omega_{spin}^2 = m a^2 \cdot \omega_{spin}^2 = m c^2. \quad (9)$$

In other words, the rest energy can be produced in full by the spinning motion of elementary particles if the peripheral speed is equal to the velocity of light. Furthermore, according to Rockenbauer, when the self-system is considered, the elementary particles behave like empty space (vacuum), and we can postulate the elementary particles as vortices defined as spinning confinements of the space.[19]

While surely his model is not complete yet, it can give an outline of vortex model of elementary particles, including equations for quarks and neutrino [19]. In short, it is quite promising alternative framework to understand spinning behavior of elementary particles.

In another perspective, Tkalya has calculated cyclo-toroid nuclear moment of particles, which may indicate the vortical structure of elementary particles [20]. It seems worth to investigate further plausibility of her approach.

Meanwhile, an alternative vortex model of elementary particles has been proposed for instance by James Tassano [21].

Concluding remarks

In a recent paper I presented a possible route from an exact analytical solution of the Navier-Stokes equations to Navier-Stokes Cosmology on Cantor Sets. But that was just an outline or a view that there is coherent hydrodynamics picture for everything in this Universe, from smallest scales to galaxies and beyond.

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It should be noted that the above presentation is not conclusive yet. However, all of these four applications seem to be intriguing enough to be investigated, therefore allow me to suggest further research and experiments.

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VC, email: victorchristianto@gmail.com

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