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Interval-valued neutrosophic soft sets and its decision making

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Abstract

In this paper, the notion of the interval valued neutrosophic soft sets ($ivn-$soft sets) is defined which is a combination of an interval valued neutrosophic sets [36] and a soft sets [30]. Our $ivn-$soft sets generalizes the concept of the soft set, fuzzy soft set, interval valued fuzzy soft set, intuitionistic fuzzy soft set, interval valued intuitionistic fuzzy soft set and neutrosophic soft set. Then, we introduce some definitions and operations on $ivn-$soft sets sets. Some properties of $ivn-$soft sets which are connected to operations have been established. Also, the aim of this paper is to investigate the decision making based on $ivn-$soft sets by level soft sets. Therefore, we develop a decision making methods and then give a example to illustrate the developed approach.

Keyword: Interval sets, soft sets, fuzzy sets, intuitionistic fuzzy sets, neutrosophic sets, level soft set.

1 Introduction

Many fields deal with the uncertain data may not be successfully modeled by the classical mathematics, since concept of uncertainty is too complicate and not clearly defined object. But they can be modeled a number of different approaches including the probability theory, fuzzy set theory [39], rough set theory [33], neutrosophic set theory [34] and some other mathematical tools. This theory have been applied in many real applications to handle uncertainty. In 1999, Molodtsov [30] succesfully proposed a completely new theory so-called *soft set theory* by using classical sets because its been pointed out that soft sets are not appropriate to deal with uncertain and fuzzy parameters. The theory is a relatively new mathematical model for dealing with uncertainty from a parametrization point of view.

After Molodtsov, there has been a rapid growth of interest in soft sets and their various applications such as; algebraic structures (e.g.[1, 2, 5, 41]), ontology (e.g.[22]), optimization (e.g.[19]), lattice (e.g.[17, 32, 37]), topology (e.g.[8, 29, 35]), perron integration, data analysis and operations research (e.g.[30, 31]), game theory (e.g.[13, 14, 30]), clustering (e.g.[4, 28]), medical diagnosis (e.g.[18, 38]), and decision making under uncertainty (e.g.[10, 16, 23]). In recent years, many interesting applications of soft set theory have been expanded by embedding the ideas of fuzzy sets (e.g. [9, 11, 12, 16, 26]), rough sets (e.g. [15]) and intuitionistic fuzzy sets (e.g. [20, 25]), interval valued intuitionistic fuzzy (e.g. [21, 40]), Neutrosophic (e.g. [24, 27]).

Intuitionistic fuzzy sets can only handle incomplete information because the sum of degree true, indeterminacy and false is one in intuitionistic fuzzy sets. But neutrosophic sets can handle the indeterminate information and inconsistent information which exists commonly in belief systems in neutrosophic set since indeterminacy is quantified explicitly and truth-membership, indeterminacy-membership and falsity-membership are independent. It is mentioned in [36]. Therefore, Maji firstly proposed neutrosophic soft sets with operations, which is free of the difficulties mentioned above, in [24]. He also, applied to decision making problems in [27]. After Maji, the studies on the neutrosophic soft set theory have been studied increasingly (e.g. [6, 7]).

From academic point of view, the neutrosophic set and operators need to be specified because is hard to be applied to the real applications. So the concept of interval neutrosophic sets [36] which can represent uncertain, imprecise, incomplete and inconsistent information was proposed. In this paper, we first define interval neutrosophic soft sets (INS-sets) which is generalizes the concept of the soft set, fuzzy soft set, interval valued fuzzy soft set, intuitionistic fuzzy soft set, interval valued intuitionistic fuzzy soft sets. Then, we introduce some definitions and operations of interval neutrosophic soft sets. Some properties of INS-sets which are connected to operations have been established. Also, the aim of this paper is to investigate the decision making based on interval valued neutrosophic soft sets. By means of level soft sets, we develop an adjustable approach to interval valued neutrosophic soft sets based decision making and a examples are provided to illustrate the developed approach.

The relationship among interval neutrosophic soft set and other soft sets is illustrated as;

$$
\begin{aligned}
\text{Soft set} \; & \subseteq \text{Fuzzy soft set} \\
& \subseteq \text{Intuitionistic fuzzy soft set (Interval valued fuzzy soft set)} \\
& \subseteq \text{Interval valued intuitionistic fuzzy soft set} \\
& \subseteq \text{Interval valued neutrosophic soft set}
\end{aligned}
$$

Therefore, interval neutrosophic soft set is a generalization other each the soft sets.

2 Preliminary

In this section, we present the basic definitions of neutrosophic set theory [36], interval neutrosophic set theory [36] and soft set theory [30] that are useful for subsequent discussions. More detailed explanations related to this subsection may be found in [6, 7, 9, 21, 24, 27, 36].

Definition 2.1 *[34] Let U be a space of points (objects), with a generic element in U denoted by u. A neutrosophic sets(N-sets) A in U is characterized by a truth-membership function T_A, a indeterminacy-membership function I_A and a falsity-membership function F_A. $T_A(u)$; $I_A(x)$ and $F_A(u)$ are real standard or nonstandard subsets of $[0, 1]$.*

There is no restriction on the sum of $T_A(u)$; $I_A(u)$ and $F_A(u)$, so $0 \leq supT_A(u) + supI_A(u) + supF_A(u) \leq 3$.

Definition 2.2 *[36] Let U be a space of points (objects), with a generic element in U denoted by u. An interval value neutrosophic set (IVN-sets) A in U is characterized by truth-membership function T_A, a indeterminacy-membership function I_A and a falsity-membership function F_A. For each point $u \in U$; T_A, I_A and $F_A \subseteq [0, 1]$.*

Thus, a IVN-sets over U can be represented by the set of

$$A = \{\langle T_A(u), I_A(u), F_A(u)\rangle/u : u \in U\}$$

Here, $(T_A(u), I_A(u), F_A(u))$ is called interval value neutrosophic number for all $u \in U$ and all interval value neutrosophic numbers over U will be denoted by $IVN(U)$.

Example 2.3 *Assume that the universe of discourse $U = \{u_1, u_2\}$ where u_1 and characterises the quality, u_2 indicates the prices of the objects. It may be further assumed that the values of u_1 and u_2 are subset of $[0, 1]$ and they are obtained from a expert person. The expert construct an interval value neutrosophic set the characteristics of the objects according to by truth-membership function T_A, a indeterminacy-membership function I_A and a falsity-membership function F_A as follows;*

$A = \{\langle[0.1, 1.0], [0.1, 0.4], [0.4, 0.7]\rangle/u_1, \langle[0.6, 0.9], [0.8, 1.0], [0.4, 0.6]\rangle/u_2\}$

Definition 2.4 *[36] Let A a interval neutrosophic sets. Then, for all $u \in U$,*

1. *A is empty, denoted $A = \widetilde{\emptyset}$, is defined by*

$$\widetilde{\emptyset} = \{< [0, 0], [1, 1], [1, 1] > /u : u \in U\}$$

2. *A is universal, denoted $A = \widetilde{E}$, is defined by*

$$\widetilde{E} = \{< [1, 1], [0, 0], [0, 0] > /u : u \in U\}$$

3. *The complement of A is denoted by $\overline{A}$ and is defined by*

$$\begin{aligned}\overline{A} = \quad & \{< [infF_A(u), supF_A(u)], [1 - supI_A(u), 1 - infI_A(u)], \\ & [infT_A(u), supT_A(u)] > /u : u \in U\}\end{aligned}$$

Definition 2.5 *[36] An interval neutrosophic set A is contained in the other interval neutrosophic set B, $A\tilde{\subseteq}B$, if and only if*

$$\begin{array}{ll} infT_A(u) \leq infT_B(u) & supI_A(u) \geq supI_B(u) \\ supT_A(u) \leq supT_B(u) & infF_A(u) \geq infF_B(u) \\ infI_A(u) \geq infI_B(u) & supF_A(u) \geq supF_B(u) \end{array}$$

for all $u \in U$.

Definition 2.6 *An interval neutrosophic number $X = (T_X, I_X, F_X)$ is larger than the other interval neutrosophic number $Y = (T_Y, I_Y, F_Y)$, denoted $X\widehat{\leq}Y$, if and only if*

$$\begin{array}{ll} infT_X \leq infT_Y & supI_X \geq supI_Y \\ supT_X \leq supT_Y & infF_X \geq infF_Y \\ infI_X \geq infI_Y & supF_X \geq supF_Y \end{array}$$

Definition 2.7 *[36] Let A and B be two interval neutrosophic sets. Then, for all $u \in U$, $a \in R^+$,*

1. *Intersection of A and B, denoted by $A\tilde{\cap}B$, is defined by*

$$\begin{array}{ll} A\tilde{\cap}B = & \{< [min(infT_A(u), infT_B(u)), min(supT_A(u), supT_B(u))], \\ & [max(infI_A(u), infI_B(u)), max(supIA(x), supI_B(u))], \\ & [max(infF_A(u), infF_B(u)), max(supF_A(u), supF_B(u))] > /u : u \in U\} \end{array}$$

2. *Union of A and B, denoted by $A\tilde{\cup}B$, is defined by*

$$\begin{array}{ll} A\tilde{\cup}B = & \{< [max(infT_A(u), infT_B(u)), max(supT_A(u), supT_B(u))], \\ & [min(infI_A(u), infI_B(u)), min(supI_A(u), supI_B(u))], \\ & [min(infF_A(u), infF_B(u)), min(supF_A(u), supF_B(u))] > /u : u \in U\} \end{array}$$

3. *Difference of A and B, denoted by $A\tilde{\setminus}B$, is defined by*

$$\begin{array}{ll} A\tilde{\setminus}B = & \{< [min(infT_A(u), infF_B(u)), min(supT_A(u), supF_B(x))], \\ & [max(infI_A(u), 1 - supI_B(u)), max(supI_A(u), 1 - infI_B(u))], \\ & [max(infF_A(u), infT_B(u)), max(supF_A(u), supT_B(u))] > /u : u \in U\} \end{array}$$

4. *Addition of A and B, denoted by $A\tilde{+}B$, is defined by*

$$\begin{array}{ll} A\tilde{+}B = & \{< [min(infT_A(u) + infT_B(u), 1), min(supT_A(u) + supT_B(u), 1)], \\ & [min(infI_A(u) + infI_B(u), 1), min(supI_A(u) + supI_B(u), 1)], \\ & [min(infF_A(u) + infF_B(u), 1), min(supF_A(u) + supF_B(u), 1)] > /u : u \in U\} \end{array}$$

5. *Scalar multiplication of A, denoted by $A\tilde{.}a$, is defined by*

$$\begin{array}{ll} A\tilde{.}a = & \{< [min(infT_A(u).a, 1), min(supT_A(u).a, 1)], \\ & [min(infI_A(u).a, 1), min(supI_A(u).a, 1)], \\ & [min(infF_A(u).a, 1), min(supF_A(u).a, 1)] > /u : u \in U\} \end{array}$$

6. *Scalar division of A, denoted by* $A\widetilde{/}a$*, is defined by*

$$\begin{aligned} A\widetilde{/}a = \quad & \{<[min(infT_A(u)/a,1), min(supT_A(u)/a,1)], \\ & [min(infI_A(u)/a,1), min(supI_A(u)/a,1)], \\ & [min(infF_A(u)/a,1), min(supF_A(u)/a,1)] > /u : u \in U\} \end{aligned}$$

7. *Truth-Favorite of A, denoted by* $\widetilde{\triangle}A$*, is defined by*

$$\begin{aligned} \widetilde{\triangle}A = \quad & \{<[min(infT_A(u)+infI_A(u),1), min(supT_A(u)+supI_A(u),1)], [0,0], \\ & [infF_A(u), supF_A(u)] > /u : u \in U\} \end{aligned}$$

8. *False-Favorite of A, denoted by* $\widetilde{\nabla}A$*, is defined by*

$$\begin{aligned} \widetilde{\nabla}A = \quad & \{<[infT_A(u), supT_A(u)], [0,0], \\ & [min(infF_A(u)+infI_A(u),1), min(supF_A(u)+supI_A(u),1)] > /u : u \in U\} \end{aligned}$$

Definition 2.8 *[30] Let U be an initial universe, $P(U)$ be the power set of U, E be a set of all parameters and $X \subseteq E$. Then a soft set F_X over U is a set defined by a function representing a mapping*

$$f_X : E \to P(U)\, such\, that\, f_X(x) = \emptyset \;if\; x \notin X$$

*Here, f_X is called approximate function of the soft set F_X, and the value $f_X(x)$ is a set called x-*element *of the soft set for all $x \in E$. It is worth noting that the sets is worth noting that the sets $f_X(x)$ may be arbitrary. Some of them may be empty, some may have nonempty intersection. Thus, a soft set over U can be represented by the set of ordered pairs*

$$F_X = \{(x, f_X(x)) : x \in E, f_X(x) \in P(U)\}$$

Example 2.9 *Suppose that $U = \{u_1, u_2, u_3, u_4, u_5, u_6\}$ is the universe contains six house under consideration in an real agent and $E = \{x_1 = cheap, x_2 = beatiful, x_3 = greensurroundings, x_4 = costly, x_5 = large\}$.*

A customer to select a house from the real agent. then, he can construct a soft set F_X that describes the characteristic of houses according to own requests. Assume that $f_X(x_1) = \{u_1, u_2\}$, $f_X(x_2) = \{u_1\}$, $f_X(x_3) = \emptyset$, $f_X(x_4) = U$, $\{u_1, u_2, u_3, u_4, u_5\}$ then the soft-set F_X is written by

$$F_X = \{(x_1, \{u_1, u_2\}), (x_2, \{u_1, u_4, u_5, u_6\}), (x_4, U), (x_5, \{u_1, u_2, u_3, u_4, u_5\})\}$$

The tabular representation of the soft set F_X is as follow:

U	u_1	u_2	u_3	u_4	u_5	u_6
x_1	1	1	0	0	0	0
x_2	1	0	0	1	1	1
x_3	0	0	0	0	0	0
x_4	1	1	1	1	1	1
x_5	1	1	1	1	1	0

Table 1: The tabular representation of the soft set F_X

Definition 2.10 *[23] Let $U = \{u_1, u_2, ..., u_k\}$ be an initial universe of objects, $E = \{x_1, x_2, ..., x_m\}$ be a set of parameters and F_X be a soft set over U. For any $x_j \in E$, $f_X(x_j)$ is a subset of U. Then, the choice value of an object $u_i \in U$ is c_i, given by $c_i = \sum_j u_{ij}$, where u_{ij} are the entries in the table of the reduct-soft-set. That is,*

$$u_{ij} = \begin{cases} 1, & u_i \in f_X(x_j) \\ 0, & u_i \notin f_X(x_j) \end{cases}$$

Example 2.11 *Consider the above Example 2.9. Clearly,*

$$c_1 = \sum_{j=1}^{5} u_{1j} = 4,$$

$$c_3 = c_6 = \sum_{j=1}^{5} u_{3j} = \sum_{j=1}^{5} u_{6j} = 2,$$

$$c_2 = c_4 = c_5 = \sum_{j=1}^{5} u_{2j} = \sum_{j=1}^{5} u_{4j} = \sum_{j=1}^{5} u_{5j} = 3$$

.

Definition 2.12 *[10] Let F_X and F_Y be two soft sets. Then,*

1. *Complement of F_X is denoted by $F_X^{\tilde{c}}$. Its approximate function $f_{X^c}(x) = U \setminus f_X(x) \quad$ for all $x \in E$*

2. *Union of F_X and F_Y is denoted by $F_X \tilde{\cup} F_Y$. Its approximate function $f_{X\tilde{\cup}F_Y}$ is defined by*

$$f_{X\tilde{\cup}Y}(x) = f_X(x) \cup f_Y(x) \quad \text{for all } x \in E.$$

3. *Intersection of F_X and F_Y is denoted by $F_X \tilde{\cap} F_Y$. Its approximate function $f_{X\widetilde{\cap}Y}$ is defined by*

$$f_{X\tilde{\cap}Y}(x) = f_X(x) \cap f_Y(x) \quad \text{for all } x \in E.$$

3 Interval-valued neutrosophic soft sets

In this section, we give interval valued neutrosophic soft sets ($ivn-$soft sets) which is a combination of an interval valued neutrosophic sets[36] and a soft sets[30]. Then, we introduce some definitions and operations of $ivn-$soft sets sets. Some properties of $ivn-$soft sets which are connected to operations have been established. Some of it is quoted from [6, 7, 10, 9, 21, 24, 27, 36].

Definition 3.1 *Let U be an initial universe set, $IVN(U)$ denotes the set of all interval valued neutrosophic sets of U and E be a set of parameters that are describe the elements of U. An interval valued neutrosophic soft sets(ivn-soft sets) over U is a set defined by a set valued function Υ_K representing a mapping*

$$\upsilon_K : E \to IVN(U)$$

It can be written a set of ordered pairs

$$\Upsilon_K = \{(x, \upsilon_K(x)) : x \in E\}$$

Here, υ_K, which is interval valued neutrosophic sets, is called approximate function of the ivn-soft sets Υ_K and $\upsilon_K(x)$ is called x-approximate value of $x \in E$. The subscript K in the υ_K indicates that υ_K is the approximate function of Υ_K.

Generally, υ_K, υ_L, υ_M,... will be used as an approximate functions of Υ_K, Υ_L, Υ_M, ..., respectively.

Note that the sets of all *ivn*-soft sets over U will be denoted by $IVNS(U)$.

Now let us give the following example for *ivn*-soft sets.

Example 3.2 *Let $U = \{u_1, u_2\}$ be set of houses under consideration and E is a set of parameters which is a neutrosophic word. Consider $E = \{x_1 = cheap, x_2 = beatiful, x_3 = greensurroundings, x_4 = costly, x_5 = large\}$. In this case, we give an (ivn-soft sets) Υ_K over U as;*

$$\begin{aligned} \Upsilon_K = \; & \{(x_1, \{\langle[0.6, 0.8], [0.8, 0.9], [0.1, 0.5]\rangle/u_1, \langle[0.5, 0.8], [0.2, 0.9], [0.1, 0.7]\rangle/u_2\}), \\ & (x_2, \{\langle[0.1, 0.4], [0.5, 0.8], [0.3, 0.7]\rangle/u_1, \langle[0.1, 0.9], [0.6, 0.9], [0.2, 0.3]\rangle/u_2\}), \\ & (x_3, \{\langle[0.2, 0.9], [0.1, 0.5], [0.7, 0.8]\rangle/u_1, \langle[0.4, 0.9], [0.1, 0.6], [0.5, 0.7]\rangle/u_2\}), \\ & (x_4, \{\langle[0.6, 0.9], [0.6, 0.9], [0.6, 0.9]\rangle/u_1, \langle[0.5, 0.9], [0.6, 0.8], [0.1, 0.8]\rangle/u_2\}), \\ & (x_5, \{\langle[0.0, 0.9], [1.0, 1.0], [1.0, 1.1]\rangle/u_1, \langle[0.0, 0.9], [0.8, 1.0], [0.2, 0.5]\rangle/u_2\})\} \end{aligned}$$

The tabular representation of the ivn-soft set Υ_K is as follow:

U	u_1	u_2
x_1	$\langle[0.6, 0.8], [0.8, 0.9], [0.1, 0.5]\rangle$	$\langle[0.5, 0.8], [0.2, 0.9], [0.1, 0.7]\rangle$
x_2	$\langle[0.1, 0.4], [0.5, 0.8], [0.3, 0.7]\rangle$	$\langle[0.1, 0.9], [0.6, 0.9], [0.2, 0.3]\rangle$
x_3	$\langle[0.2, 0.9], [0.1, 0.5], [0.7, 0.8]\rangle$	$\langle[0.4, 0.9], [0.1, 0.6], [0.5, 0.7]\rangle$
x_4	$\langle[0.6, 0.9], [0.6, 0.9], [0.6, 0.9]\rangle$	$\langle[0.5, 0.9], [0.6, 0.8], [0.1, 0.8]\rangle$
x_5	$\langle[0.0, 0.9], [1.0, 1.0], [1.0, 1.1]\rangle$	$\langle[0.0, 0.9], [0.8, 1.0], [0.2, 0.5]\rangle$

Table 1: The tabular representation of the *ivn*-soft set Υ_K

Definition 3.3 *Let $\Upsilon_K \in IVNS(U)$. If $\upsilon_K(x) = \widetilde{\emptyset}$ for all $x \in E$, then N is called an empty ivn-soft set, denoted by $\widehat{\emptyset}$.*

Definition 3.4 *Let $\Upsilon_K \in IVNS(U)$. If $\upsilon_K(x) = \widetilde{E}$ for all $x \in E$, then Υ_K is called a universal ivn-soft set, denoted by $\Upsilon_{\Upsilon_{\widehat{E}}}$.*

Example 3.5 *Assume that* $U = \{u_1, u_2\}$ *is a universal set and* $E = \{x_1, x_2, x_3, x_4, x_5\}$ *is a set of all parameters. Consider the tabular representation of the* $\Upsilon_{\widehat{\emptyset}}$ *is as follows;*

U	u_1	u_2
x_1	$\langle[0.0, 0.0], [1.0, 1.0], [1.0, 1.0]\rangle$	$\langle[0.0, 0.0], [1.0, 1.0], [1.0, 1.0]\rangle$
x_2	$\langle[0.0, 0.0], [1.0, 1.0], [1.0, 1.0]\rangle$	$\langle[0.0, 0.0], [1.0, 1.0], [1.0, 1.0]\rangle$
x_3	$\langle[0.0, 0.0], [1.0, 1.0], [1.0, 1.0]\rangle$	$\langle[0.0, 0.0], [1.0, 1.0], [1.0, 1.0]\rangle$
x_4	$\langle[0.0, 0.0], [1.0, 1.0], [1.0, 1.0]\rangle$	$\langle[0.0, 0.0], [1.0, 1.0], [1.0, 1.0]\rangle$
x_5	$\langle[0.0, 0.0], [1.0, 1.0], [1.0, 1.0]\rangle$	$\langle[0.0, 0.0], [1.0, 1.0], [1.0, 1.0]\rangle$

Table 2: The tabular representation of the *ivn*-soft set $\Upsilon_{\widehat{\emptyset}}$

The tabular representation of the $\Upsilon_{\widehat{E}}$ *is as follows;*

U	u_1	u_2
x_1	$\langle[1.0, 1.0], [0.0, 0.0], [0.0, 0.0]\rangle$	$\langle[1.0, 1.0], [0.0, 0.0], [0.0, 0.0]\rangle$
x_2	$\langle[1.0, 1.0], [0.0, 0.0], [0.0, 0.0]\rangle$	$\langle[1.0, 1.0], [0.0, 0.0], [0.0, 0.0]\rangle$
x_3	$\langle[1.0, 1.0], [0.0, 0.0], [0.0, 0.0]\rangle$	$\langle[1.0, 1.0], [0.0, 0.0], [0.0, 0.0]\rangle$
x_4	$\langle[1.0, 1.0], [0.0, 0.0], [0.0, 0.0]\rangle$	$\langle[1.0, 1.0], [0.0, 0.0], [0.0, 0.0]\rangle$
x_5	$\langle[1.0, 1.0], [0.0, 0.0], [0.0, 0.0]\rangle$	$\langle[1.0, 1.0], [0.0, 0.0], [0.0, 0.0]\rangle$

Table 3: The tabular representation of the *ivn*-soft set $\Upsilon_{\widehat{E}}$

Definition 3.6 *Let* $\Upsilon_K, \Upsilon_L \in IVNS(U)$*. Then,* Υ_K *is an ivn-soft subset of* Υ_L*, denoted by* $\Upsilon_K \widehat{\subseteq} \Upsilon_L$*, if* $\upsilon_K(x) \widetilde{\subseteq} \upsilon_L(x)$ *for all* $x \in E$*.*

Example 3.7 *Assume that* $U = \{u_1, u_2\}$ *is a universal set and* $E = \{x_1, x_2, x_3, x_4, x_5\}$ *is a set of all parameters. Consider the tabular representation of the* Υ_K *is as follows;*

U	u_1	u_2
x_1	$\langle[0.5, 0.7], [0.8, 0.9], [0.2, 0.5]\rangle$	$\langle[0.3, 0.6], [0.3, 0.9], [0.2, 0.8]\rangle$
x_2	$\langle[0.0, 0.3], [0.6, 0.8], [0.3, 0.9]\rangle$	$\langle[0.1, 0.8], [0.8, 0.9], [0.3, 0.5]\rangle$
x_3	$\langle[0.1, 0.7], [0.4, 0.5], [0.8, 0.9]\rangle$	$\langle[0.2, 0.5], [0.5, 0.7], [0.6, 0.8]\rangle$
x_4	$\langle[0.2, 0.4], [0.7, 0.9], [0.6, 0.9]\rangle$	$\langle[0.3, 0.9], [0.6, 0.9], [0.3, 0.9]\rangle$
x_5	$\langle[0.0, 0.2], [1.0, 1.0], [1.0, 1.0]\rangle$	$\langle[0.0, 0.1], [0.9, 1.0], [0.2, 0.9]\rangle$

Table 4: The tabular representation of the *ivn*-soft set Υ_K

The tabular representation of the Υ_L *is as follows;*

U	u_1	u_2
x_1	$\langle[0.6, 0.8], [0.8, 0.9], [0.1, 0.5]\rangle$	$\langle[0.5, 0.8], [0.2, 0.9], [0.1, 0.7]\rangle$
x_2	$\langle[0.1, 0.4], [0.5, 0.8], [0.3, 0.7]\rangle$	$\langle[0.1, 0.9], [0.6, 0.9], [0.2, 0.3]\rangle$
x_3	$\langle[0.2, 0.9], [0.1, 0.5], [0.7, 0.8]\rangle$	$\langle[0.4, 0.9], [0.1, 0.6], [0.5, 0.7]\rangle$
x_4	$\langle[0.6, 0.9], [0.6, 0.9], [0.6, 0.9]\rangle$	$\langle[0.5, 0.9], [0.6, 0.8], [0.1, 0.8]\rangle$
x_5	$\langle[0.0, 0.9], [1.0, 1.0], [1.0, 1.0]\rangle$	$\langle[0.0, 0.9], [0.8, 1.0], [0.2, 0.5]\rangle$

Table 5: The tabular representation of the *ivn*-soft set Υ_L

Clearly, by Definition 3.6, we have $\Upsilon_K \widehat{\subseteq} \Upsilon_L$.

Remark 3.8 $\Upsilon_K \widehat{\subseteq} \Upsilon_L$ *does not imply that every element of* Υ_K *is an element of* Υ_L *as in the definition of the classical subset.*

Proposition 3.9 *Let* $\Upsilon_K, \Upsilon_L, \Upsilon_M \in IVNS(U)$. *Then,*

1. $\Upsilon_K \widehat{\subseteq} \Upsilon_{\widehat{E}}$
2. $\Upsilon_{\widehat{\emptyset}} \widehat{\subseteq} \Upsilon_K$
3. $\Upsilon_K \widehat{\subseteq} \Upsilon_K$
4. $\Upsilon_K \widehat{\subseteq} \Upsilon_L$ *and* $\Upsilon_L \widehat{\subseteq} \Upsilon_M \Rightarrow \Upsilon_K \widehat{\subseteq} \Upsilon_M$

Proof 3.10 *They can be proved easily by using the approximate function of the ivn-soft sets.*

Definition 3.11 *Let* $\Upsilon_K, \Upsilon_L \in IVNS(U)$. *Then,* Υ_K *and* Υ_L *are ivn-soft equal, written as* $\Upsilon_K \widehat{=} \Upsilon_L$, *if and only if* $\upsilon_K(x) = \upsilon_L(x)$ *for all* $x \in E$.

Proposition 3.12 *Let* $\Upsilon_K, \Upsilon_L, \Upsilon_M \in IVNS(U)$. *Then,*

1. $\Upsilon_K \widehat{=} \Upsilon_L$ *and* $\Upsilon_L = \Upsilon_M \Leftrightarrow \Upsilon_K = \Upsilon_M$
2. $\Upsilon_K \widehat{\subseteq} \Upsilon_L$ *and* $\Upsilon_L \widehat{\subseteq} \Upsilon_K \Leftrightarrow \Upsilon_K = \Upsilon_L$

Proof 3.13 *The proofs are trivial.*

Definition 3.14 *Let* $\Upsilon_K \in IVNS(U)$. *Then, the complement* $\Upsilon_K^{\widehat{c}}$ *of* Υ_K *is an ivn-soft set such that*

$$\upsilon_K^{\widehat{c}}(x) = \overline{\upsilon_K}(x), \text{ for all } x \in E.$$

Example 3.15 *Consider the above Example 3.7, the complement* $\Upsilon_L^{\widehat{c}}$ *of* Υ_L *can be represented into the following table;*

U	u_1	u_2
x_1	$\langle [0.1, 0.5], [0.1, 0.2], [0.6, 0.8] \rangle$	$\langle [0.1, 0.7], [0.1, 0.8], [0.5, 0.8] \rangle$
x_2	$\langle [0.3, 0.7], [0.2, 0.5], [0.1, 0.4] \rangle$	$\langle [0.2, 0.3], [0.1, 0.4], [0.1, 0.9] \rangle$
x_3	$\langle [0.7, 0.8], [0.5, 0.9], [0.2, 0.9] \rangle$	$\langle [0.5, 0.7], [0.4, 0.9], [0.4, 0.9] \rangle$
x_4	$\langle [0.6, 0.9], [0.1, 0.4], [0.6, 0.9] \rangle$	$\langle [0.1, 0.8], [0.2, 0.4], [0.5, 0.9] \rangle$
x_5	$\langle [1.0, 1.0], [0.0, 0.0], [0.0, 0.9] \rangle$	$\langle [0.2, 0.5], [0.0, 0.2], [0.0, 0.9] \rangle$

Table 6: The tabular representation of the *ivn*-soft set $\Upsilon_L^{\widehat{c}}$

Proposition 3.16 *Let* $\Upsilon_K \in IVNS(U)$. *Then,*

1. $(\Upsilon_K^{\widehat{c}})^{\widehat{c}} = \Upsilon_K$

2. $\Upsilon_{\widehat{\emptyset}}^{\widehat{c}} = \Upsilon_{\widehat{E}}$

3. $\Upsilon_{\widehat{E}}^{\widehat{c}} = \Upsilon_{\widehat{\emptyset}}$

Proof 3.17 *By using the fuzzy approximate functions of the ivn-soft set, the proofs can be straightforward.*

Theorem 3.18 *Let $\Upsilon_K \in IVNS(U)$. Then, $\Upsilon_K \widehat{\subseteq} \Upsilon_L \Leftrightarrow \Upsilon_L^{\widehat{c}} \widehat{\subseteq} \Upsilon_K^{\widehat{c}}$*

Proof 3.19 *By using the fuzzy approximate functions of the ivn-soft set, the proofs can be straightforward.*

Definition 3.20 *Let $\Upsilon_K, \Upsilon_L \in IVNS(U)$. Then, union of Υ_K and Υ_L, denoted $\Upsilon_K \widehat{\cup} \Upsilon_L$, is defined by*

$$\upsilon_{K\widehat{\cup}L}(x) = \upsilon_K(x) \widetilde{\cup} \upsilon_L(x) \quad \text{for all } x \in E.$$

Example 3.21 *Consider the above Example 3.7, the union of Υ_K and Υ_L, denoted $\Upsilon_K \widehat{\cup} \Upsilon_L$, can be represented into the following table;*

U	u_1	u_2
x_1	$\langle[0.6, 0.8], [0.8, 0.9], [0.1, 0.5]\rangle$	$\langle[0.5, 0.8], [0.2, 0.9], [0.1, 0.7]\rangle$
x_2	$\langle[0.1, 0.4], [0.5, 0.8], [0.3, 0.7]\rangle$	$\langle[0.1, 0.9], [0.6, 0.9], [0.2, 0.3]\rangle$
x_3	$\langle[0.2, 0.9], [0.1, 0.5], [0.7, 0.8]\rangle$	$\langle[0.4, 0.9], [0.1, 0.6], [0.5, 0.7]\rangle$
x_4	$\langle[0.6, 0.9], [0.6, 0.9], [0.6, 0.9]\rangle$	$\langle[0.5, 0.9], [0.6, 0.8], [0.1, 0.8]\rangle$
x_5	$\langle[0.0, 0.9], [1.0, 1.0], [1.0, 1.0]\rangle$	$\langle[0.0, 0.9], [0.8, 1.0], [0.2, 0.5]\rangle$

Table 7: The tabular representation of the *ivn*-soft set $\Upsilon_K \widehat{\cup} \Upsilon_L$

Theorem 3.22 *Let $\Upsilon_K, \Upsilon_L \in IVNS(U)$. Then, $\Upsilon_K \widehat{\cup} \Upsilon_L$ is the smallest ivn$-$soft set containing both Υ_K and Υ_L.*

Proof 3.23 *The proofs can be easily obtained from Definition 3.20.*

Proposition 3.24 *Let $\Upsilon_K, \Upsilon_L, \Upsilon_M \in IVNS(U)$. Then,*

1. $\Upsilon_K \widehat{\cup} \Upsilon_K = \Upsilon_K$

2. $\Upsilon_K \widehat{\cup} \Upsilon_{\widehat{\emptyset}} = \Upsilon_K$

3. $\Upsilon_K \widehat{\cup} \Upsilon_{\widehat{E}} = \Upsilon_{\widehat{E}}$

4. $\Upsilon_K \widehat{\cup} \Upsilon_L = \Upsilon_L \widehat{\cup} \Upsilon_K$

5. $(\Upsilon_K \widehat{\cup} \Upsilon_L) \widehat{\cup} \Upsilon_M = \Upsilon_K \widehat{\cup} (\Upsilon_L \widehat{\cup} \Upsilon_M)$

Proof 3.25 *The proofs can be easily obtained from Definition 3.20.*

Definition 3.26 *Let $\Upsilon_K, \Upsilon_L \in IVNS(U)$. Then, intersection of Υ_K and Υ_L, denoted $\Upsilon_K \widehat{\cap} \Upsilon_L$, is defined by*

$$\upsilon_{K\widehat{\cap}L}(x) = \upsilon_K(x) \widetilde{\cap} \upsilon_L(x) \quad \text{for all } x \in E.$$

Example 3.27 *Consider the above Example 3.7, the intersection of Υ_K and Υ_L, denoted $\Upsilon_K \widehat{\cap} \Upsilon_L$, can be represented into the following table;*

U	u_1	u_2
x_1	$\langle [0.5, 0.7], [0.8, 0.9], [0.2, 0.5] \rangle$	$\langle [0.3, 0.6], [0.3, 0.9], [0.2, 0.8] \rangle$
x_2	$\langle [0.0, 0.3], [0.6, 0.8], [0.3, 0.9] \rangle$	$\langle [0.1, 0.8], [0.8, 0.9], [0.3, 0.5] \rangle$
x_3	$\langle [0.1, 0.7], [0.4, 0.5], [0.8, 0.9] \rangle$	$\langle [0.2, 0.5], [0.5, 0.7], [0.6, 0.8] \rangle$
x_4	$\langle [0.2, 0.4], [0.7, 0.9], [0.6, 0.9] \rangle$	$\langle [0.3, 0.9], [0.6, 0.9], [0.3, 0.9] \rangle$
x_5	$\langle [0.0, 0.2], [1.0, 1.0], [1.0, 1.0] \rangle$	$\langle [0.0, 0.1], [0.9, 1.0], [0.2, 0.9] \rangle$

Table 8: The tabular representation of the *ivn*-soft set $\Upsilon_K \widehat{\cap} \Upsilon_L$

Proposition 3.28 *Let $\Upsilon_K, \Upsilon_L \in IVNS(U)$. Then, $\Upsilon_K \widehat{\cap} \Upsilon_L$ is the largest ivn− soft set containing both Υ_K and Υ_L.*

Proof 3.29 *The proofs can be easily obtained from Definition 3.26.*

Proposition 3.30 *Let $\Upsilon_K, \Upsilon_L, \Upsilon_M \in IVNS(U)$. Then,*

1. $\Upsilon_K \widehat{\cap} \Upsilon_K = \Upsilon_K$
2. $\Upsilon_K \widehat{\cap} \Upsilon_{\widehat{\emptyset}} = \Upsilon_{\widehat{\emptyset}}$
3. $\Upsilon_K \widehat{\cap} \Upsilon_{\widehat{E}} = \Upsilon_K$
4. $\Upsilon_K \widehat{\cap} \Upsilon_L = \Upsilon_L \widehat{\cap} \Upsilon_K$
5. $(\Upsilon_K \widehat{\cap} \Upsilon_L) \widehat{\cap} \Upsilon_M = \Upsilon_K \widehat{\cap} (\Upsilon_L \widehat{\cap} \Upsilon_M)$

Proof 3.31 *The proof of the Propositions 1- 5 are obvious.*

Remark 3.32 *Let $\Upsilon_K \in IVNS(U)$. If $\Upsilon_K \neq \Upsilon_{\widehat{\emptyset}}$ or $\Upsilon_K \neq \Upsilon_{\widehat{E}}$, then $\Upsilon_K \widehat{\cup} \Upsilon_K^{\widehat{c}} \neq \Upsilon_{\widehat{E}}$ and $\Upsilon_K \widehat{\cap} \Upsilon_K^{\widehat{c}} \neq \Upsilon_{\widehat{\emptyset}}$.*

Proposition 3.33 *Let $\Upsilon_K, \Upsilon_L \in IVNS(U)$. Then, De Morgan's laws are valid*

1. $(\Upsilon_K \widehat{\cup} \Upsilon_L)^{\widehat{c}} = \Upsilon_K^{\widehat{c}} \widehat{\cap} \Upsilon_L^{\widehat{c}}$
2. $(\Upsilon_K \widehat{\cap} \Upsilon_L)^{\widehat{c}} = \Upsilon_K^{\widehat{c}} \widehat{\cup} \Upsilon_L^{\widehat{c}}$.

Proof 3.34 *The proofs can be easily obtained from Definition 3.14, Definition 3.20 and Definition 3.26.*

Proposition 3.35 *Let $\Upsilon_K, \Upsilon_L, \Upsilon_M \in IVNS(U)$. Then,*

1. $\Upsilon_K \widehat{\cup} (\Upsilon_L \widehat{\cap} \Upsilon_M) = (\Upsilon_K \widehat{\cup} \Upsilon_L) \widehat{\cap} (\Upsilon_K \widehat{\cup} \Upsilon_M)$
2. $\Upsilon_K \widehat{\cap} (\Upsilon_L \widehat{\cup} \Upsilon_M) = (\Upsilon_K \widehat{\cap} \Upsilon_L) \widehat{\cup} (\Upsilon_K \widehat{\cap} \Upsilon_M)$
3. $\Upsilon_K \widehat{\cup} (\Upsilon_K \widehat{\cap} \Upsilon_L) = \Upsilon_K$
4. $\Upsilon_K \widehat{\cap} (\Upsilon_K \widehat{\cup} \Upsilon_L) = \Upsilon_K$

Proof 3.36 *The proofs can be easily obtained from Definition 3.20 and Definition 3.26.*

Definition 3.37 *Let $\Upsilon_K, \Upsilon_L \in IVNS(U)$. Then, OR operator of Υ_K and Υ_L, denoted $\Upsilon_K \widehat{\bigvee} \Upsilon_L$, is defined by a set valued function Υ_O representing a mapping*

$$\upsilon_O : E \times E \to IVN(U)$$

where

$$\upsilon_O(x, y) = \upsilon_K(x) \widetilde{\cup} \upsilon_L(y) \quad \text{for all } (x, y) \in E \times E.$$

Definition 3.38 *Let $\Upsilon_K, \Upsilon_L \in IVNS(U)$. Then, AND operator of Υ_K and Υ_L, denoted $\Upsilon_K \widehat{\bigwedge} \Upsilon_L$, is defined by is defined by a set valued function Υ_A representing a mapping*

$$\upsilon_A : E \times E \to IVN(U)$$

where

$$\upsilon_A(x, y) = \upsilon_K(x) \widetilde{\cap} \upsilon_L(y) \quad \text{for all } (x, y) \in E \times E.$$

Proposition 3.39 *Let $\Upsilon_K, \Upsilon_L, \Upsilon_M \in IVNS(U)$. Then,*

1. $(\Upsilon_K \widehat{\bigvee} \Upsilon_L)^{\widehat{c}} = \Upsilon_K^{\widehat{c}} \widehat{\bigwedge} \Upsilon_L^{\widehat{c}}$
2. $(\Upsilon_K \widehat{\bigwedge} \Upsilon_L)^{\widehat{c}} = \Upsilon_K^{\widehat{c}} \widehat{\bigvee} \Upsilon_L^{\widehat{c}}$.
3. $(\Upsilon_K \widehat{\bigvee} \Upsilon_L) \widehat{\bigvee} \Upsilon_M = \Upsilon_K \widehat{\bigvee} (\Upsilon_L \widehat{\bigvee} \Upsilon_M)$
4. $(\Upsilon_K \widehat{\bigwedge} \Upsilon_L) \widehat{\bigwedge} \Upsilon_M = \Upsilon_K \widehat{\bigwedge} (\Upsilon_L \widehat{\bigwedge} \Upsilon_M)$

Proof 3.40 *The proof of the Propositions 1- 4 are obvious.*

Definition 3.41 *Let $\Upsilon_K, \Upsilon_L \in IVNS(U)$. Then, difference of Υ_K and Υ_L, denoted $\Upsilon_K \widehat{\setminus} \Upsilon_L$, is defined by*

$$\upsilon_{K \widehat{\setminus} L}(x) = \upsilon_K(x) \widetilde{\setminus} \upsilon_L(x) \quad \text{for all } x \in E.$$

Definition 3.42 *Let $\Upsilon_K, \Upsilon_L \in IVNS(U)$. Then, addition of Υ_K and Υ_L, denoted $\Upsilon_K \widehat{+} \Upsilon_L$, is defined by*

$$\upsilon_{K \widehat{+} L}(x) = \upsilon_K(x) \widetilde{+} \upsilon_L(x) \quad \text{for all } x \in E.$$

Proposition 3.43 *Let $\Upsilon_K, \Upsilon_L, \Upsilon_M \in IVNS(U)$. Then,*

1. $\Upsilon_K(x)\widehat{+}\Upsilon_L(x)\widehat{=}\Upsilon_L(x)\widehat{+}\Upsilon_K(x)$
2. $(\Upsilon_K(x)\widehat{+}\Upsilon_L(x))\widehat{+}\Upsilon_M(x) = \Upsilon_K(x)\widehat{+}(\Upsilon_L(x)\widehat{+}\Upsilon_M(x))$

Proof 3.44 *The proofs can be easily obtained from Definition 3.42.*

Definition 3.45 *Let $\Upsilon_K \in IVNS(U)$. Then, scalar multiplication of Υ_K, denoted $a\widehat{\times}\Upsilon_K$, is defined by*

$$a\widehat{\times}\Upsilon_K = a\tilde{.}\upsilon_K \quad \text{for all } x \in E.$$

Proposition 3.46 *Let $\Upsilon_K, \Upsilon_L, \Upsilon_M \in IVNS(U)$. Then,*

1. $\Upsilon_K(x)\widehat{\times}\Upsilon_L(x)=\Upsilon_L(x)\widehat{\times}\Upsilon_K(x)$
2. $(\Upsilon_K(x)\widehat{\times}\Upsilon_L(x))\widehat{\times}\Upsilon_M(x) = \Upsilon_K(x)\widehat{\times}(\Upsilon_L(x)\widehat{\times}\Upsilon_M(x))$

Proof 3.47 *The proofs can be easily obtained from Definition 3.45.*

Definition 3.48 *Let $\Upsilon_K \in IVNS(U)$. Then, scalar division of Υ_K, denoted $\Upsilon_K\widehat{/}a$, is defined by*

$$\Upsilon_K\widehat{/}a = \Upsilon_K\widetilde{/}a \quad \text{for all } x \in E.$$

Example 3.49 *Consider the above Example 3.7, for $a = 5$, the scalar division of Υ_K, denoted $\Upsilon_K\widehat{/}5$, can be represented into the following table;*

U	u_1	u_2
x_1	$\langle[0.1, 0.14], [0.16, 0.18], [0.04, 0.1]\rangle$	$\langle[0.06, 0.12], [0.15, 0.18], [0.04, 0.16]\rangle$
x_2	$\langle[0.0, 0.06], [0.12, 0.16], [0.16, 0.18]\rangle$	$\langle[0.02, 0.16], [0.16, 0.18], [0.15, 0.25]\rangle$
x_3	$\langle[0.02, 0.14], [0.08, 0.1], [0.16, 0.18]\rangle$	$\langle[0.04, 0.1], [0.1, 0.14], [0.12, 0.16]\rangle$
x_4	$\langle[0.04, 0.08], [0.14, 0.18], [0.12, 0.18]\rangle$	$\langle[0.15, 0.18], [0.12, 0.18], [0.06, 0.18]\rangle$
x_5	$\langle[0.0, 0.04], [0.2, 0.2], [0.2, 0.2]\rangle$	$\langle[0.0, 0.05], [0.18, 0.2], [0.04, 0.18]\rangle$

Table 9: The tabular representation of the *ivn*-soft set $\Upsilon_K\widehat{/}5$

Definition 3.50 *Let $\Upsilon_K \in IVNS(U)$. Then, truth-Favorite of Υ_K, denoted $\widehat{\triangle}\Upsilon_K$, is defined by*

$$\widehat{\triangle}\Upsilon_K = \widetilde{\triangle}\upsilon_K \quad \text{for all } x \in E.$$

Example 3.51 *Consider the above Example 3.7, the truth-Favorite of Υ_K, denoted $\widehat{\triangle}\Upsilon_K$, can be represented into the following table;*

U	u_1	u_2
x_1	$\langle[1.0, 1.0], [0.0, 0.0], [0.2, 0.5]\rangle$	$\langle[0.6, 1.0], [0.0, 0.0], [0.2, 0.8]\rangle$
x_2	$\langle[0.6, 1.0], [0.0, 0.0], [0.3, 0.9]\rangle$	$\langle[0.9, 1.0], [0.0, 0.0], [0.3, 0.5]\rangle$
x_3	$\langle[0.5, 1.0], [0.0, 0.0], [0.8, 0.9]\rangle$	$\langle[0.7, 1.0], [0.0, 0.0], [0.6, 0.8]\rangle$
x_4	$\langle[0.9, 1.0], [0.0, 0.0], [0.6, 0.9]\rangle$	$\langle[0.9, 1.0], [0.0, 0.0], [0.3, 0.9]\rangle$
x_5	$\langle[1.0, 1.0], [0.0, 0.0], [1.0, 1.0]\rangle$	$\langle[0.9, 1.0], [0.0, 0.0], [0.2, 0.9]\rangle$

Table 10: The tabular representation of the *ivn*-soft set $\widehat{\triangle}\Upsilon_K$

Proposition 3.52 *Let* $\Upsilon_K, \Upsilon_L \in IVNS(U)$*. Then,*

1. $\widehat{\triangle}\widehat{\triangle}\Upsilon_K = \widehat{\triangle}\Upsilon_K$
2. $\widehat{\triangle}(\Upsilon_K \widehat{\cup} \Upsilon_K) \widehat{\subseteq} \widehat{\triangle}\Upsilon_K \widehat{\cup} \widehat{\triangle}\Upsilon_K$
3. $\widehat{\triangle}(\Upsilon_K \widehat{\cap} \Upsilon_K) \widehat{\subseteq} \widehat{\triangle}\Upsilon_K \widehat{\cap} \widehat{\triangle}\Upsilon_K$
4. $\widehat{\triangle}(\Upsilon_K \widehat{+} \Upsilon_K) = \widehat{\triangle}\Upsilon_K \widehat{+} \widehat{\triangle}\Upsilon_K$

Proof 3.53 *The proofs can be easily obtained from Definition 3.20, Definition 3.26 and Definition 3.50.*

Definition 3.54 *Let* $\Upsilon_K \in IVNS(U)$*. Then, False-Favorite of* Υ_K*, denoted* $\widehat{\triangledown}\Upsilon_K$*, is defined by*

$$\widehat{\triangledown}\Upsilon_K = \widetilde{\triangledown}\upsilon_K \quad \textit{for all } x \in E.$$

Example 3.55 *Consider the above Example 3.7, the False-Favorite of* Υ_K*, denoted* $\widehat{\triangledown}\Upsilon_K$*, can be represented into the following table;*

U	u_1	u_2
x_1	$\langle[0.5, 0.7], [0.0, 0.0], [1.0, 1.0]\rangle$	$\langle[0.3, 0.6], [0.0, 0.0], [0.5, 1.0]\rangle$
x_2	$\langle[0.0, 0.3], [0.0, 0.0], [0.9, 1.0]\rangle$	$\langle[0.1, 0.8], [0.0, 0.0], [1.0, 1.0]\rangle$
x_3	$\langle[0.1, 0.7], [0.0, 0.0], [1.0, 1.0]\rangle$	$\langle[0.2, 0.5], [0.0, 0.0], [1.0, 1.0]\rangle$
x_4	$\langle[0.2, 0.4], [0.0, 0.0], [1.0, 1.0]\rangle$	$\langle[0.3, 0.9], [0.0, 0.0], [0.9, 1.0]\rangle$
x_5	$\langle[0.0, 0.2], [0.0, 0.0], [1.0, 1.0]\rangle$	$\langle[0.0, 0.1], [0.0, 0.0], [1.0, 1.0]\rangle$

Table 11: The tabular representation of the *ivn*-soft set $\widehat{\triangledown}\Upsilon_K$

Proposition 3.56 *Let* $\Upsilon_K, \Upsilon_L \in IVNS(U)$*. Then,*

1. $\widehat{\triangledown}\widehat{\triangledown}\Upsilon_K \widehat{=} \widehat{\triangledown}\Upsilon_K$
2. $\widehat{\triangledown}(\Upsilon_K \widehat{\cup} \Upsilon_K) \widehat{\subseteq} \widehat{\triangledown}\Upsilon_K \widehat{\cup} \widehat{\triangledown}\Upsilon_K$
3. $\widehat{\triangledown}(\Upsilon_K \widehat{\cap} \Upsilon_K) \widehat{\subseteq} \widehat{\triangledown}\Upsilon_K \widehat{\cap} \widehat{\triangledown}\Upsilon_K$
4. $\widehat{\triangledown}(\Upsilon_K \widehat{+} \Upsilon_K) = \widehat{\triangledown}\Upsilon_K \widehat{+} \widehat{\triangledown}\Upsilon_K$

Proof 3.57 *The proofs can be easily obtained from Definition 3.20, Definition 3.26 and Definition 3.54.*

Theorem 3.58 *Let* P *be the power set of all ivn− soft sets defined in the universe U. Then* $(P, \widehat{\cap}, \widehat{\cup})$ *is a distributive lattice.*

Proof 3.59 *The proofs can be easily obtained by showing properties; idempotency, commutativity, associativity and distributivity*

4 $ivn-$soft set based decision making

In this section, we present an adjustable approach to $ivn-$soft set based decision making problems by extending the approach to interval-valued intuitionistic fuzzy soft set based decision making. [40]. Some of it is quoted from [20, 23, 36, 40].

Definition 4.1 *Let $\Upsilon_K \in IVNS(U)$. Then a* relation form *of Υ_K is defined by*

$$R_{\Upsilon_K} = \{(r_{\Upsilon_K}(x,u)/(x,u)) : r_{\Upsilon_K}(x,u) \in IVN(U), x \in E, u \in U\}$$

where

$r_{\Upsilon_K} : E \times U \to IVN(U)$ *and* $r_{\Upsilon_K}(x,u) = \upsilon_{K(x)}(u)$ *for all* $x \in E$ *and* $u \in U$.

That is, $r_{\Upsilon_K}(x,u) = \upsilon_{K(x)}(u)$ *is characterized by truth-membership function* T_K*, a indeterminacy-membership function* I_K *and a falsity-membership function* F_K*. For each point* $x \in E$ *and* $u \in U$*;* T_K*,* I_K *and* $F_K \subseteq [0,1]$.

Example 4.2 *Consider the above Example 3.7, then,* $r_{\Upsilon_K}(x,u) = \upsilon_{K(x)}(u)$ *can be given as follows*

$\upsilon_{K(x_1)}(u_1) = \langle[0.6,0.8],[0.8,0.9],[0.1,0.5]\rangle,$
$\upsilon_{K(x_1)}(u_2) = \langle[0.5,0.8],[0.2,0.9],[0.1,0.7]\rangle,$
$\upsilon_{K(x_2)}(u_1) = \langle[0.1,0.4],[0.5,0.8],[0.3,0.7]\rangle,$
$\upsilon_{K(x_2)}(u_1) = \langle[0.1,0.9],[0.6,0.9],[0.2,0.3]\rangle,$
$\upsilon_{K(x_3)}(u_1) = \langle[0.2,0.9],[0.1,0.5],[0.7,0.8]\rangle,$
$\upsilon_{K(x_3)}(u_2) = \langle[0.4,0.9],[0.1,0.6],[0.5,0.7]\rangle,$
$\upsilon_{K(x_4)}(u_1) = \langle[0.6,0.9],[0.6,0.9],[0.6,0.9]\rangle,$
$\upsilon_{K(x_4)}(u_2) = \langle[0.5,0.9],[0.6,0.8],[0.1,0.8]\rangle,$
$\upsilon_{K(x_5)}(u_1) = \langle[0.0,0.9],[1.0,1.0],[1.0,1.0]\rangle,$
$\upsilon_{K(x_5)}(u_2) = \langle[0.0,0.9],[0.8,1.0],[0.2,0.5]\rangle.$

Zhang et al.[40] introduced level-soft set and different thresholds on different parameters in interval-valued intuitionistic fuzzy soft sets. Taking inspiration these definitions we give level-soft set and different thresholds on different parameters in $ivn-$soft sets.

Definition 4.3 *Let $\Upsilon_K \in IVNS(U)$. For $\alpha,\beta,\gamma \subseteq [0,1]$, the (α,β,γ)-level soft set of Υ_K is a crisp soft set, denoted $(\Upsilon_K; <\alpha,\beta,\gamma>)$, defined by*

$$(\Upsilon_K; <\alpha,\beta,\gamma>) = \{(x_i, \{u_{ij} : u_{ij} \in U, \mu(u_{ij}) = 1\}) : x_i \in E\}$$

where,

$$\mu(u_{ij}) = \begin{cases} 1, & (\alpha,\beta,\gamma)\widehat{\leq}\upsilon_{K(x_i)}(u_j) \\ 0, & others \end{cases}$$

for all $u_j \in U$.

Obviously, the definition is an extension of level soft sets of interval-valued intuitionistic fuzzy soft sets [40].

Remark 4.4 *In Definition 4.3,* $\alpha = (\alpha_1, \alpha_2) \subseteq [0, 1]$ *can be viewed as a given least threshold on degrees of truth-membership,* $\beta = (\beta_1, \beta_2) \subseteq [0, 1]$ *can be viewed as a given greatest threshold on degrees of indeterminacy-membership and* $\gamma = (\gamma_1, \gamma_2) \subseteq [0, 1]$ *can be viewed as a given greatest threshold on degrees of falsity-membership. If* $(\alpha, \beta, \gamma)\widehat{\leq}\upsilon_{K(x_i)}(u)$*, it shows that the degree of the truth-membership of* u *with respect to the parameter* x *is not less than* α*, the degree of the indeterminacy-membership of* u *with respect to the parameter* x *is not more than* γ *and the degree of the falsity-membership of* u *with respect to the parameter* x *is not more than* β*. In practical applications of inv−soft sets, the thresholds* α*,* β*,* γ *are pre-established by decision makers and reflect decision makers requirements on truth-membership levels, indeterminacy-membership levels and falsity-membership levels, respectively.*

Example 4.5 *Consider the above Example 3.7.*

Clearly the $([0.3, 0.4], [0.3, 0.5], [0.1, 0.2])$*-level soft set of* Υ_K *as follows*

$$(\Upsilon_K; < [0.3, 0.4], [0.3, 0.5], [0.1, 0.2] >) = \{(x_1, \{u_1\}), (x_4, \{u_1, u_2\})\}$$

Note 4.6 *In some practical applications the thresholds* α, β, γ *decision makers need to impose different thresholds on different parameters. To cope with such problems, we replace a constant value the thresholds by a function as the thresholds on truth-membership values, indeterminacy-membership values and falsity-membership values, respectively.*

Theorem 4.7 *Let* $\Upsilon_K, \Upsilon_L \in IVNS(U)$*. Then,*

1. $(\Upsilon_K; < \alpha_1, \beta_1, \gamma_1 >)$ *and* $(\Upsilon_K; < \alpha_2, \beta_2, \gamma_2 >)$ *are* $< \alpha_1, \beta_1, \gamma_1 >$*-level soft set and* $< \alpha_2, \beta_2, \gamma_2 >$*-level soft set of* Υ_K*, respectively.*
   If $< \alpha_2, \beta_2, \gamma_2 > \widehat{\leq} < \alpha_1, \beta_1, \gamma_1 >$*, then we have*
   $(\Upsilon_K; < \alpha_1, \beta_1, \gamma_1 >)\tilde{\subseteq}(\Upsilon_K; < \alpha_2, \beta_2, \gamma_2 >)$*.*

2. $(\Upsilon_K; < \alpha, \beta, \gamma >)$ *and* $(\Upsilon_L; < \alpha, \beta, \gamma >)$ *are* $< \alpha, \beta, \gamma$*-level soft set* Υ_K *and* Υ_L*, respectively.*
   If $\Upsilon_K \widehat{\subseteq} \Upsilon_L$*, then we have* $(\Upsilon_K; < \alpha, \beta, \gamma >)\tilde{\subseteq}(\Upsilon_L; < \alpha, \beta, \gamma >)$*.*

Proof 4.8 *The proof of the theorems are obvious.*

Definition 4.9 *Let* $\Upsilon_K \in IVNS(U)$*. Let an interval-valued neutrosophic set* $< \alpha, \beta, \gamma >_{\Upsilon_K}: A \to IVN(U)$ *in* U *which is called a threshold interval-valued neutrosophic set. The level soft set of* Υ_K *with respect to* $< \alpha, \beta, \gamma >_{\Upsilon_K}$ *is a crisp soft set, denoted by* $(\Upsilon_K; < \alpha, \beta, \gamma >_{\Upsilon_K})$*, defined by;*

$$(\Upsilon_K; < \alpha, \beta, \gamma >_{\Upsilon_K}) = \{(x_i, \{u_{ij} : u_{ij} \in U, \mu(u_{ij}) = 1\}) : x_i \in E\}$$

where,

$$\mu(u_{ij}) = \begin{cases} 1, & < \alpha, \beta, \gamma >_{\Upsilon_K}(x_i)\widehat{\leq}\upsilon_{K(x_i)}(u_j) \\ 0, & others \end{cases}$$

for all $u_j \in U$*.*

Obviously, the definition is an extension of level soft sets of interval-valued intuitionistic fuzzy soft sets [40].

Remark 4.10 *In Definition 4.9,* $\alpha = (\alpha_1, \alpha_2) \subseteq [0,1]$ *can be viewed as a given least threshold on degrees of truth-membership,* $\beta = (\beta_1, \beta_2) \subseteq [0,1]$ *can be viewed as a given greatest threshold on degrees of indeterminacy-membership and* $\gamma = (\gamma_1, \gamma_2) \subseteq [0,1]$ *can be viewed as a given greatest threshold on degrees of falsity-membership of* u *with respect to the parameter* x.

If $< \alpha, \beta, \gamma >_{\Upsilon_K}(x_i) \widehat{\leq} \upsilon_{K(x_i)}(u)$ *it shows that the degree of the truth-membership of* u *with respect to the parameter* x *is not less than* α*, the degree of the indeterminacy-membership of* u *with respect to the parameter* x *is not more than* γ *and the degree of the falsity-membership of* u *with respect to the parameter* x *is not more than* β.

Definition 4.11 *Let* $\Upsilon_K \in IVNS(U)$*. Based on* Υ_K*, we can define an interval-valued neutrosophic set* $< \alpha, \beta, \gamma >^{avg}_{\Upsilon_K}: A \to IVN(U)$ *by*

$$< \alpha, \beta, \gamma >^{avg}_{\Upsilon_K}(x_i) = \sum_{u \in U} \upsilon_{K(x_i)}(u)/|U|$$

for all $x \in E$.

The interval-valued neutrosophic set $< \alpha, \beta, \gamma >^{avg}_{\Upsilon_K}$ is called the avg-threshold of the $ivn-$soft set Υ_K. In the following discussions, the avg-level decision rule will mean using the avg-threshold and considering the avg-level soft set in $ivn-$soft sets based decision making.

Let us reconsider the $ivn-$soft set Υ_K in Example 3.7. The avg-threshold $< \alpha, \beta, \gamma >^{avg}_{\Upsilon_K}$ of Υ_K is an interval-valued neutrosophic set and can be calculated as follows:

$$< \alpha, \beta, \gamma >^{avg}_{\Upsilon_K}(x_1) = \sum_{i=1}^{2} \upsilon_{K(x_1)}(u_i)/|U| = \langle [0.55, 0.8], [0.5, 0.9], [0.1, 0.6] \rangle$$

$$< \alpha, \beta, \gamma >^{avg}_{\Upsilon_K}(x_2) = \sum_{i=1}^{2} \upsilon_{K(x_2)}(u_i)/|U| = \langle [0.1, 0.65], [0.55, 0.85], [0.25, 0.5] \rangle$$

$$< \alpha, \beta, \gamma >^{avg}_{\Upsilon_K}(x_3) = \sum_{i=1}^{2} \upsilon_{K(x_3)}(u_i)/|U| = \langle [0.15, 0.9], [0.1, 0.55], [0.6, 0.75] \rangle$$

$$< \alpha, \beta, \gamma >^{avg}_{\Upsilon_K}(x_4) = \sum_{i=1}^{2} \upsilon_{K(x_4)}(u_i)/|U| = \langle [0.55, 0.9], [0.6, 0.85], [0.35, 0.85] \rangle$$

$$< \alpha, \beta, \gamma >^{avg}_{\Upsilon_K}(x_5) = \sum_{i=1}^{2} \upsilon_{K(x_5)}(u_i)/|U| = \langle [0.0, 0.9], [0.9, 1.0], [0.6, 0.75] \rangle$$

Therefore, we have

$$< \alpha, \beta, \gamma >^{avg}_{\Upsilon_K} = \{\langle[0.55, 0.8], [0.5, 0.9], [0.1, 0.6]\rangle/x_1, \langle[0.1, 0.65], [0.55, 0.85], [0.25, 0.5]\rangle/x_2, \langle[0.15, 0.9], [0.1, 0.55], [0.6, 0.75]\rangle/x_3, \langle[0.55, 0.9], [0.6, 0.85], [0.35, 0.85]\rangle/x_4, \langle[0.0, 0.9], [0.9, 1.0], [0.6, 0.75]\rangle/x_5\}$$

Example 4.12 *Consider the above Example 3.7. Clearly;*

$$(\Upsilon_K; < \alpha, \beta, \gamma >^{avg}_{\Upsilon_K}) = \{(x_5, \{u_2\})\}$$

Definition 4.13 *Let $\Upsilon_K \in IVNS(U)$. Based on Υ_K, we can define an interval-valued neutrosophic set $< \alpha, \beta, \gamma >^{Mmm}_{\Upsilon_K}: A \to IVN(U)$ by*

$$< \alpha, \beta, \gamma >^{Mmm}_{\Upsilon_K} = \{< [max_{u\in U}\{infT_{\upsilon_{K(x_i)}(u)}\}, max_{u\in U}\{supT_{\upsilon_{K(x_i)}(u)}\}], [min_{u\in U}\{infI_{\upsilon_{K(x_i)}(u)}\}, min_{u\in U}\{supI_{\upsilon_{K(x_i)}(u)}\}], [min_{u\in U}\{infF_{\upsilon_{K(x_i)}(u)}\}, min_{u\in U}\{supF_{\upsilon_{K(x_i)}(u)}\}] > /x_i : x_i \in E\}$$

The interval-valued neutrosophic set $< \alpha, \beta, \gamma >^{Mmm}_{\Upsilon_K}$ is called the max-min-min-threshold of the $ivn-$soft set Υ_K. In what follows the Mmm-level decision rule will mean using the max-min-min-threshold and considering the Mmm-level soft set in $ivn-$soft sets based decision making.

Definition 4.14 *Let $\Upsilon_K \in IVNS(U)$. Based on Υ_K, we can define an interval-valued neutrosophic set $< \alpha, \beta, \gamma >^{mmm}_{\Upsilon_K}: A \to IVN(U)$ by*

$$< \alpha, \beta, \gamma >^{mmm}_{\Upsilon_K} = \{< [min_{u\in U}\{infT_{\upsilon_{K(x_i)}(u)}\}, min_{u\in U}\{supT_{\upsilon_{K(x_i)}(u)}\}], [min_{u\in U}\{infI_{\upsilon_{K(x_i)}(u)}\}, min_{u\in U}\{supI_{\upsilon_{K(x_i)}(u)}\}], [min_{u\in U}\{infF_{\upsilon_{K(x_i)}(u)}\}, min_{u\in U}\{supF_{\upsilon_{K(x_i)}(u)}\}] > /x_i : x_i \in E\}$$

The interval-valued neutrosophic set $< \alpha, \beta, \gamma >^{mmm}_{\Upsilon_K}$ is called the min-min-min-threshold of the $ivn-$soft set Υ_K. In what follows the mmm-level decision rule will mean using the min-min-min-threshold and considering the mmm-level soft set in $ivn-$soft sets based decision making.

Theorem 4.15 *Let $\Upsilon_K \in IVNS(U)$. Then, $(\Upsilon_K; < \alpha, \beta, \gamma >^{avg}_{\Upsilon_K})$, $(\Upsilon_K; < \alpha, \beta, \gamma >^{Mmm}_{\Upsilon_K}), (\Upsilon_K; < \alpha, \beta, \gamma >^{mmm}_{\Upsilon_K}), (\Upsilon_K; < \alpha, \beta, \gamma >^{MMM}_{\Upsilon_K}))$ are the avg-level soft set, Mmm-level soft set, mmm-level soft set, MMM-level soft set of $\Upsilon_K \in IVNS(U)$, respectively. Then,*

1. $(\Upsilon_K; < \alpha, \beta, \gamma >^{Mmm}_{\Upsilon_K})\tilde{\subseteq}(\Upsilon_K; < \alpha, \beta, \gamma >^{avg}_{\Upsilon_K})$
2. $(\Upsilon_K; < \alpha, \beta, \gamma >^{Mmm}_{\Upsilon_K})\tilde{\subseteq}(\Upsilon_K; < \alpha, \beta, \gamma >^{mmm}_{\Upsilon_K})$

Proof 4.16 *The proof of the theorems are obvious.*

Theorem 4.17 *Let $\Upsilon_K, \Upsilon_L \in IVNS(U)$. Then,*

1. *Let* $<\alpha_1,\beta_1,\gamma_1>_{\Upsilon_K}$ *and* $<\alpha_2,\beta_2,\gamma_2>_{\Upsilon_K}$ *be two threshold interval-valued neutrosophic sets. Then,* $(\Upsilon_K;<\alpha_1,\beta_1,\gamma_1>_{\Upsilon_K})$ *and* $(\Upsilon_K;<\alpha_2,\beta_2,\gamma_2>_{\Upsilon_K})$ *are* $<\alpha_1,\beta_1,\gamma_1>_{\Upsilon_K}$*-level soft set and* $<\alpha_2,\beta_2,\gamma_2>_{\Upsilon_K}$*-level soft set of* Υ_K*, respectively.*

   If $<\alpha_2,\beta_2,\gamma_2>_{\Upsilon_K}\widehat{\leq}<\alpha_1,\beta_1,\gamma_1>_{\Upsilon_K}$*, then we have*

   $(\Upsilon_K;<\alpha_1,\beta_1,\gamma_1>_{\Upsilon_K})\tilde{\subseteq}(\Upsilon_K;<\alpha_2,\beta_2,\gamma_2>_{\Upsilon_K})$.

2. *Let* $<\alpha,\beta,\gamma>_{\Upsilon_K}$ *be a threshold interval-valued neutrosophic sets.*

   Then, $(\Upsilon_K;<\alpha,\beta,\gamma>_{\Upsilon_K})$ *and* $(\Upsilon_L;<\alpha,\beta,\gamma>_{\Upsilon_K})$ *are* $<\alpha,\beta,\gamma$*-level soft set* Υ_K *and* Υ_L*, respectively.*

   If $\Upsilon_K\widehat{\subseteq}\Upsilon_L$*, then we have* $(\Upsilon_K;<\alpha,\beta,\gamma>_{\Upsilon_K})\tilde{\subseteq}(\Upsilon_L;<\alpha,\beta,\gamma>_{\Upsilon_K})$.

Proof 4.18 *The proof of the theorems are obvious.*

Now, we construct an $ivn-$soft set decision making method by the following algorithm;

Algorithm:

1. Input the $ivn-$soft set Υ_K,

2. Input a threshold interval-valued neutrosophic set $<\alpha,\beta,\gamma>^{avg}_{\Upsilon_K}$ (or $<\alpha,\beta,\gamma>^{Mmm}_{\Upsilon_K}, <\alpha,\beta,\gamma>^{mmm}_{\Upsilon_K}$) by using avg-level decision rule (or Mmm-level decision rule, mmm-level decision rule) for decision making.

3. Compute avg-level soft set $(\Upsilon_K;<\alpha,\beta,\gamma>^{avg}_{\Upsilon_K})$ (or Mmm-level soft set $((\Upsilon_K;<\alpha,\beta,\gamma>^{Mmm}_{\Upsilon_K})$, mmm-level soft set $(\Upsilon_K;<\alpha,\beta,\gamma>^{mmm}_{\Upsilon_K})))$

4. Present the level soft set $(\Upsilon_K;<\alpha,\beta,\gamma>^{avg}_{\Upsilon_K})$ (or the level soft set$((\Upsilon_K;<\alpha,\beta,\gamma>^{Mmm}_{\Upsilon_K}$, the level soft set $(\Upsilon_K;<\alpha,\beta,\gamma>^{mmm}_{\Upsilon_K})))$ in tabular form.

5. Compute the choice value c_i of u_i for any $u_i\in U$,

6. The optimal decision is to select u_k if $c_k=max_{u_i\in U}c_i$.

Remark 4.19 *If* k *has more than one value then any one of* u_k *may be chosen.*

If there are too many optimal choices in Step 6, we may go back to the second step and change the threshold (or decision rule) such that only one optimal choice remains in the end.

Remark 4.20 *The aim of designing the Algorithm is to solve* $ivn-$*soft sets based decision making problem by using level soft sets. Level soft sets construct bridges between* $ivn-$*soft sets and crisp soft sets. By using level soft sets, we need not treat* $ivn-$*soft sets directly but only cope with crisp soft sets derived from them after choosing certain thresholds or decision strategies such as the mid-level or the topbottom-level decision rules. By the Algorithm, the choice value of an object in a level soft set is in fact the number of fair attributes which belong to that object on the premise that the degree of the truth-membership* *of u with respect to the parameter x is not less than truth-membership levels, the degree of the indeterminacy-membership of u with respect to the parameter x is not more than indeterminacy-membership levels and the degree of the falsity-membership of u with respect to the parameter x is not more than falsity-membership levels.*

Example 4.21 *Suppose that a customer to select a house from the real agent. He can construct a ivn−soft set Υ_K that describes the characteristic of houses according to own requests. Assume that $U = \{u_1, u_2, u_3, u_4, u_5, u_6\}$ is the universe contains six house under consideration in an real agent and $E = \{x_1 = cheap, x_2 = beatiful, x_3 = greensurroundings, x_4 = costly, x_5 = large\}$.*

Now, we can apply the method as follows:

1. *Input the ivn−soft set Υ_K as,*

U	u_1	u_2
x_1	$\langle[0.5, 0.7], [0.8, 0.9], [0.2, 0.5]\rangle$	$\langle[0.3, 0.6], [0.3, 0.9], [0.2, 0.8]\rangle$
x_2	$\langle[0.0, 0.3], [0.6, 0.8], [0.3, 0.9]\rangle$	$\langle[0.1, 0.8], [0.8, 0.9], [0.3, 0.5]\rangle$
x_3	$\langle[0.1, 0.7], [0.4, 0.5], [0.8, 0.9]\rangle$	$\langle[0.2, 0.5], [0.5, 0.7], [0.6, 0.8]\rangle$
x_4	$\langle[0.2, 0.4], [0.7, 0.9], [0.6, 0.9]\rangle$	$\langle[0.3, 0.9], [0.6, 0.9], [0.3, 0.9]\rangle$
x_5	$\langle[0.0, 0.2], [1.0, 1.0], [1.0, 1.0]\rangle$	$\langle[0.0, 0.1], [0.9, 1.0], [0.2, 0.9]\rangle$
U	u_3	u_4
x_1	$\langle[0.5, 0.8], [0.8, 0.9], [0.3, 0.9]\rangle$	$\langle[0.1, 0.9], [0.5, 0.9], [0.2, 0.4]\rangle$
x_2	$\langle[0.9, 0.9], [0.2, 0.3], [0.3, 0.5]\rangle$	$\langle[0.7, 0.9], [0.1, 0.3], [0.5, 0.6]\rangle$
x_3	$\langle[0.8, 0.9], [0.1, 0.7], [0.6, 0.8]\rangle$	$\langle[0.8, 0.9], [0.1, 0.2], [0.5, 0.6]\rangle$
x_4	$\langle[0.6, 0.9], [0.6, 0.9], [0.6, 0.9]\rangle$	$\langle[0.5, 0.9], [0.6, 0.8], [0.5, 0.8]\rangle$
x_5	$\langle[0.8, 0.9], [0.0, 0.4], [0.7, 0.7]\rangle$	$\langle[0.7, 0.9], [0.5, 1.0], [0.6, 0.5]\rangle$
U	u_5	u_6
x_1	$\langle[0.6, 0.8], [0.8, 0.9], [0.1, 0.5]\rangle$	$\langle[0.5, 0.8], [0.2, 0.9], [0.1, 0.7]\rangle$
x_2	$\langle[0.1, 0.4], [0.5, 0.8], [0.3, 0.7]\rangle$	$\langle[0.1, 0.9], [0.6, 0.9], [0.2, 0.3]\rangle$
x_3	$\langle[0.2, 0.9], [0.1, 0.5], [0.7, 0.8]\rangle$	$\langle[0.4, 0.9], [0.1, 0.6], [0.5, 0.7]\rangle$
x_4	$\langle[0.6, 0.9], [0.6, 0.9], [0.6, 0.9]\rangle$	$\langle[0.5, 0.9], [0.6, 0.8], [0.1, 0.8]\rangle$
x_5	$\langle[0.0, 0.9], [1.0, 1.0], [1.0, 1.0]\rangle$	$\langle[0.0, 0.9], [0.8, 1.0], [0.2, 0.5]\rangle$

Table 12: The tabular representation of the *ivn*-soft set $\widehat{\bigtriangledown}\Upsilon_K$

2. *Input a threshold interval-valued neutrosophic set $< \alpha, \beta, \gamma >^{avg}_{\Upsilon_K}$ by using avg-level decision rule for decision making as;*

$$< \alpha, \beta, \gamma >^{avg}_{\Upsilon_K} = \{\langle[0.41, 0.76], [0.56, 0.9], [0.18, 0.63]\rangle/x_1, \langle[0.31, 0.7], [0.46, 0.66], [0.31, 0.58]\rangle/x_2, \langle[0.41, 0.8], [0.21, 0.53], [0.61, 0.76]\rangle/x_3, \langle[0.45, 0.81], [0.61, 0.86], [0.45, 0.86]\rangle/x_4, \langle[0.25, 0.65], [0.7, 0.9], [0.61, 0.76]\rangle/x_5\}$$

3. *Compute avg-level soft set $(\Upsilon_K; < \alpha, \beta, \gamma >^{avg}_{\Upsilon_K})$ as;*

$$(\Upsilon_K; < \alpha, \beta, \gamma >^{avg}_{\Upsilon_K}) = \{(x_2, \{u_3\}), (x_3, \{u_4\}), (x_4, \{u_6\}, (x_5, \{u_3\})\}$$

4. Present the level soft set $(\Upsilon_K; <\alpha,\beta,\gamma>^{avg}_{\Upsilon_K})$ in tabular form as;

U	u_1	u_2	u_3	u_4	u_5	u_6
x_1	0	0	0	0	0	0
x_2	0	0	1	0	0	0
x_3	0	0	0	1	0	0
x_4	0	0	0	0	0	1
x_5	0	0	1	0	0	0

Table 13: The tabular representation of the soft set F_X

5. Compute the choice value c_i of u_i for any $u_i \in U$ as;

$$c_1 = c_2 = c_5 = \sum_{j=1}^{5} u_{1j} = \sum_{j=1}^{6} u_{2j} = \sum_{j=1}^{5} h_{5j} = 0,$$

$$c_4 = c_6 = \sum_{j=1}^{5} u_{4j} = \sum_{j=1}^{6} h_{6j} = 1$$

$$c_3 = \sum_{j=1}^{5} u_{3j} = 2$$

6. The optimal decision is to select u_3 since $c_3 = max_{u_i \in U} c_i$.

Note that this decision making method can be applied for group decision making easily with help of the Definition 3.37 and Definition 3.38.

5 Conclusion

In this paper, the notion of the interval valued neutrosophic soft sets ($ivn-$soft sets) is defined which is a combination of an interval valued neutrosophic sets[36] and a soft sets[30]. Then, we introduce some definitions and operations of $ivn-$soft sets sets. Some properties of $ivn-$soft sets which are connected to operations have been established. Finally, we propose an adjustable approach by using level soft sets and illustrate this method with some concrete examples. This novel proposal proves to be feasible for some decision making problems involving $ivn-$soft sets. It can be applied to problems of many fields that contain uncertainty such as computer science, game theory, and so on.

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