Linear weighted averaging method on SVN-sets and its sensitivity analysis based on multi-attribute decision making problems

Irfan Deli

Muallim Rıfat Faculty of Education, 7 Aralık University, 79000 Kilis, Turkey

irfandeli@kilis.edu.tr

February 6, 2015

Abstract

In this paper, we develop a method of multi-attribute decision-making with both weights and attribute ratings expressed by single valued neutrosophic sets(SVN-sets). The method is called linear weighted averaging method of SVN-sets. Then, we present a sensitivity analysis of attribute weights which give changing intervals of attribute weights in which the ranking order of the alternatives is required to remain unchanging. Finally, validity and applicability of the proposed method are illustrated with a real application.

Keyword 0.1 *Neutrosophic set, single valued neutrosophic numbers, linear weighted averaging method, sensitivity analysis, multi-attribute decision making.*

1 Introduction

Since Atanassov [1] proposed intuitionistic fuzzy set theory, some generalized applications have been proposed and studied to handle vagueness in [5, 8, 15, 18, 20]. The intuitionistic fuzzy set considers only the degree of membership and non-membership which is one sum of degree of membership and non-membership. To handle such situations, Smaradanche [25, 26] introduced the notions of neutrosophic sets which allow to incorporate simultaneously the truth-membership degree, indeterminacy-membership degree and falsity-membership degree of each element, as a generalization of the notion of a intuitionistic fuzzy set. Intuitionistic fuzzy set otherwise neutrosophic set is characterized by a the truth-membership degree, indeterminacy-membership degree and falsity-membership degree of each element so that $^-0 \leq$ truth-membership degree+ indeterminacy-membership degree + falsity-membership degree $\leq 3^+$ where $^-0 \leq$ truth-membership degree $\leq 1^+$, $^-0 \leq$ indeterminacy-membership degree$\leq 1^+$ and $^-0 \leq$falsity-membership degree $\leq 1^+$. After Smaradanche, Wang et al. [28] introduced the definition of single valued neutrosophic set to apply the idea of neutrosophic set to real life application in scientific and engineering problems. The neutrosophic sets has been applied to many different fields, such as; supplier selection [27], medical diagnoses [31], mage thresholding [6, 14], image segmentation [13], matrices [7], decision making [2, 3, 4, 9, 10, 11], topological spaces [24], clustering [35, 36], and so on.

In multiple-attribute decision-making problems, decision maker usually need to compare a set of alternatives by using attributes with different weight. Thereafter, the decision maker need to rank the given alternatives. But the ranking of alternatives with neutrosophic sets is a significant issue. Methodically to rank alternatives with neutrosophic sets, one neutrosophic element needs to be compared with the others, but it is arduous to determine clearly which of them is larger or smaller. Therefore, many methods have been raised in literature to rank alternatives with neutrosophic sets (e.g. [22, 23, 29, 30, 32, 34, 37]).

Feng [12] said that "Decision may change with the changes of time, conditions or environments. The problem how changes of ratings of alternatives on attributes or attribute weights affect final decision results is of useness in theoretical and practical research. In other words, we highly concern with what conditions should be satisfied if the final decision results are required to remain unchanging. These problems are called sensitivity analysis". In this study we extend the linear weighted averaging method and sensitivity analysis as well as applications of intuitionistic fuzzy sets by given [12, 16, 17, 19] to neutrosophic sets. Therefore, we organize the rest of the paper as follows: in the following section, we present preliminary definitions of intuitionistic fuzzy set, neutrosophic sets and single valued neutrosophic sets. In Section 3, we proposed a method of multi-attribute decision-making with both weights and attribute ratings expressed by single valued neutrosophic sets(SVN-sets). In Section 4, we introduced a sensitivity analysis of attribute weights by using the linear weighted averaging method. In Section 5, validity and applicability of the proposed method are illustrated with a real application. In last section, conclusion are presented.

2 Preliminaries

Definition 2.1 *[?] Let E be a universe. Then a fuzzy set X over E is defined by*

$$X = \{(\mu_X(x)/x) : x \in E\}$$

where μ_X is called membership function of X and defined by $\mu_X : E \to [0.1]$. For each $x \in E$, the value $\mu_X(x)$ represents the degree of x belonging to the fuzzy set X.

Definition 2.2 *[1] Let E be a universe. An intuitionistic fuzzy set K over E is defined by*

$$K = \{< x, \mu_K(x), \gamma_K(x) >:\ x \in E\}$$

where $\mu_K : E \to [0,1]$ *and* $\gamma_K : E \to [0,1]$ *such that* $0 \leq \mu_K(x) + \gamma_K(x) \leq 1$ *for any* $x \in E$*. For each* $x \in E$*, the values* $\mu_K(x)$ *and* $\gamma_K(x)$ *are the degree of membership and degree of non-membership of* x*, respectively.*

Definition 2.3 *[25] Let* E *be a universe. A neutrosophic sets* A *over* E *is defined by*

$$A = \{< x, (T_A(x), I_A(x), F_A(x)) >: x \in E\}.$$

where $T_A(x)$*,* $I_A(x)$ *and* $F_A(x)$ *are called truth-membership function, indeterminacy-membership function and falsity-membership function, respectively. They are respectively defined by*

$$T_A : E \to]^-0, 1^+[, \quad I_A : E \to]^-0, 1^+[, \quad F_A : E \to]^-0, 1^+[$$

such that $0^- \leq T_A(x) + I_A(x) + F_A(x) \leq 3^+$*.*

Definition 2.4 *[28] Let* E *be a universe. An SVN-set over* E *is a neutrosophic set over* E*, but the truth-membership function, indeterminacy-membership function and falsity-membership function are respectively defined by*

$$T_A : E \to [0,1], \quad I_A : E \to [0,1], \quad F_A : E \to [0,1]$$

such that $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$*.*

Definition 2.5 *[22] Let* A *and* B *be two SVN-sets. Then the operations of SVN-sets can be defined as follows:*

1. $A + B = \{< x, (T_A(x) + T_B(x) - T_A(x)T_B(x), I_A(x)I_B(x), F_A(x)F_B(x)) >: x \in E\}$
2. $A.B = \{< x, (T_A(x)T_B(x), I_A(x) + I_B(x) - I_A(x)I_B(x), F_A(x) + F_B(x) - F_A(x)F_B(x)) >: x \in E\}$
3. $kA = \{< x, (1 - (1 - T_A(x))^k, I_A(x)^k, F_A(x)^k) >: x \in E\}$
4. $A^k =< \{x, (T_A(x)^k, 1 - (1 - I_A(x))^k, 1 - (1 - F_A(x))^k) >: x \in E\}$

where $k \in R$*.*

Definition 2.6 *[22] Let* $X = \{x_1, x_2, ..., x_n\}$ *be a set of alternatives,* $U = \{o_1, o_2, ..., o_m\}$ *be the set of attributes. The ratings (or evaluations) of alternatives* $x_j \in X (j = 1, 2, ..., n)$ *on attributes* $o_i \in O(o_1, o_2, ..., o_m)$ *are expressed with SVN-sets* $A_{ij} = \langle T_{ij}, I_{ij}, F_{ij} \rangle$*, where* $T_A : E \to [0,1], \quad I_A : E \to [0,1], \quad F_A : E \to [0,1]$

such that $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$. *Then*

$$[A_{ij}]_{m\times n} = \begin{array}{cc} & \begin{array}{cccc} x_1 & x_2 & \cdots & x_n \end{array} \\ \begin{array}{c} o_1 \\ o_2 \\ \vdots \\ o_m \end{array} & \begin{pmatrix} \langle T_{11}, I_{11}, F_{11}\rangle & \langle T_{12}, I_{12}, F_{12}\rangle & \cdots & \langle T_{1n}, I_{1n}, F_{1n}\rangle \\ \langle T_{21}, I_{21}, F_{21}\rangle & \langle T_{22}, I_{22}, F_{22}\rangle & \cdots & \langle T_{2n}, I_{2n}, F_{2n}\rangle \\ \vdots & \vdots & \ddots & \vdots \\ \langle T_{m1}, I_{m1}, F_{m1}\rangle & \langle T_{m2}, I_{m2}, F_{m2}\rangle & \cdots & \langle T_{mn}, I_{mn}, F_{mn}\rangle \end{pmatrix} \end{array}$$

is called an SVN-multi-criteria decision making matrix.

In [22], they assumed that the weights ω_i of the attributes $o_i \in O(i = 1, 2, ..., m)$ are a fuzzy concept fuzzy concept, which is difficult to be precisely determined in real applications. Therefore, by taking inspiration [27], we assume that weight of each attribute $o_i \in O(i = 1, 2, ..., m)$ is expressed with the SVN-set $\omega_i = \langle \alpha_i, \beta_i, \gamma_i\rangle$, where $\alpha_i \in [0, 1]$, $\beta_i \in [0, 1]$, $\gamma_i \in [0, 1]$ such that $0 \leq \alpha_i + \beta_i + \gamma_i \leq 3$, which is truth-degree, indeterminacy-degree and falsity-degree of the attribute $o_i \in O$, respectively. The weights of all m attributes is also written shortly as follows:

$$\omega = (\omega_1, \omega_2, ..., \omega_m) = (\langle \alpha_1, \beta_1, \gamma_1\rangle, \langle \alpha_2, \beta_2, \gamma_2\rangle, ..., \langle \alpha_m, \beta_m, \gamma_m\rangle)$$

which is called the SVN-weight vector.

Definition 2.7 *Let* $A_{ij} = \langle T_{ij}, I_{ij}, F_{ij}\rangle$ $(i = 1, 2, ..., m; j = 1, 2, ..., n)$ *be a SVN-multi-criteria decision making matrix and* $\omega_i = \langle \alpha_i, \beta_i, \gamma_i\rangle$ $(i = 1, 2, ..., m$ *be a SVN-weight vector. Then, the products of the SVN-sets* A_{ij} *and* ω_i*, denoted by* $\bar{A}_{ij} = \langle \bar{T}_{ij}, \bar{I}_{ij}, \bar{F}_{ij}\rangle$*, is given as*

$$[\bar{A}_{ij}]_{m\times n} = \begin{array}{cc} & \begin{array}{cccc} x_1 & x_2 & \cdots & x_n \end{array} \\ \begin{array}{c} o_1 \\ o_2 \\ \vdots \\ o_m \end{array} & \begin{pmatrix} \langle \bar{T}_{11}, \bar{I}_{11}, \bar{F}_{11}\rangle & \langle \bar{T}_{12}, \bar{I}_{12}, \bar{F}_{12}\rangle & \cdots & \langle T_{1n}, \bar{I}_{1n}, \bar{F}_{1n}\rangle \\ \langle \bar{T}_{21}, \bar{I}_{21}, \bar{F}_{21}\rangle & \langle \bar{T}_{22}, \bar{I}_{22}, \bar{F}_{22}\rangle & \cdots & \langle T_{2n}, \bar{I}_{2n}, \bar{F}_{2n}\rangle \\ \vdots & \vdots & \ddots & \vdots \\ \langle \bar{T}_{m1}, \bar{I}_{m1}, \bar{F}_{m1}\rangle & \langle \tilde{T}_{m2}, \bar{I}_{m2}, \bar{F}_{m2}\rangle & \cdots & \langle T_{mn}, \bar{I}_{mn}, \bar{F}_{mn}\rangle \end{pmatrix} \end{array}$$

where

$$\langle \bar{T}_{ij}, \bar{I}_{ij}, \bar{F}_{ij}\rangle = \omega_i A_{ij} = \langle \alpha_i, \beta_i, \gamma_i\rangle \langle T_{ij}, I_{ij}, F_{ij}\rangle = \langle \alpha_i T_{ij}, \beta_i + I_{ij} - \beta_i I_{ij}, \gamma_i + F_{ij} - \gamma_i F_{ij}\rangle$$

which is called weighted SVN-multi-criteria decision making matrix.

Definition 2.8 *Let* $\bar{A}_{ij} = \langle \bar{T}_{ij}, \bar{I}_{ij}, \bar{F}_{ij}\rangle$ $(i = 1, 2, ..., m; j = 1, 2, ..., n)$ *be a weighted SVN-multi-criteria decision making matrix. Then, comprehensive evaluation, denoted* V_j*, of each alternative* $x_i \in X(j =$

$1, 2, ..., n)$ *is given by;*

$$V_j = \sum_{i=1}^{m} \langle \bar{T}_{ij}, \bar{I}_{ij}, \bar{F}_{ij} \rangle = \langle T_j, I_j, F_j \rangle$$

Definition 2.9 *[21] Let $\bar{A}_{ij} = \langle \bar{T}_{ij}, \bar{I}_{ij}, \bar{F}_{ij} \rangle$ $(i = 1, 2, ..., m; j = 1, 2, ..., n)$ be a weighted SVN-multi-criteria decision making matrix and $V_j = \sum_{i=1}^{m} \langle \bar{T}_{ij}, \bar{I}_{ij}, \bar{F}_{ij} \rangle = \langle T_j, I_j, F_j \rangle$ be a comprehensive evaluation. Then,*

1. *score function of V_j (j=1,2,...,n), denoted $s(V_j)$, defined as;*

$$s(V_j) = 2 + \bar{T}_j - \bar{F}_j - \bar{I}_j$$

2. *accuracy function of V_j (j=1,2,...,n), denoted $a(V_j)$, defined as;*

$$a(V_j) = \bar{T}_j - \bar{F}_j$$

and also,

1. *If $s(V_j) < s(V_i)$, then V_i is smaller than V_2, denoted by $V_1 < V_2$*
2. *If $s(V_j) = s(V_i;$*
   (a) If $a(V_j) < a(V_i)$, then V_1 is smaller than V_2, denoted by $V_1 < V_2$
   (b) If $s(V_j) = s(V_i)$, then $\tilde{a}_1$ and $\tilde{a}_2$ are the same, denoted by $V_1 = V_2$

According to the scoring function ranking method of SVN-sets, the ranking order of the set of the alternatives can be generated and the best alternative can be determined.

3 Sensitivity analysis of the linear weighted averaging method for multiattribute decision-making with SVN-sets

In this section, we present sensitivity analysis of attribute weights in the linear weighted averaging method of multiattribute decision-making with SVN-sets.

Decision may change with the changes of time, conditions or environments. The problem how changes of ratings of alternatives on attributes or attribute weights affect final decision results is of useness in theoretical and practical research. In other words, we highly concern with what conditions should be satisfied if the final decision results are required to remain unchanging. These problems are called sensitivity analysis. [12].

Definition 3.1 ***(Sensitivity analysis)*** *Let $A_{ij} = \langle T_{ij}, I_{ij}, F_{ij} \rangle$ $(i = 1, 2, ..., m; j = 1, 2, ..., n)$ be a SVN-multi-criteria decision making matrix, $\omega_i = \langle \alpha_i, \beta_i, \gamma_i \rangle$ $(i = 1, 2, ..., m$ be a SVN-weight vector and $\omega' =$*

$(\omega_1, \omega_2, ..., \omega_m) = (\langle\alpha_1, \beta_1, \gamma_1\rangle, \langle\alpha_2, \beta_2, \gamma_2\rangle, ..., \langle\alpha_k + \Delta\alpha_k, \beta_k + \Delta\beta_k, \gamma_k + \Delta\gamma_k\rangle, ..., \langle\alpha_m, \beta_m, \gamma_m\rangle)^T$ *be a changed SVN-weight vector where* $\Delta\alpha_k$, $\Delta\beta_k$ *and* $\Delta\gamma_k$ *are increments of* α_k, β_k *and* γ_k, *respectively. Then, comprehensive evaluation* V_j *of the alternative* x_j *can be rewritten as follows:*

$$\begin{aligned} V_j &= \textstyle\sum_{i=1, i\neq k}^{m} \omega_i A_{ij} + \omega_k A_{kj} \\ &= \langle x_j, y_j, z_j\rangle + \langle \alpha_k T_{kj}, \beta_k + I_{kj} - \beta_k I_{kj}, \gamma_k + F_{kj} - \gamma_k F_{kj}\rangle \\ &= \langle x_j + \alpha_k T_{kj} - x_j \alpha_k T_{kj}, y_j(\beta_k + I_{kj} - \beta_k I_{kj}), z_j(\gamma_k + F_{kj} - \gamma_k F_{kj})\rangle \end{aligned}$$

where

$$\langle x_j, y_j, z_j\rangle = \sum_{i=1, i\neq k}^{m} \omega_i A_{ij}$$

and

$$\omega_k A_{kj} = \langle \alpha_k, \beta_k, \gamma_k\rangle \langle T_{kj}, I_{kj}, F_{kj}\rangle = \langle \alpha_k T_{kj}, \beta_k + I_{kj} - \beta_k I_{kj}, \gamma_k + F_{kj} - \gamma_k F_{kj}\rangle$$

Therefore, we have:

$$\begin{aligned} T_j &= \quad x_j + \alpha_k T_{kj} - x_j \alpha_k T_{kj}, \\ I_j &= \quad y_j(\beta_k + I_{kj} - \beta_k I_{kj}) \\ &\textit{and} \\ F_j &= \quad z_j(\gamma_k + F_{kj} - \gamma_k F_{kj}). \end{aligned}$$

Likewise, the changed comprehensive evaluation $V_j^{'}$ of the alternative x_j with the weight change of the attribute o_k can be calculated as follows;

$$\begin{aligned} V_j^{'} &= \langle x_j, y_j, z_j\rangle + \langle (\alpha_k + \Delta\alpha_k) T_{kj}, \beta_k + \Delta\beta_k + I_{kj} - (\beta_k + \Delta\beta_k) I_{kj}, \gamma_k + \Delta\gamma_k + F_{kj} - (\gamma_k + \Delta\gamma_k) F_{kj}\rangle \\ &= \langle x_j + \alpha_k T_{kj} + \Delta\alpha_k T_{kj} - x_j \alpha_k T_{kj} - x_j \Delta\alpha_k T_{kj}, y_j(\beta_k + I_{kj} - \beta_k I_{kj}) + y_j(\Delta\beta_k - \Delta\beta_k I_{kj}), \\ &\quad z_j(\gamma_k + F_{kj} - \gamma_k F_{kj}) + z_j(\Delta\gamma_k - \Delta\gamma_k F_{kj})\rangle \\ &= \langle T_j + \Delta\alpha_k T_{kj}(1 - x_j), I_j + \Delta\beta_k y_j(1 - I_{kj}), F_j + \Delta\gamma_k z_j(1 - F_{kj})\rangle \end{aligned}$$

where

$$\begin{aligned} \omega^{'} A_{kj} &= \langle \alpha_k + \Delta\alpha_k, \beta_k + \Delta\beta_k, \gamma_k + \Delta\gamma_k\rangle \langle T_{kj}, I_{kj}, F_{kj}\rangle \\ &= \langle (\alpha_k + \Delta\alpha_k) T_{kj}, \beta_k + \Delta\beta_k + I_{kj} - (\beta_k + \Delta\beta_k) I_{kj}, \gamma_k + \Delta\gamma_k + F_{kj} - (\gamma_k + \Delta\gamma_k) F_{kj}\rangle \end{aligned}$$

In a similar way, the changed comprehensive evaluations $V_s^{'}$ and $V_t^{'}$ of the alternatives x_s and x_t with the weight change of the attribute o_k can be calculated as follows:

$$\begin{aligned} V_s^{'} &= \langle x_s, y_s, z_s\rangle + \langle (\alpha_k + \Delta\alpha_k) T_{ks}, \beta_k + \Delta\beta_k + I_{ks} - (\beta_k + \Delta\beta_k) I_{ks}, \gamma_k + \Delta\gamma_k + F_{ks} - (\gamma_k + \Delta\gamma_k) F_{ks}\rangle \\ &= \langle T_s + \Delta\alpha_k T_{ks}(1 - x_s), I_s + \Delta\beta_k y_s(1 - I_{ks}), F_s + \Delta\gamma_k z_s(1 - F_{ks})\rangle \end{aligned}$$

and

$$\begin{aligned} V_t^{'} &= \langle x_t, y_t, z_t\rangle + \langle(\alpha_k+\Delta\alpha_k)T_{kt}, \beta_k+\Delta\beta_k+I_{kt}-(\beta_k+\Delta\beta_k)I_{kt}, \gamma_k+\Delta\gamma_k+F_{kt}-(\gamma_k+\Delta\gamma_k)F_{kt}\rangle \\ &= \langle T_t+\Delta\alpha_k T_{kt}(1-x_t), I_t+\Delta\beta_k y_t(1-I_{kt}), F_t+\Delta\gamma_k z_t(1-F_{kt})\rangle \end{aligned}$$

respectively, where

$$\begin{aligned} T_s = &\quad x_s+\alpha_k T_{ks}-x_s\alpha_k T_{ks}, \\ I_s = &\quad y_s(\beta_k+I_{ks}-\beta_k I_{ks}) \\ F_s = &\quad z_s(\gamma_k+F_{ks}-\gamma_k F_{ks}), \\ T_t = &\quad x_t+\alpha_k T_{kt}-x_t\alpha_k T_{kt}, \\ I_t = &\quad y_t(\beta_k+I_{kt}-\beta_k I_{kt}) \\ &and \\ F_t = &\quad z_t(\gamma_k+F_{kt}-\gamma_k F_{kt}). \end{aligned}$$

Then, we can calculate the scores of $V_j^{'}$, $V_s^{'}$, and $V_t^{'}$ as follows:

$$\begin{aligned} s(V_j^{'}) &= 2+T_j-I_j-F_j+\Delta\alpha_k T_{kj}(1-x_j)-\Delta\beta_k y_j(1-I_{kj})-\Delta\gamma_k z_j(1-F_{kj}) \\ s(V_s^{'}) &= 2+T_s-I_s-F_s+\Delta\alpha_k T_{ks}(1-x_s)-\Delta\beta_k y_s(1-I_{ks})-\Delta\gamma_k z_s(1-F_{ks}) \\ s(V_t^{'}) &= 2+T_t-I_t-F_t+\Delta\alpha_k T_{kt}(1-x_t)-\Delta\beta_k y_t(1-I_{kt})-\Delta\gamma_k z_t(1-F_{kt}) \end{aligned}$$

Also, we can obtain the accuracies of $V_j^{'}$, $V_s^{'}$, and $V_t^{'}$ as follows:

$$\begin{aligned} a(V_j^{'}) &= T_j-F_j+\Delta\alpha_k T_{kj}(1-x_j)-\Delta\gamma_k z_j(1-F_{kj}) \\ a(V_s^{'}) &= T_s-F_s+\Delta\alpha_k T_{ks}(1-x_s)-\Delta\gamma_k z_s(1-F_{ks}) \\ a(V_t^{'}) &= T_t-F_t+\Delta\alpha_k T_{kt}(1-x_t)-\Delta\gamma_k z_t(1-F_{kt}) \end{aligned}$$

Without loss of generality, assume that the first ranking order of the three alternatives x_j, x_s and x_t is $x_j > x_s > x_t$. When the weight ω_t of the attribute o_k is changed to $\omega_t^{'}$, if the ranking order of the alternatives x_j, x_s and x_t are required to remain unchanging, then according to the scoring function ranking method of SVN-sets, the scores and accuracies of $V_j^{'}$, $V_s^{'}$ and $V_t^{'}$ should satisfy either

1. $s(V_j^{'}) > s(V_s^{'})$ and $s(V_s^{'}) > s(V_t^{'})$

   or

2. $s(V_j^{'}) = s(V_s^{'})$, $s(V_s^{'}) = s(V_t^{'})$, $a(V_j^{'}) > a(V_s^{'})$,and $a(V_s^{'}) > a(V_t^{'})$.

Finally, we have the systems of inequalities as follows:

1.

$$\begin{aligned}
&s(V_j^{'}) > s(V_s^{'})\\
&s(V_s^{'}) > s(V_t^{'})\\
&0 \le \alpha_k + \Delta\alpha_k + \beta_k + \Delta\beta_k + \gamma_k + \Delta\gamma_k \le 3,\\
&0 \le \alpha_k + \Delta\alpha_k \le 1\\
&0 \le \beta_k + \Delta\beta_k \le 1\\
&0 \le \gamma_k + \Delta\gamma_k \le 1
\end{aligned}$$

2.

$$\begin{aligned}
&s(V_j^{'}) = s(V_s^{'})\\
&s(V_s^{'}) = s(V_t^{'})\\
&a(V_j^{'}) > a(V_s^{'})\\
&a(V_s^{'}) > a(V_t^{'})\\
&0 \le \alpha_k + \Delta\alpha_k + \beta_k + \Delta\beta_k + \gamma_k + \Delta\gamma_k \le 3,\\
&0 \le \alpha_k + \Delta\alpha_k \le 1\\
&0 \le \beta_k + \Delta\beta_k \le 1\\
&0 \le \gamma_k + \Delta\gamma_k \le 1
\end{aligned}$$

and

Solving either 1. or 2., we can obtain the changing ranges $\Delta\alpha_k$, $\Delta\beta_k$ and $\Delta\gamma_k$ of the weight ω_k of the attribute o_k. Namely, if the weight ω_k takes any value between $\langle\alpha_k, \beta_k, \gamma_k\rangle$; and $\langle\alpha_k + \Delta\alpha_k, \beta_k + \Delta\beta_k, \gamma_k + \Delta\gamma_k\rangle$, then, the ranking order of the alternatives still remains unchanging.

4 SVN-Linear Weighted Averaging Method

In this section, we give a method, is called linear weighted averaging method, for sensitivity analysis of SVN-weights of the attributes;

Let $X = (x_1, x_2, ..., x_n)$ be a set of alternatives, $U = (u_1, u_2, ..., u_m)$ be the set of attributes.

Algorithm:

Step 1. Construct the SVN-multi-attribute decision making matrix $[A_{ij}]_{m\times n}$;

Step 2. Determine the SVN-weight vector $\omega = (\langle\alpha_i, \beta_i, \gamma_i\rangle)_{mx1}$

Step 3. Compute the weighted the SVN-multi-attribute decision making matrix $[\bar{A}_{ij}]_{m\times n}$;

Step 4. Compute the comprehensive evaluations V_j of the alternatives $x_j \in X(j = 1, 2, ..., n)$;

Step 5. Rank the comprehensive evaluations $V_j(j = 1, 2, ..., n)$;

Step 6. Find the sensitivity analysis of SVN-weights of the attributes in the linear weighted averaging method;

5 Application

In this section, we cite the commonly used example [22] and extend it to the SVN-sets.

An investment company wants to invest a sum of money in the best option considering three criteria which are denoted by $O = (o_1 = \text{risk analysis}, o_2 = \text{growth analysis}, o_3 = \text{environmental impact analysis})$. The company has set up a panel which has to choose between three possible alternatives for investing the money which are denoted by $X = (x_1 = \text{car company}, x_2 = \text{food company}, x_3 = \text{computer company})$

Step 1. Construct the SVN-multi-attribute decision making matrix $[A_{ij}]_{3\times 3}$ as;

$$[A_{ij}]_{3\times 3} = \begin{pmatrix} \langle 0.7, 0.1, 0.8\rangle & \langle 0.7, 0.6, 0.8\rangle & \langle 0.1, 0.4, 0.7\rangle \\ \langle 0.5, 0.2, 0.8\rangle & \langle 0.4, 0.2, 0.3\rangle & \langle 0.2, 0, 1, 0.9\rangle \\ \langle 0.1, 0.1, 0.6\rangle & \langle 0.8, 0.5, 0.4\rangle & \langle 0, 6, 0.3, 0.7\rangle \end{pmatrix}$$

Step 2. Determine the SVN-weight vector as

$$\omega = (\langle 0.2, 0.9, 0.8\rangle, \langle 0.8, 0.4, 0.9\rangle, \langle 0.7, 0.6, 0.3\rangle)$$

Step 3. Compute the weighted the SVN-multi-attribute decision making matrix $[\bar{A}_{ij}]_{m\times n}$ as;

$$[\bar{A}_{ij}]_{3\times 3} = \begin{pmatrix} \langle 0.14, 0.91, 0.96\rangle & \langle 0.14, 0.96, 0.96\rangle & \langle 0.02, 0.94, 0.94\rangle \\ \langle 0.40, 0.52, 0.98\rangle & \langle 0.32, 0.52, 0.93, \rangle & \langle 0.16, 0, 46, 0.99\rangle \\ \langle 0.07, 0.64, 0.72\rangle & \langle 0.56, 0.80, 0.58\rangle & \langle 0, 42, 0.72, 0.79\rangle \end{pmatrix}$$

Step 4. Compute the comprehensive evaluations V_j of the alternatives $x_j \in X (j = 1, 2, ..., n)$ as;

$$\begin{aligned} V_1 &= \langle 1 - (1 - 0.14)(1 - 0.40)(1 - 0.07), 0.91 \times 0.52 \times 0.64, 0.96 \times 0.98 \times 0.72\rangle \\ &= \langle 0.52012, 0.30285, 0.67738\rangle \end{aligned}$$

$$\begin{aligned} V_2 &= \langle 1 - (1 - 0.14)(1 - 0.32)(1 - 0.56), 0.96 \times 0.52 \times 0.80, 0.96 \times 0.93 \times 0.58\rangle \\ &= \langle 0.74269, 0.39936, 0.51782\rangle \end{aligned}$$

and

$$\begin{aligned} V_3 &= \langle 1 - (1 - 0.02)(1 - 0.16)(1 - 0.42), 0.94 \times 0.46 \times 0.72, 0.94 \times 0.99 \times 0.79\rangle \\ &= \langle 0.52254, 0.31133, 0.73517\rangle \end{aligned}$$

respectively.

Step 5. Rank the comprehensive evaluations $V_j(j = 1, 2, ..., n)$ as;

$$s(V_1) = 1.53990$$
$$s(V_2) = 1.82550$$
$$and$$
$$s(V_3) = 1.47604$$

respectively. Then, it is obvious that the best alternative is x_2 and the ranking order of the three alternatives is $x_2 > x_1 > x_3$.

Step 6. Find the sensitivity analysis of SVN-weights of the attributes in the linear weighted averaging method as;

Firstly, we assume that only weight $\omega_2 = \langle\alpha_2, \beta_2, \gamma_2\rangle$ of the attribute o_2 is changed to the weight $\bar{\omega}_2 = \langle\alpha_2 + \Delta\alpha_2, \beta_2 + \Delta\beta_2, \gamma_2 + \Delta\gamma_2\rangle$ and the weights of other attributes $o_i(i = 1, 3)$ remain the same as the original weights ω_1. Then, we hava the system of inequalities with respect to $\Delta\alpha_2$, $\Delta\beta_2$ and $\Delta\gamma_2$ is obtained as follows: Finally, we have the systems of inequalities as follows:

$$\begin{aligned}
&s(V_2^{'}) > s(V_1^{'})\\
&s(V_1^{'}) > s(V_3^{'})\\
&0 \leq \alpha_2 + \Delta\alpha_2 + \beta_2 + \Delta\beta_2 + \gamma_2 + \Delta\gamma_2 \leq 3,\\
&0 \leq 0.8 + \Delta\alpha_2 \leq 1\\
&0 \leq 0.4 + \Delta\beta_2 \leq 1\\
&0 \leq 0.9 + \Delta\gamma_2 \leq 1
\end{aligned}$$

where

$$\begin{aligned}
s(V_1^{'}) &= 2 + T_1 - I_1 - F_1 + \Delta\alpha_2 T_{21}(1 - x_1) - \Delta\beta_2 y_1(1 - I_{21}) - \Delta\gamma_2 z_1(1 - F_{21})\\
&= 1.35176 + 0.31992\Delta\alpha_2 - 0.27955\Delta\beta_2 - 0.01382\Delta\gamma_2\\
s(V_2^{'}) &= 2 + T_2 - I_2 - F_2 + \Delta\alpha_2 T_{22}(1 - x_2) - \Delta\beta_2 y_2(1 - I_{22}) - \Delta\gamma_2 z_2(1 - F_{22})\\
&= 1.38793 + 0.12109\Delta\alpha_2 - 0.36864\Delta\beta_2 - 0.03898\Delta\gamma_2\\
s(V_3^{'}) &= 2 + T_3 - I_3 - F_3 + \Delta\alpha_2 T_{23}(1 - x_3) - \Delta\beta_2 y_3(1 - I_{23}) - \Delta\gamma_2 z_3(1 - F_{23})\\
&= 1.30498 + 0.09094\Delta\alpha_2 - 0.36547\Delta\beta_2 - 0.00743\Delta\gamma_2
\end{aligned}$$

and where

$$T_1 = 0.45614$$
$$I_1 = 0.41467$$
$$F_1 = 0.68982$$
$$T_2 = 0.71847$$
$$I_2 = 0.46694$$
$$F_2 = 0.86360$$
$$T_3 = 0.50436$$
$$I_3 = 0.45752$$
$$F_3 = 0.74186$$

which can be simplified into the system of inequalities as follows:

$$0.03628 - 0.19883\Delta\alpha_2 - 0.08909\Delta\beta_2 - 0.02515\Delta\gamma_2 > 0$$
$$0,04667 + 0,22898\Delta\alpha_2 + 0,08592\Delta\beta_2 - 0,00640\Delta\gamma_2 > 0$$
$$-0.8 \leq \Delta\alpha_2 \leq 0.2$$
$$-0.4 \leq \Delta\beta_2 \leq 0.6$$
$$-0.9 \leq \Delta\gamma_2 \leq 0.1$$

Some solutions of the system is given by Fig. 1.

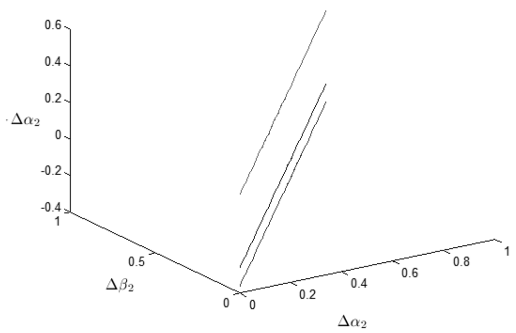

Figure 1: Some solutions of the system

Likewise,we assume that only weight $\omega_1 = \langle\alpha_1, \beta_1, \gamma_1\rangle$ of the attribute o_1 is changed to the weight $\bar{\omega}_1 = \langle\alpha_1 + \Delta\alpha_1, \beta_1 + \Delta\beta_1, \gamma_1 + \Delta\gamma_1\rangle$ and the weights of other attributes $o_i(i = 2, 3)$ remain the same as the original weights or that only weight $\omega_3 = \langle\alpha_3, \beta_3, \gamma_3\rangle$ of the attribute o_3 is changed to the weight $\bar{\omega}_3 = \langle\alpha_3 + \Delta\alpha_3, \beta_3 + \Delta\beta_3, \gamma_3 + \Delta\gamma_3\rangle$ and the weights of other attributes $o_i(i = 1, 2)$ remain the same as the original weights, then the solutions can easily be made in a similar way for o_1 and o_3.

6 Conclusion

This paper presents a method of multi-attribute decision-making with both weights and attribute ratings expressed by single valued neutrosophic sets(SVN-sets). Then, we developed a sensitivity analysis of attribute weights by using the linear weighted averaging method. This analysis give changing intervals of attribute weights in which the ranking order of the alternatives is required to remain unchanging. It is easily seen that the proposed the method can be extended to attribute ratings in a straightforward manner. More effective methods of SVN-sets will be investigated in the near future and applied this concepts to game theory, algebraic structure, optimization and so on.

References

[1] K.T. Atanassov, Intuitionistic Fuzzy Sets, Pysica-Verlag A Springer-Verlag Company, New York, 1999.

[2] S. Broumi, I. Deli, F. Smarandache, Interval valued neutrosophic parameterized soft set theory and its decision making, Journal of New Results in Science 7 (2014) 58–71.

[3] S. Broumi, I. Deli, F. Smarandache, Relations on Interval Valued Neutrosophic Soft Sets, Journal of New Results in Science 5 (2014) 01–20.

[4] S. Broumi, I. Deli, F. Smarandache, Distance and Similarity Measures of Interval Neutrosophic Soft Sets, Critical Review, Center for Mathematics of Uncertainty, Creighton University, USA, 8 (2014) 14–31.

[5] T.Y. Chen, H.P. Wang, Y.Y. Lu, "A multicriteria group decision-making approach based on interval-valued intuitionistic fuzzy sets, a comparative perspective" Exp. Syst. Appl. 38/6 (2011) 7647-7658.

[6] H. D. Cheng and Y. Guo, A new neutrosophic approach to mage thresholding, New Mathematics and Natural Computation, 4/3 (2008) 291–308.

[7] I. Deli and S. Broumi, Neutrosophic Soft Matrices and NSM-decision Making, Journal of Intelligent and Fuzzy Systems, DOI:10.3233/IFS-141505.

[8] I. Deli, N. Çağman, Intuitionistic fuzzy parameterized soft set theory and its decision making, Applied Soft Computing 28 (2015) 109-113.

[9] I. Deli, Y. Tokta and S. Broumi, Neutrosophic Parameterized Soft Relations and Their Applications, Neutrosophic Sets and Systems, 4 (2014) 25–34.

[10] I. Deli and S. Broumi, Neutrosophic soft relations and some properties, Annals of Fuzzy Mathematics and Informatics 9(1) (2015) 169–182.

[11] I. Deli, S. Broumi and M. Ali, Neutrosophic Soft Multi-Set Theory and Its Decision Making, Neutrosophic Sets and Systems, 5 (2014) 65–76.

[12] D. F. Li, "Decision and Game Theory in Management With Intuitionistic Fuzzy Sets." *Studies in Fuzziness and Soft Computing* Volume 308, springer (2014).

[13] Y. Guo, A. Şengür, A novel image segmentation algorithm based on neutrosophic similarity clustering, Applied Soft Computing 25 (2014) 391–398.

[14] Y. Guo, A. Şengür, J. Ye, A novel image thresholding algorithm based on neutrosophic similarity score, Measurement 58 (2014) 175–186.

[15] D.F. Li, J.X. Nan, An extended weighted average method for MADM using intuitionistic fuzzy sets and sensitivity analysis, Crit View, V (2011) 5–25.

[16] D.F. Li, G.H. Chen, Z.G. Huang, Linear programming method for multiattribute group decision making using IF sets, Inf. Sci. 180(9) (2010) 1591–1609.

[17] D.F. Li, TOPSIS-based nonlinear-programming methodology for multiattribute decision making with interval-valued intuitionistic fuzzy sets. IEEE Trans. Fuzzy Syst., 18(2) (2010) 299–311.

[18] D.F. Li, Multiattribute decision making models and methods using intuitionistic fuzzy sets. J. Comput. Syst. Sci., 70(1) (2005) 73–85.

[19] D.F. Li, Extension of the LINMAP for multiattribute decision making under Atanassovs intuitionistic fuzzy environment, Fuzzy Optim. Decis. Making, 7(1) (2008) 17–34.

[20] D.F. Li, Y.C. Wang, Mathematical programming approach to multiattribute decision making under intuitionistic fuzzy environments. Int. J. Uncertainty Fuzziness Knowl. Based Syst. 16(4) (2008) 557–577.

[21] P. Liu, Y. Chu, Y. Li, and Y. Chen, Some Generalized Neutrosophic Number Hamacher Aggregation Operators and Their Application to Group Decision Making, International Journal of Fuzzy Systems, 16/2, (2014) 242–255.

[22] J.J. Peng, J.Q. Wang, H.Y. Zhang, X.H. Chen, An outranking approach for multi-criteria decision-making problemswith simplified neutrosophic sets, Applied Soft Computing 25 (2014) 336-346

[23] J.J. Peng, J.Q. Wang, X.H. Wu, J. Wang, X.H. Chen, Multi-valued Neutrosophic Sets and Power Aggregation Operators with Their Applications in Multi-criteria Group Decision-making Problems, International Journal of Computational Intelligence Systems, 8/2 (2014) 345–363.

[24] A. A. Salama and S. A. Alblowi, "Neutrosophic Set and Neutrosophic Topological Spaces." *IOSR Journal of Mathematics,* 3/4 (2012) 31–35.

[25] F. Smarandache, "A Unifying Field in Logics. Neutrosophy: Neutrosophic Probability, Set and Logic". Rehoboth: American Research Press, (1998).

[26] F. Smarandache, "Neutrosophic set, a generalisation of the intuitionistic fuzzy sets." *Int. J. Pure Appl. Math.* 24 (2005) 287-297.

[27] Rdvan Şahin, M. Yiğider, A Multi-criteria neutrosophic group decision making metod based TOPSIS for supplier selection, 2014, http://arxiv.org/abs/1412.5077.

[28] H. Wang, F. Y. Smarandache, Q. Zhang and R. Sunderraman, "Single valued neutrosophic sets", *Multispace and Multistructure* 4 (2010) 410–413.

[29] J. Ye, "A multicriteria decision-making method using aggregation operators for simplified neutrosophic sets." *Journal of Intelligent and Fuzzy Systems,* 26 (2014) 2459–2466.

[30] J. Ye, Single Valued Neutrosophic Cross-Entropy for Multicriteria Decision Making Problems, Applied Mathematical Modelling 38 (2014) 1170-1175

[31] J. Ye, Improved cosine similarity measures of simplified neutrosophic sets for medical diagnoses, Artificial Intelligence in Medicine (2014), http://dx.doi.org/10.1016/j.artmed.2014.12.007

[32] J. Ye, Vector Similarity Measures of Simplified Neutrosophic Sets and Their Application in Multicriteria Decision Making, International Journal of Fuzzy Systems, 16/2 (2014) 204-211.

[33] J. Ye, Trapezoidal neutrosophic set and its application to multiple attribute decision-making, Neural Comput. and Appl., DOI:10.1007/s00521-014-1787-6.

[34] J. Ye, Similarity measures between interval neutrosophic sets and their applications in multicriteria decision-making, Journal of Intelligent and Fuzzy Systems 26 (2014) 165–172.

[35] J. Ye, Clustering Methods Using Distance-Based Similarity Measures of Single-Valued Neutrosophic Sets, Journal of Intelligent Systems, 23(4) (2014) 379–389.

[36] J. Ye, Single-Valued Neutrosophic Minimum Spanning Tree and Its Clustering Method, Journal of Intelligent Systems, 23(3) (2014) 311–324.

[37] H.Y. Zhang, J.Q. Wang, and X.H. Chen, Interval Neutrosophic Sets and Their Application in Multi-criteria Decision Making Problems, Hindawi Publishing Corporation, Scientific World Journal (2014) 1–15.