Size of atoms in RW universe: a correction to Dirac equation for curved space-time

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Abstract.
Dirac equation for curved space-time is solved in Cartesian coordinates giving a result which is contrary to the generally accepted notion in cosmology that space expansion does not impact size of atoms. The solution shows that size of atoms reduces in proportion to the square of scale factor in an expanding universe. Work of McVittie, who solved the same equation with a different result is reviewed, highlighting his mistake and why he got an incorrect result. As the result is completely counter-intuitive a correction to the Dirac equation for curved space-time is attempted, which is desired due to many other reasons as well. The corrected equation when solved, shows that size of atoms changes in proportion to square of scale factor. Same result has been reported earlier as well using Schrödinger equation indicating that correction proposed is in right direction. Gravitational redshift formula is derived using the new equation, which matches with general relativistic prediction. The new equation proposed in this paper is thus indeed correct Dirac equation for curved space-time. Validity of standard model of cosmology is questioned due to this discovery.

Keywords: Dirac Equation, cosmology, RW metric

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1. Introduction

In standard model (also known as Concordance model or $ΛCDM$ model) of cosmology, the redshift in light coming from galaxies is explained as increase in wavelength of photons due to increased distances with passage of time. Calculated using geodesics of a photon, this increase is proportional to the value of scale factor at the epoch of emission [1]. Cosmological redshift versus distance relation derived from this relation is compared against observed luminosities and redshifts of Type Ia Supernovae, which is then claimed as the proof of the standard model [2, 3, 4, 5, 6, 7, 8, 9, 10]. But behind all this success is the implicit assumption that size of atoms and molecules does not change with changing scale factor, because if it does then the relation between redshift and scale factor and all other relations derived from that will not hold. It is therefore of fundamental importance that we independently check this assumption. Size of an atom is nothing but the size of electron orbit as per Bohr’s model and the radius of electron orbit around the nucleus is determined by using the expression of classical electron orbit

$$E_b = -\frac{Ze^2}{8\pi\epsilon_0 r}$$

where $E_b$ is the binding energy of the electron, which is the total relativistic energy $E$ of the electron minus its rest energy $E_0 = mc^2$.

$$E_b = E - E_0 = E - mc^2$$

As expected when $r \to \infty$ then $E_b \to 0$ and $E = mc^2$. Obviously radius of electron orbits and hence size of atoms depends on the binding energy and hence total relativistic energy of the electron. Arnold Sommerfeld was the first one to derive the relativistic solution of atomic energy levels [11].

$$E_b = -mc^2 \left[ 1 - \frac{1}{\sqrt{1 + \frac{z^2\alpha^2}{(n+\frac{1}{2})^2 - z^2\alpha^2}}} \right]$$

Where $n$ is the principal quantum number and $j$ is the angular momentum number. Above equation can be simplified to second order as

$$E_b \approx -mc^2 \frac{z^2\alpha^2}{2n^2} \left[ 1 + \frac{z^2\alpha^2}{n^2} \left( \frac{n}{j + \frac{3}{2}} - \frac{3}{4} \right) \right]$$

This is equivalent to Dirac equation which is based on more generally accepted Quantum mechanics while Sommerfeld equation was based on old quantum theory which was not self-consistent. For this reason I must solve Dirac equation for hydrogen atom in curved space time to confirm if indeed size of atoms does change with changing scale factor. This is what is done exactly in section 2 where we get a totally unexpected and counter-intuitive result demanding a review of the Dirac equation for curved space. As the result is contrary to expectations I review the result obtained by McVittie and highlight the mistake in his derivation in section 3. In section 4, I justify the need for a correction to Dirac equation for curved space-time and derive the corrected equation from energy-momentum relation in curved space-time. In section 5, I solve the corrected equation which again gives a result contrary to expectations of standard model. The corrected equation is solved for Schwarzschild metric in section 6 to confirm that it gives the correct result as expected.
2. Dirac equation solution for RW universe in Cartesian coordinates

The covariant Dirac equation in the presence of a central potential is given as [12, 13, 14, 15, 16]

\[
(i\hbar\gamma^\mu e^\mu_a \left[ \frac{\partial}{\partial x^\mu} - \Gamma_\mu(x) \right] - \gamma^\mu eA_\mu - mc) \psi(x) = 0 \tag{5}
\]

Here \(\Gamma_\mu(x)\) are the spin connections to be determined. \(A_\mu\) is the electromagnetic four potential and \(e\) is charge of an electron. \(\gamma^a\) denotes the standard flat-space Dirac metrics, which satisfies \(\gamma^a, \gamma^b = 2\eta^{ab}I_4\) and inverse Vierbein has components, \(e^\mu_a\), that satisfy the orthonormality conditions

\[
e^\mu_a(x)e^a_\nu(x) = \delta^\mu_\nu \tag{6}
\]

To keep things simple I will work in Cartesian coordinates and assume a flat universe \((k = 0)\). The metric is then

\[
\begin{pmatrix}
-1 & 0 & 0 & 0 \\
0 & R^2(t) & 0 & 0 \\
0 & 0 & R^2(t) & 0 \\
0 & 0 & 0 & R^2(t)
\end{pmatrix} \tag{7}
\]

The Vierbein and inverse Vierbein fields in the case of RW metric are thus

\[
e^\alpha_\mu(x) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & R(t) & 0 & 0 \\ 0 & 0 & R(t) & 0 \\ 0 & 0 & 0 & R(t) \end{pmatrix} \tag{8}
\]

\[
e^\mu_a(x) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{1}{R(t)} & 0 & 0 \\ 0 & 0 & \frac{1}{R(t)} & 0 \\ 0 & 0 & 0 & \frac{1}{R(t)} \end{pmatrix} \tag{9}
\]

All non-zero spin connections are calculated as (see [17] for details)

\[
\Gamma_j = \frac{\dot{R}}{2} \gamma^0 \gamma^j \tag{10}
\]

As the rate of change of scale factor \(R\) is very slow, we can safely ignore all terms involving time derivative of the scale factor

\[
\dot{R} \approx 0 \tag{11}
\]

This implies that spin connection can also be assumed as negligible

\[
\Gamma_j \approx 0 \tag{12}
\]

Dirac equation is thus simplified as

\[
\left( \gamma^0 \frac{i\hbar\partial}{\partial x^0} + \frac{1}{R(t)} \gamma^j \frac{i\hbar\partial}{\partial x^j} - \gamma^0 eA_0 - mc \right) \psi(x) = 0 \tag{13}
\]

Multiplying both sides by \(c\gamma^0\) and re-arranging we get

\[
\frac{c}{R(t)} \gamma^0 \gamma^j \left( -\frac{i\hbar\partial}{\partial x^j} \right) \psi(x)
\]
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\[ \psi(x) = \left( \begin{array}{c} \chi \\ \phi \end{array} \right) \tag{15} \]

Straightforward simplification and using the definition of energy and momentum operators, gives a set of two equations

\[ c \vec{\sigma} \cdot \vec{p} \phi(x) = -R(t) \left( \frac{mc^2 - E}{\hbar c} + \frac{z\alpha}{r} \right) \chi(x) \tag{16} \]

and

\[ c \vec{\sigma} \cdot \vec{p} \chi(x) = R(t) \left( \frac{mc^2 + E}{\hbar c} - \frac{z\alpha}{r} \right) \phi(x) \tag{17} \]

Defining \( m' = R(t)m, E' = R(t) \) and \( \alpha' = R(t)\alpha \) we get the equation in the standard form

\[ c \vec{\sigma} \cdot \vec{p} \phi(x) = - \left( \frac{m'c^2 - E'}{\hbar c} + \frac{z\alpha'}{r} \right) \chi(x) \tag{18} \]

and

\[ c \vec{\sigma} \cdot \vec{p} \chi(x) = \left( \frac{m'c^2 + E'}{\hbar c} - \frac{z\alpha'}{r} \right) \phi(x) \tag{19} \]

Solving these equations (see [18] or [19] for detailed calculations), we get the energy of hydrogen

\[ E' = \sqrt{1 + \frac{m'^2}{1 + \frac{z^2 \alpha'^2}{(n + \sqrt{(j + \frac{1}{2})^2 - z^2 \alpha'^2})^2}}} \tag{20} \]

Replacing back values of \( m' = R(t)m, E' = R(t) \) and \( \alpha' = R(t)\alpha \) we get

\[ E = \sqrt{1 + \frac{mc^2}{1 + \frac{R^2(t)z^2 \alpha^2}{(n + \sqrt{(j + \frac{1}{2})^2 - R^2(t)z^2 \alpha^2})^2}}} \tag{21} \]

Using 2 and 21 we get the binding energy as

\[ E_b \approx -\frac{mc^2 z \alpha^2 R^2(t)}{2n^2} \left[ 1 + \frac{2^4 z^2 \alpha^2 R^2(t)}{n^2} \left( \frac{n}{j + \frac{1}{2}} \right)^2 \right] \tag{22} \]

Ignoring higher order terms we get

\[ E_b \approx -\frac{mc^2 z \alpha^2 R^2(t)}{2n^2} \tag{23} \]
Comparing with 1 we get the following relation between radius of electron orbitals and scale factor

\[ r \propto \frac{1}{R^2} \] (24)

This result clearly violates the generally accepted notion in cosmology that space expansion does not impact size of atoms [20, 21, 22, 23]. Also it is completely counter-intuitive as one expects the size of atoms in an expanding universe to either increase or remain constant. Thus there is no justification for the solution in which the size of atoms reduces with expanding space. This and many other issues prompt one to seek corrections to the Dirac equation for curved space-time.

3. Review of McVittie’s work

Before we seek to correct the Dirac equation for curved space-time we must check how other scholars have calculated the energy levels of hydrogen to remain constant in an expanding universe. What mistake did they make exactly to miss the simple result derived in previous section? McVittie was one of the first persons to solve the Dirac equation for RW universe in my knowledge. I therefore pick his paper [16] for the analysis as it is likely that all other authors that followed him made the same mistake. Since McVittie followed same steps as Darwin who had solved the Dirac equation for hydrogen in flat space-time earlier [19], I will be frequently referring to Darwin as well.

Dirac equation for Hydrogen atom in flat space-time is a set of wave equations in spherical harmonics and radial distance, which after separation reads in radial part as (equation 8.3 of Darwin)

\[
\left( A^2 + \frac{\gamma}{r} \right) F + \frac{dG}{dr} - \frac{k}{r} G = 0 \\
\left( B^2 - \frac{\gamma}{r} \right) G + \frac{dF}{dr} - \frac{k + 2}{r} F = 0
\]

Here \( A, B \) and \( \gamma \) are given as

\[
A^2 = \frac{2\pi}{\hbar} \left( mc + \frac{W}{c} \right) \\
B^2 = \frac{2\pi}{\hbar} \left( mc - \frac{W}{c} \right) \\
\gamma = \frac{2\pi e^2}{\epsilon_0 \hbar}
\]

and \( r \) is the spatial distance of electron from the nucleus. Apart from the electric potential term (\( \gamma/r \)), \( r \) appears in other terms of the equations as can be seen above. Also the radial wave functions \( F \) and \( G \) are functions in \( r \) (equation 8.4 of Darwin).

\[
F = e^{-\lambda r} \left( a_0 r^\beta + a_1 r^{\beta-1} + a_2 r^{\beta-2} + \cdots \right) \\
G = e^{-\lambda r} \left( b_0 r^\beta + b_1 r^{\beta-1} + b_2 r^{\beta-2} + \cdots \right)
\]

In all the places it appears in the equations it represents the same physical quantity, i.e. the proper distance of electron from the nucleus. Now in curved space-time the Dirac equation is similar in form to the one above (equations 30 and 31 of McVittie)

\[
\left( A^2 - \frac{\gamma}{r} \right) F + \frac{dG}{dr} - \frac{k}{r} G = 0
\]
\[(B^2 + \frac{\bar{\gamma}}{r})G + \frac{dF}{dr} - \frac{k + 2}{r}F = 0\]

As expected radial wave functions \(F\) and \(G\) are also functions of the same form (equations 32 and 33 of McVittie)

\[F(r) = e^{-\lambda r} \left( a_0 r^\beta + a_1 r^{\beta-1} + a_2 r^{\beta-2} + \cdots \right)\]

\[G(r) = e^{-\lambda r} \left( b_0 r^\beta + b_1 r^{\beta-1} + b_2 r^{\beta-2} + \cdots \right)\]

Naturally \(r\) here still represents the same physical quantity as in flat space-time (i.e. proper distance from the nucleus), else one would expect to multiply all \(r\) terms with some correctional term. At first glance it seems that even for the potential term \((\bar{\gamma}/r)\), \(r\) is used as proper distance only but in reality this is not the case. While calculating the potential term \(\phi_0\), it is used as coordinate distance and multiplied with \(\alpha(1 + \eta)\) to get the proper distance (equation 23 and before that in McVittie)

\[r = \sqrt{x^2 + y^2 + z^2}\]

\[\phi_0 = \frac{2\pi e^2}{\hbar c} \frac{1}{\alpha(1 + \eta)r}\]

This potential term is then included in the Dirac equation and it so happens on simplification that the final term \(\bar{\gamma}/r\) does not have the multiplication factor \(\alpha(1 + \eta)\), making it look like as if \(r\) is used consistently in the Dirac equation, while in reality it is not. McVittie in his paper treated \(r\) as coordinate distance for potential term \(\phi_0\) and multiplied it with a factor \(\alpha(1 - \eta)\), but did not multiply with the same factor for other terms like \(F(r)\) and \(G(r)\) within the same equation. This dual treatment of a single variable within the same equation artificially suppressed the \(\alpha^2\) term in the final solution and it can be easily shown that due to this mistake only McVittie got the result where energy of the electron does not change in RW universe. If we use \(r\) correctly (as proper distance in all terms) then we will get the same result as I have derived above. For Schwarzschild metric also the result derived by McVittie no longer holds if \(r\) is used as proper distance in all terms.

One can argue here that variable \(r\) represents coordinate distance in Dirac equations and not the proper distance. An equation in physics is just a representation of phenomena of the real world so of-course we can chose to represent the reality in a way we want, as long as we chose a representation which is consistent. But an equation is also used to predict behavior in the real word so it is critical that the representation is reversible, i.e. we should be able to calculate the measurable quantities from the equation, which can then be verified against the measurements made in real word. This demands that all the terms used in an equation are represented in exactly same way else it will not be possible to predict anything. For example if we chose to express one or more terms of an equation in coordinate distance, then all other terms in the equation should also be expressed in terms of coordinate distance only so that we can use a formula to convert from coordinate distances to physical distances and predict the measurable quantities for all terms of the equation. It is therefore totally inappropriate to convert the coordinate distance to proper distance for only one term in the equation and leave all other terms unchanged, which is exactly what is done by McVittie.
4. Correction in Dirac equation for curved space-time

Apart from the obvious contradictory result in section 2, there are many other issues with the Dirac equation in curved space-time as explained below.

(i) While deriving the Dirac equation for curved space-time the standard metrics are contracted with inverse Vierbeins to "write them in terms of global coordinates" [section 6.2][15]. But Inverse Vierbeins are used to move from a coordinate frame (global coordinates) to an inertial frame of reference (local coordinates) [15, 12, 1], and not the other way round. Dirac equation in flat space-time is already in an inertial frame of reference and hence contracting the Dirac metrics with inverse Vierbeins does not make any sense. In fact the contraction should be done with Vierbeins (to move to global coordinates) and not the inverse-Vierbeins.

(ii) Tetrad formalism consists of setting up local inertial coordinates at each point on the curved manifold and Spin connection is used to formulate the covariant derivatives. Spin connection answers the questions "how a vector located at one point with components in the local basis at that point, parallel transports to another point on the manifold with a new basis" [section 2.5][15]. Since Dirac equation for curved space-time is supposed to be in global coordinates as discussed above, it's doubly wrong to then use the Spin Connection.

(iii) Dirac equation in flat space-time is derived from the energy-momentum relation in flat space-time \( P^\alpha \eta_{\alpha \beta} P^\beta = (mc)^2 \). It is therefore only natural to derive Dirac equation from the energy-momentum relation in curved space-time \( P^\alpha g_{\alpha \beta} P^\beta = (mc)^2 \). But 5 is not derived from energy-momentum relation in curved space-time. In fact we can show that the equation represents a relation \( \Sigma \alpha \beta P^\alpha g^{\alpha \beta} P^\beta + f(x) = (mc)^2 \), which is completely wrong. Here \( f(x) \) is some unknown function of \( x^\mu \) which comes into existence because of presence of Spin Connection in 5.

(iv) Dirac equation in curved space-time does not reduce to a unique Hamiltonian (and energy operator) as expected [24]. In fact it can be shown that this problem occurs precisely due to use of Spin connection (which is uncalled for anyway as discussed above).

To derive the correct equation for curved space-time I start from energy-momentum relation in general relativity [25]

\[
P^\alpha g_{\alpha \beta} P^\beta = (mc)^2
\]  

(25)

Using the relation between metric and Vierbeins \( g_{\mu \nu} = e^a_\mu e^b_\nu \eta_{ab} \) we get

\[
e^a_\mu e^b_\nu \eta_{ab} P^\mu P^\nu = (mc)^2
\]  

(26)

From this equation it is straightforward to derive the Dirac equation in curved space-time (in global coordinates) following the same method as followed by Dirac himself [26].

\[
\left( i\hbar \gamma^\mu e^a_\mu \frac{\partial}{\partial x^\mu} - mc \right) \psi(x) = 0
\]  

(27)

Here \( \gamma^\mu \) are the standard Dirac metrics as earlier. Although Dirac spinor looks like a four component vector but strictly speaking, this is just a representation and Dirac spinor cannot be treated as a geometric object like vectors or tensors. In reality
Dirac equation is an equation of scalar functions only. This is evident from the fact that three out of four complex components of the Dirac spinor can be algebraically eliminated from the Dirac equation, yielding a partial differential equation of the fourth order for the remaining complex component [27]. As individual components of Dirac spinor are complex scalar functions and since partial derivative is the covariant derivative for scalar functions, replacing partial derivative with covariant derivative (containing the spin connection) is not needed in the equation above.

It can be shown that solutions of 27 are similar to the equation in flat space-time except for the normalization factor. We can also express this equation in local coordinates (tetrad frame) by contracting energy-momentum with inverse Vierbeins. Interestingly doing this we get back the standard Dirac equation in flat space-time. Since tetrad frame is the local observer frame, what this implies is that any effect of gravitational field on energy of a particle will not be measurable by a local observer. This hints at the fact that all measures of scale should change along with any change (expansion or contraction) of space. It is easy to show that above equation is locally Lorentz invariant and does not have any of the issues (which exist in 5) discussed above. Assuming this to be the correct Dirac equation for curved space-time, I solve it for Hydrogen atom using the RW metric and find out how binding energies of electrons in the atom (and hence the size of atoms) change with changing scale factor.

5. Energy and size of atoms using the corrected Dirac equation in RW universe

In presence of a central potential 27 reads as

\[
\left(\frac{i\hbar \gamma^\mu e^\mu_\alpha}{\partial_x} - \gamma^\mu eA_\mu - mc\right)\psi(x) = 0
\]  

(28)

Using 8 Dirac equation is thus

\[
\left(\gamma^0 \frac{i\hbar \partial}{\partial x^0} + R(t)\gamma^j \frac{i\hbar \partial}{\partial x^j} - \gamma^0 eA_0 - mc\right)\psi(x) = 0
\]  

(29)

Following the same steps as in 2 we get

\[
c \sigma \cdot \tilde{p} \phi(x) = -\frac{1}{R(t)} \left(\frac{mc^2 - E}{\hbar c} + \frac{z\alpha}{r}\right)\chi(x)
\]  

(30)

and

\[
c \sigma \cdot \tilde{p} \chi(x) = \frac{1}{R(t)} \left(\frac{mc^2 + E}{\hbar c} - \frac{z\alpha}{r}\right)\phi(x)
\]  

(31)

Defining \( m' = m/R(t), E' = E/R(t) \) and \( \alpha' = \alpha/R(t) \) we get the equation in the standard form

\[
c \sigma \cdot \tilde{p} \phi(x) = -\left(\frac{m' c^2 - E'}{\hbar c} + \frac{z\alpha'}{r}\right)\chi(x)
\]  

(32)

and

\[
c \sigma \cdot \tilde{p} \chi(x) = \left(\frac{m' c^2 + E'}{\hbar c} - \frac{z\alpha'}{r}\right)\phi(x)
\]  

(33)
Solving these equations, and replacing back values of $m', E'$ and $\alpha'$ we get

$$E = mc^2 \sqrt{1 + \frac{z^2 \alpha^2}{R^2(t)}} \left( n + \sqrt{\left(j + \frac{1}{2}\right)^2 - \frac{z^2 \alpha^2}{R^2(t)}} \right)^2 $$

(34)

Using 2 and 34 we get the binding energy as

$$E_b \approx \frac{-mc^2 z^2 \alpha^2}{2R^2(t)n^2} \left[ 1 + \frac{z^2 \alpha^2}{R^2(t)n^2} \left( \frac{n}{j + \frac{1}{2}} - \frac{3}{4} \right) \right]$$

(35)

Ignoring higher order terms we get

$$E_b \approx \frac{-mc^2 z^2 \alpha^2}{2R^2(t)n^2}$$

(36)

Comparing with 1 we get the following relation between radius of electron orbitals and scale factor

$$r \propto R^2$$

(37)

This implies that size of atoms will vary directly proportional to square of scale factor. Even though this result makes better sense than before, it still does not conform to the generally accepted notion that size of atoms does not change with changing scale factor.

6. Solution using Schwarzschild metric

We cannot claim the new equation derived in this paper to be correct Dirac equation unless we can show that this equation correctly predicts the energy levels in case of Schwarzschild metric as well. The metric is given as (Defining $R^2 = x^2 + y^2 + z^2$, $R' = (1 - 2GM/c^2R)^{-1/2}$ and assuming Cartesian coordinates centered at origin of mass) [28, 25]

$$
\begin{pmatrix}
\frac{1}{R^2} & 0 & 0 & 0 \\
0 & R^2 & 0 & 0 \\
0 & 0 & R^2 & 0 \\
0 & 0 & 0 & R^2
\end{pmatrix}
$$

(38)

The Vierbeins are thus

$$e^a_\mu = \begin{pmatrix}
\frac{1}{R^2} & 0 & 0 & 0 \\
0 & R' & 0 & 0 \\
0 & 0 & R' & 0 \\
0 & 0 & 0 & R'
\end{pmatrix}
$$

(39)

Dirac equation then becomes

$$\left( \frac{1}{R^2} \gamma^0 \frac{i\hbar}{\partial x^0} + R'. \gamma^1 \frac{i\hbar}{\partial x^1} - \gamma^0 eA_0 - mc \right) \psi(x) = 0$$

(40)

Following the same steps as in 2 we get

$$c \vec{\sigma} \cdot \vec{p} \phi(x) = -\frac{1}{R^2} \left( \frac{mc^2 - E'/R'}{\hbar c} + \frac{z\alpha}{r} \right) \chi(x)$$

(41)
and
\[ c \mathcal{P} \cdot \mathcal{P} \chi(x) = \frac{1}{R'} \left( \frac{mc^2 + E/R'}{\hbar c} - \frac{z\alpha}{r} \right) \phi(x) \] (42)

Defining \( m' = m/R', E' = E/R'^2 \) and \( \alpha' = \alpha/R' \) we get the equation in the standard form
\[ c \mathcal{P} \cdot \mathcal{P} \phi(x) = -\left( \frac{m'c^2 - E'}{\hbar c} + \frac{z\alpha'}{r} \right) \chi(x) \] (43)

and
\[ c \mathcal{P} \cdot \mathcal{P} \chi(x) = \left( \frac{m'c^2 + E'}{\hbar c} - \frac{z\alpha'}{r} \right) \phi(x) \] (44)

Solving these equations, and replacing back values of \( m', E' \) and \( \alpha' \) we get
\[ E = \frac{mc^2 R'}{\sqrt{1 + \frac{z^2\alpha^2}{R'^2(n + \sqrt{(j + \frac{1}{2})^2 - \frac{z^2\alpha^2}{R'^2}})^2}}} \] (45)

Using 2 and 45 we get the binding energy as
\[ E_b \approx -\frac{mc^2 z^2\alpha^2}{R'n^2} \left[ \frac{1}{1 + \frac{z^2\alpha^2}{R'^2n^2} \left( \frac{n}{j + \frac{1}{2}} - \frac{3}{4} \right)} \right] \] (46)

Ignoring higher order terms and replacing the value of \( R' \) we get
\[ E_b \approx -\frac{mc^2 z^2\alpha^2(1 - 2GM/c^2R)^{1/2}}{n^2} \] (47)

Using the fact that energy of photon absorbed or emitted by the electron is the difference in binding energies of the orbitals we get change in frequency due to gravity as
\[ \nu_g \approx \left( 1 - 2GM/c^2R \right)^{1/2} \] (48)

Where \( \nu_g \) is the frequency of a photon in the gravitational well while \( \nu_\infty \) is the frequency at infinite distance from the source of gravitation

Using the relation \( 1 + z = \nu_\infty/\nu_g \) we get the gravitational redshift as
\[ z = \left( 1 - 2GM/c^2R \right)^{-1/2} - 1 \] (49)

Which is same as predicted in general relativity [25, page 659]
7. Discussion and conclusion

The notion that space expansion in standard model does not impact the size of atoms has been proved wrong by solving the currently accepted Dirac equation for curved space-time. But how come then other authors like McVittie got a result which conforms to the very notion? To understand this we need to answer the following question: "does the variable \( r \) used in simplified Dirac equation represent the proper distance from the nucleus or the coordinate distance?". Actually it should not matter as long as we are using the same definition for all terms in the equation. So if we are treating \( r \) as proper distance then there is no need to multiply it with some factor in any of the terms. But if we are treating it as coordinate distance then we must either multiply all terms containing \( r \) with a factor to express all terms in proper distance or we should let all terms remain in coordinate distance only. Multiplying the variable in only one term to express that term in proper distance and letting all other terms remain in coordinate distance is totally inappropiate. McVittie multiplied \( r \) with a factor of \( \alpha(1-\eta) \) in potential term only and did not do the same for other terms. This is clearly wrong meaning that results derived by him are incorrect. Once \( r \) is treated consistently we get the same result as derived in section 2.

Due to the counter-intuitive result and many other reasons mentioned in section 4 a new Dirac equation for curved space-time is derived from energy-momentum relation in general relativity. The new equation for curved space-time shows that size of atoms does change proportional to square of scale factor. The new equation correctly predicts the gravitational redshift reinforcing the claim that the new equation is in-fact the correct equation.

Change in size of atoms with changing scale factor is equally bad for standard model irrespective of whether this change is proportional, or inversely proportional to the change in scale factor. If we take the first result derived in section 2 where the size changes inversely proportional to the square of scale factor (\( r \propto 1/R^2(t) \)) then we should see much higher redshifts in lights from far off galaxies and Supernovae than observed. On the other hand if we assume the result where size changes proportional to the square of scale factor (\( r \propto R^2(t) \)) then we should see blue shifts and not redshifts. Not only this any change in size of atoms with time means all our measures of length will also change with time. If that happens then an observer will measure speed of light to be different at different epochs. This raises serious questions about the validity of standard model of cosmology.

It is not the first time such a conclusion has been drawn about change in size of atoms with changing scale factor. It has been shown earlier as well by using Schrödinger equation solution in curved space time that size of atoms varies directly in proportion to square of scale factor in RW universe [29, 30, 31]. Unfortunately this result has been ignored by the scientific community for many years, but the fact that same result has been repeated using a completely different equation, gives one a confidence that indeed 27 is the correct Dirac equation for curved space-time.

References


