SOME BASIC CONCEPTS OF FRACTIONAL CALCULUS

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Theory I

Abstract

In this research section, a definition for Fractional Derivative is presented.

Theory

Definition For Fractional Derivative

Given

\[ 0 < \alpha < 1 \text{ and } N \in \{1, 2, 3, 4, \ldots\} \] i.e., a set of positive integers from 1 onwards,

We know that a derivative for a function is given by

\[
 f'(x) = \frac{df(x)}{dx} = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}
\]

(1)

In the same spirit, we define the Fractional Derivative in the following fashion:

\[
 f^{1+\alpha}(x) = \frac{d^{1+\alpha}f(x)}{dx} = \lim_{\Delta x \to 0} \frac{f'(x + \alpha\Delta x) - f'(x)}{\alpha\Delta x}
\]

(2)

\[
 f^{2+\alpha}(x) = \frac{d^{2+\alpha}f(x)}{dx} = \lim_{\Delta x \to 0} \frac{f'(x + \alpha\Delta x) - f'(x)}{\alpha\Delta x}
\]

(3)

And similarly, we write

\[
 f^{N+\alpha}(x) = \frac{d^{N+\alpha}f(x)}{dx} = \lim_{\Delta x \to 0} \frac{\underbrace{f(x + \alpha\Delta x) - f(x)}_{\text{times } N}}{\alpha\Delta x}
\]

(4)

Here, the notation, \( f'(x) = \frac{df(x)}{dx} \) \text{ and } \( f^{N+\alpha}(x) = \frac{d^{N+\alpha}f(x)}{dx} \)

(5)

\[
 f^{\overbrace{\underline{\ldots \ldots \ldots \cdot}}^{N \text{ times}}} (x) = \frac{d^N f(x)}{dx^N}
\]

(6)
Theory II

Abstract

In this research section, a method to calculate Fractional Integral is detailed.

1 Theory

For the Integral

\[ \int_{a}^{b} f(x) d^\beta x, \quad \text{where } 0 < \beta < 1 \]  

(7)

we consider the Riemann sum of the kind

**Definition 1:**

\[
\int_{a}^{b} f(x) d^\beta x = \lim_{n \to \infty} \sum_{i=1}^{n} \left\{ f \left( a + \left( \frac{b-a}{n+\beta} \right) \right) \left( \frac{b-a}{n+\beta} \right) \right\} \]

(8)

\[ dx = \left( \frac{b-a}{n+\beta} \right) \]  

(9)

For the Integral

\[ \int_{a}^{b} f(x) d^{1+\beta} x, \quad \text{where } 0 < \beta < 1 \]  

(10)

\[
\int_{a}^{b} f(x) d^{1+\beta} x = \int_{a}^{b} p(x) d^\beta x \]

(11)

where

\[ p(x) = \int f(x) dx \]  

(12)
\[
\int_{a}^{b} f(x) d^{1+\beta} x = \int_{a}^{b} \left( \int_{a}^{b} f(x) dx \right) d^{\beta} x
\]

(13)

Say, for \(\int_{a}^{b} f(x) d^{N+\beta} x\)

(14)

\[
\int_{a}^{b} f(x) d^{N+\beta} x = \int_{a}^{b} \left( \int \left( \int \left( \int f(x) dx \right) dx \right) dx \right) \cdots dx d^{1+\beta} x
\]

(15)

Where \(N\) is a positive Integer

If

\[
g(x) = \left( \int \left( \int \left( \int f(x) dx \right) dx \right) dx \right) \cdots dx
\]

(16)

Then

\[
\int_{a}^{b} f(x) d^{N+\beta} x = \int_{a}^{b} g(x) d^{1+\beta} x
\]

(17)

2 A Definition for Functional Integration

The Integral

\[
\int_{a}^{b} f(x) d^{h(x)} x
\]

(18)

Is a curve where we find the Integral for every co-ordinate of \(h(x)\) for every co-ordinate of \(x\) if \(h(x)\) is a function of \(x\)

References

1. ‘Calculus’ by Thomas & Finney.
2. ‘Calculus’ by Apostol.
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Note

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