

Gedankenexperiment for degree of flatness, or lack of, in early Universe conditions

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Abstract. This document will from first principles delineate the degree of flatness, or deviations from, in early universe models. We will, afterwards, make comparison with recent results we have looked at concerning metric tensor fluctuations and comment upon the role of what early universe gravitational energy may play a role in the presumed deviation from flat space results. Note that $N \sim S_{initial(graviton)} \sim 10^{37}$ will be tied into the presumed results for initial state density, in ways we will comment upon. Leading to observations as to leading to Eq.(25) of this document as to GW, from relic conditions. The deviations from flat space may help confirm the conclusions given by Buchert, Carfora, Kolb, and Wiltshire allegedly refuting the claim by Green and Wald that “ the standard FLRW model approximates our Universe extremely well on all scales, except close to strong field astrophysical objects” , as well as give additional analysis appropriate for adding detail to expanding experimental procedures for investigating non FLRW models such as the Polynomial Inflation models as given by Kobayashi, and Seto, as well as other nonstandard cosmologies, as brought up by Corda, and other researchers. As well as improve upon post Bicep 2 measurements which will avoid GW signatures from interstellar dust, as opposed to relic GW. We hope that our approach may help in the differentiation between different cosmology models. Most importantly, our procedure may help, with refinement of admissible frequency range, avoid the problem of BICEP 2, which had its presumed GW signals from presumed relic conditions identical to dust induced frequencies, as so identified by the Planck collaboration in reference [25] which we comment upon in the conclusion.

Key words; HUP, stress energy tensor, quantum bounce, Infinite quantum statistics, heavy Gravity

i. Introduction

We will start off first, with a description of the following equation which we will derive in the next section. We discuss the implications of a deviation from flat space, with a description of what $S_{initial(graviton)} \sim 10^{37}$ in the aftermath of a quantum bounce implies [1], and what we should be looking forward in terms of structure formation afterwards. The relevant equation we will be working with is from the time component of the Stress energy Tensor which we will write up as, if $V^{(3)}$ is a statement of volume and if $\Lambda_{initial-value} = \Lambda_{today}$ for the cosmological “constant”.

$$\begin{aligned} T_{00} = \rho_{Energy-density} &= \frac{-(g_{00}=1)}{16 \cdot \pi} \cdot \left(\frac{3\tilde{k}_{Curvature-measure}}{a_{initial-scale-factor}^2} + \Lambda_{initial-value} \right) \\ \Leftrightarrow \tilde{k}_{Curvature-measure} &= -\frac{a_{initial-scale-factor}^2}{3} \times \left[\left(16 \cdot \pi \cdot \left(\rho_{Energy-density} = \left[\frac{S_{initial-Entropy} \cdot m_{graviton}}{V^{(3)}} \right] \right) \right) + \Lambda_{initial-value} \right] \end{aligned} \quad (1)$$

In picking this, we are using Ng infinite quantum statistics [1] as a counting factor and likely for $V^{(3)}$ have a Planck length cubed, volume as a starting point, if so then, the mass of the graviton, will be important as well as some considerations given if $\Lambda_{initial-value}$ stays the same, to the present era, or if it has quintessence [2,3], a topic we will bring up. In delineating Eq.(1) above, we will examining the following from first principle, while keeping in mind that [4]

$$\mathfrak{R} = \text{Riemann} - \text{scalar} = -\frac{6 \cdot \tilde{k}_{\text{Curvature-measure}}}{(a_{\text{initial-value}})^2} \quad (2)$$

Then

$$\begin{aligned} T_{uv} &= \frac{1}{\sqrt{-|g|}} \cdot \frac{\delta S_{\text{action}}}{\delta g_{uv}} = \left(\frac{\delta L_{GR}}{\delta g_{uv}} - \frac{g_{uv}}{2} \cdot L_{GR} \right) \\ &\& \\ L_{GR} &= -\frac{\mathfrak{R}}{16\pi} + \frac{\Lambda_{\text{initial-value}}}{8\pi} \\ \Leftrightarrow T_{uv} &= -\frac{g_{uv}}{2} \cdot L_{GR} = \frac{g_{uv}}{2} \cdot \left(\frac{\mathfrak{R}}{16\pi} - \frac{\Lambda_{\text{initial-value}}}{8\pi} \right) \\ \xrightarrow[\substack{(u,v) \rightarrow (0,0) \\ g_{uv} \rightarrow g_{00}=1}]{\text{Energy-density}} \rho_{\text{Energy-density}} &= T_{00} = \left(\frac{\mathfrak{R}}{32\pi} - \frac{\Lambda_{\text{initial-value}}}{16\pi} \right) \end{aligned} \quad (3)$$

If we make the substitution of $\rho_{\text{Energy-density}} = \left[\frac{S_{\text{initial-Entropy}} \cdot m_{\text{graviton}}}{V^{(3)}} \right]$ Then the results, above follow for Eq.(1)

Our supposition is, that if $S_{\text{initial(graviton)}} \sim 10^{37}$ [1,5] is used, as well as $a_{\text{initial-scale-factor}}^2 \sim 10^{-110}$ [5], and $V^{(3)} \sim l_p = \text{Planck-length, cubed}$, and $m_{\text{graviton}} \sim 10^{-62}$ grams [6] then we have an almost but not zero negative value for the $\tilde{k}_{\text{Curvature-measure}}$ value, we will from here discuss its implications and what it says physically.

2. Implications as to choosing $S_{\text{initial(graviton)}} \sim 10^{37}$ for our problem: Where it comes from

First of all, this non zero initial value of the entropy is consistent with a quantum bounce, as can be postulated through LQG, as by [5,7] but it says more than that. In reality the very small value for the $\tilde{k}_{\text{Curvature-measure}}$ in the aftermath of the quantum bounce, with $a_{\text{initial-scale-factor}}^2 \sim 10^{-110}$, has some very interesting implications for information transfer from a prior to a present universe which we will be brought up next. We start with what Turok [8] wrote up as to the initial starting point of analysis, as to where he described the cosmological evolution to describe a perfect bounce," in which the universe passes smoothly through the initial singularity. In what we analyze for our purposes, we have that the 2nd order perturbative term of $h^{T(n)}$ for cosmological perturbations obey, here with a 2nd order contribution we can set as

$$\psi^{(2)}(\eta, x) \sim \frac{A^2}{12} \cdot \left[\exp\left(-\frac{2}{\sqrt{3}} i \tilde{k}_0 \eta\right) \right] \cdot (1 + 2 \cos(2k_0 x)) + \dots \quad (4)$$

Which is a 2nd order perturbative term for the equation for the evolution of h , if $J^n(\eta, x)$ is nonlinear[8]

$$\frac{\partial^2 h^{T(n)}}{\partial \eta^2} + \frac{2}{\eta} \cdot \frac{\partial h^{T(n)}}{\partial \eta} - \frac{\partial^2 h^{T(n)}}{\partial x^2} = -J^n(\eta, x) \quad (5)$$

Then setting a conformal time as approaching early universe conditions requires that

$$\eta \xrightarrow{a \rightarrow a_{INITIAL} \sim 10^{-55}} -10^\xi; \xi \approx \text{very big} \neq \infty \quad (6)$$

Our supposition is, then that we have the following for well behaved GW and early cosmological perturbations being viable, in the face of cosmological evolution with modifying the formalism of Turok [8] to obtain

$$\tilde{k}_0 |\eta| \sim \tilde{k}_0 \times 10^\xi < 1/\varepsilon \Leftrightarrow \tilde{k}_0 < 10^{-\xi}/\varepsilon \quad (7)$$

In practical terms near the initial expansion point it would mean that near the beginning of cosmological expansion we would have an initial energy density of the order of

$$\rho(\text{initial} - \text{energy} - \text{density}) \sim \hbar \cdot 10^{-\xi} / l_p^3 \varepsilon \quad (8)$$

If so then , if we assume that gravitons, of initial mass about 10^{-62} grams, i.e. and that we have Planck mass of about 10^{-5} grams, if gravitons were the only 'information' passed into a new universe, making use of the following expression for the initiation of quantum effects, i.e. by Haggard and Rovelli [7]

$$r \sim \frac{7}{3} m \quad (9)$$

Then, we would have, the initiation of quantum effects as of about[8]

$$r_{\text{entropy-gravitons.contribution}} \sim \frac{7}{3} \times S(\text{entropy-count}) \times 10^{-57} \times l_p \quad (10)$$

Then by making use of Eq.(10) we could, by dimensional analysis, start the comparison by setting values from Eq. (7) and Eq. (10) to obtain

$$10^{-\varepsilon}/\varepsilon \sim \frac{7}{3} \times S(\text{entropy-count}) \times 10^{-57} \quad (11)$$

So that to first order, a graviton count, for a radii of about the order of l_p would be

$$S(\text{entropy-count}) \sim 10^{57} \times \frac{3}{7} \times 10^{-\varepsilon}/\varepsilon \quad (12)$$

Depending upon $\varepsilon < \tilde{k}_0 \cdot \eta < 1/\varepsilon$, this will then lead to a condition for which Eq. (4) vanishes, with

$$10^{-\varepsilon}/\varepsilon \sim 10^{-20} \quad (13)$$

Eq.(13) would put restrictions upon the following, namely

3. Considerations of what could lead to Eq.(4), i.e. 2nd order perturbation to cosmological evolution, vanishing

The simple short course as to the radius achieving its starting point to being quantum mechanical in its effects, from the big bang initiating from a quantum bounce is to have the following threshold for quantum effects to be in action, to the vanishing of Eq.(1). Here the quantum effects start with a value

$$r(\text{quantum-effects}) \sim (10^{-\varepsilon}/\varepsilon) \times l_p \quad (14)$$

If Eq.(4) is zero due to $x = r(\text{quantum-effects})$ and we want Eq.(4) to vanish, it leads to the following for the vanishing of the 2nd order perturbative effect, with λ the critical value of wavelength for which Eq.(4) vanishes, i.e. hence ,

$$\begin{aligned} \cos(k_0 \cdot r(\text{quantum-effects})) &= -1/2 \\ \Leftrightarrow k_0 \cdot r(\text{quantum-effects}) &= \frac{2\pi}{3} \\ \Leftrightarrow k_0 &= \left(\frac{2\pi}{3} \times \frac{\varepsilon}{l_p} \times 10^{\varepsilon} \right) \sim \frac{2\pi}{\lambda} \\ \Leftrightarrow \lambda &\sim \frac{3 \cdot l_p}{\varepsilon} \times 10^{-\varepsilon} \end{aligned} \quad (15)$$

It means that there is the following interval may be our best Quantum Mechanical perturbative indicator in terms of Eq.(4) , that is

$$\frac{l_p}{\varepsilon} \times 10^{-\xi} < x < \frac{3 \cdot l_p}{\varepsilon} \times 10^{-\xi} \quad (16)$$

4. Comparing the variance in position given in Eq.(16) with modified HUP

Note this very small value of x comes from a scale factor, if [9] $z \sim 10^{55} \Leftrightarrow a_{scale-factor} \sim 10^{-55}$, i.e. 55 orders of magnitude smaller than what would normally consider, but here note that the scale factor is not zero, so we do not have a space – time singularity. Then

$$\begin{aligned} \frac{l_p}{\varepsilon} \times 10^{-\xi} < (x = \Delta l) < \frac{3 \cdot l_p}{\varepsilon} \times 10^{-\xi} \\ \Delta l \cdot \Delta p \geq \frac{\hbar}{2} \end{aligned} \quad (17)$$

We will next discuss the implications of this point in the next section, of a nonzero smallest scale factor

We will be using the approximation given by Unruh [10], of a generalization we will write as

$$\begin{aligned} (\Delta l)_{ij} &= \frac{\delta g_{ij}}{g_{ij}} \cdot \frac{l}{2} \\ (\Delta p)_{ij} &= \Delta T_{ij} \cdot \delta t \cdot \Delta A \end{aligned} \quad (18)$$

If we use the following, from the Robertson-Walker metric[3,4].

$$\begin{aligned} g_{tt} &= 1 \\ g_{rr} &= \frac{-a^2(t)}{1 - k \cdot r^2} \\ g_{\theta\theta} &= -a^2(t) \cdot r^2 \\ g_{\phi\phi} &= -a^2(t) \cdot \sin^2 \theta \cdot d\phi^2 \end{aligned} \quad (19)$$

Following Unruh [10] , write then, an uncertainty of metric tensor as, with the following inputs

$$a^2(t) \sim 10^{-110}, r \equiv l_p \sim 10^{-35} \text{ meters} \quad (20)$$

Then, if $\Delta T_{ii} \sim \Delta \rho$ [3,4,5]

$$\begin{aligned}
V^{(4)} &= \delta t \cdot \Delta A \cdot r \\
\delta g_{tt} \cdot \Delta T_{tt} \cdot \delta t \cdot \Delta A \cdot \frac{r}{2} &\geq \frac{\hbar}{2} \\
\Leftrightarrow \delta g_{tt} \cdot \Delta T_{tt} &\geq \frac{\hbar}{V^{(4)}}
\end{aligned} \tag{21}$$

5. The questions of Nonstandard Cosmologies

It is noteworthy that, there have been numerous attempts to vet and prove a modification [11] for the Friedman Walker cosmology, which has been correction to the large scale inhomogeneity raised as a possibility by [12,13], as the approach taken by Wald, and Green, a summary can be seen in the statement that as given by [13] that , “*Our framework requires the metric to be close to a "background metric", but allows arbitrarily large stress-energy fluctuations on small scales*” . We have the situation in defining $V^{(4)}$ that the x component may be defining a situation through Eq.(16) with $V^{(4)} = \delta t \cdot \Delta A \cdot r$, and with the r in $V^{(4)}$ re defined as x as in Eq.(16) which may be validating [11] (especially in the above quote from [11] given above) , whereas Eq. (21) may be in fact satisfying what is quoted in [13]. Having said that, the issues of the nature of determining if there is or not if there are conditions allowing for quantization in the genesis of GR, as given by [14] in the quote that “*On the other hand, one can define Extended Theories of Gravity those semiclassical theories where the Lagrangian is modified, in respect to the standard Einstein-Hilbert gravitational Lagrangian, adding high-order terms in the curvature invariants (terms like R^2 ) or terms with scalar fields non minimally coupled to geometry (terms like $\phi^2 R$)*”, allows for conditions giving more structure to the terms in the Pre Planckian possible quantization of GR we give as $\delta g_{tt} \cdot \Delta T_{tt} \geq \frac{\hbar}{V^{(4)}}$. Note. inputs into the terms δg_{tt} , and ΔT_{tt} may

determine if the quote taken about the admissibility of adding in higher order terms in the curvature as alluded to in [14] above is accurate, and that the definition of classical versions of inputs eventually quantized and put into δg_{tt} , and ΔT_{tt} , reflects a purely minimum contribution to terms in the curvature as given in Eq.(2) ,which will in turn affect the magnitude of Eq.(1) If Eq. (1) is small, it is likely that the higher order terms as in the quote from [14] “*the curvature invariants (terms like R^2 ) or terms with scalar fields non minimally coupled to geometry (terms like $\phi^2 R$)*”, do not play a large role, and that we do not have to talk about extended gravity. If Eq.(1) is not small, then extended gravity.

6. The matter of GW, as ascertained through Reference [15] and Extended Gravity

We start off with a quote from [15] which neatly summarizes up the interesting issues of GW research we should keep in mind; “*Omni-directional gravitational wave background radiation could arise from fundamental processes in the early Universe, or from the superposition of a large number of signals with a point-like origin. Examples of the former include parametric amplification of gravitational vacuum fluctuations during the inflationary era , termination of inflation through axion decay or resonant preheating, Pre-Big Bang models inspired by string theory, and phase transitions in the early Universe; the observation of a primordial background would give access to energy scales of $10^9 - 10^{10} GeV$, well beyond the reach of particle accelerators on Earth*” . Note, [16] has, in page 66 the datum that if there exist an inflaton field or fields , that as stated “*At the end of our simulation, at $t=300m$ the fields were noticeably non Gaussian*” . As stated in [16] this leads to a rapid increase in turbulent interacting scalar waves. I.e. one could, unless we are careful still, even if we have a primordial signal, have through the turbulence, due to preheating a stochastic background , and we go

through the needle in a haystack problem. The author in [17] wrote in the section called **technical problems which need to be addressed in order to improve the quality of research for relic signals ."** *The upshot as claimed by Corda is for that range of GW that $\varphi \geq 10^{-5}$ grams as a lower bound for the inflaton field. If so, then the inflaton field may have a different lower bound if, as an example one looks at $10\text{Hz} < f < 10\text{KHz}$, even if one looks at $H \sim 10^{22} \text{ Hz}$. The lower bound of the inflaton field becomes especially significant, if as an example inflaton fields are connected with initial entropy conditions which Beckwith picked as $n \sim \text{particle-count} \sim 10^5$ ".* The upshot with the frequency, to this range, $10\text{Hz} < f < 10\text{KHz}$ will affect the size of the initial scale factor, admissible to the perturbation of the δg_{ii} term which will be important as to the setting of frequency.

7. Conclusion. Need to avoid measuring Dust. i.e. how to avoid BICEP 2's Mistake

Eq. (21) may, with refinements of $r=x$, in the four dimensional Volume give the new HUP, in our problem, its impact upon GW generation and its relevance to Bicep 2, the search for validation of nonstandard cosmologies, and GW searches. If from Giovannini [18] we can write

$$\delta g_{ii} \sim a^2(t) \cdot \phi \ll 1 \quad (25)$$

Refining the inputs from Eq.(26) means more study as to the possibility of a non zero minimum scale factor, as well as the nature of ϕ as specified by Giovannini [18]. Then we will assert that if $r=x$ then if we use $\frac{l_p}{\varepsilon} \times 10^{-\varepsilon} < (x=r) < \frac{3 \cdot l_p}{\varepsilon} \times 10^{-\varepsilon}$ and then the volume $V^{(4)} = \delta t \cdot \Delta A \cdot r$, as used in [3,4,5]

$$(\delta t \cdot \Delta A) \times \frac{l_p}{\varepsilon} \times 10^{-\varepsilon} < V^{(4)} < (\delta t \cdot \Delta A) \times \frac{3 \cdot l_p}{\varepsilon} \times 10^{-\varepsilon} \quad (26)$$

This Eq. (26) will be put into $\delta g_{ii} \cdot \Delta T_{ii} \geq \frac{\hbar}{V^{(4)}}$, if $\Delta T_{ii} \sim \Delta \rho$, it means that $\delta g_{ii} \cdot \Delta T_{ii} \geq \frac{\hbar}{V^{(4)}}$ that this is defined for all x as to where and when $\frac{l_p}{\varepsilon} \times 10^{-\varepsilon} < (x=r) < \frac{3 \cdot l_p}{\varepsilon} \times 10^{-\varepsilon}$ holds, with the lower value for x signifying the spatial range of x for which quantum mechanics is valid, with three times that value connected as to when the perturbative methods break down. Thereby influencing the range of values for $V^{(4)} = \delta t \cdot \Delta A \cdot r$ in $\delta g_{ii} \cdot \Delta T_{ii} \geq \frac{\hbar}{V^{(4)}}$. Furthermore we have, if there is an eventual weak field approximation according to Katti [4] gravitational spin off according to $g_{ij} = \eta_{ij} + h_{ij}$, with a gravitational wave signal according to, if $V^{(3)} = \Delta A \cdot r$ [4]

$$h_{kl}(x^i) = -\frac{4G}{c^2} \cdot \int_{V^{(3)}} \frac{T_{kl}^*(x^i)}{R} \cdot d^3x' \equiv -\frac{4G}{c^2} \cdot \int_{V^{(3)}} \frac{T_{kl}^*(x^i)}{[\eta_{ij} \cdot (x^i - x'^i) \cdot (x^j - x'^j)]^{1/2}} \cdot d^3x' \quad (27)$$

If the contribution from Pre-Planckian to Planckian is due to the stress energy tensor as given in $\Delta T_{ii} \sim \Delta \rho$ form [5], it means that the relevant relic GW signal will be of the form, with D^{ij} a small quadrupole tensor. This with space-time which is almost flat according to Eq. (1) initially as the genesis of the GW which may be analyzed with a dominant contribution coming from [4]

$$\begin{aligned}
h_{00}(x^i) &= -\frac{4G}{c^2} \cdot \int_{V^{(3)}} \frac{T_{00}^*(x'^i)}{R} \cdot d^3x' \\
&\sim -\frac{4G}{c^2} \cdot \int_{V^{(3)}} \frac{\rho_{Energy-density}(x'^i)}{R} \cdot d^3x' \\
&\sim -\frac{4G}{c^2} \cdot \int_{V^{(3)}} \frac{\left(\frac{\mathfrak{R}}{32\pi} - \frac{\Lambda_{initial-value}}{16\pi} \right)}{R} \cdot d^3x'
\end{aligned} \tag{28}$$

The importance of Eq.(28) is in giving a compliment to [18] as to the problem of relic gravitational waves, which the author views is extremely important. Also, this borrows from [19]. A correct rendering of Eq.(28) would be to determine additional experimental constraints which may determine if detection of early universe gravitational waves is feasible with LIGO technology, or if there is a requirement for other detectors. Certainly, what is wished by this inquiry is to avoid the problems associated with BICEP 2, through a refinement of methods as given in [20]. In addition, of special note would be to avoid picking up interstellar dust effects upon Gravitational waves, which has been a primary reason for the development of methodologies as given in [20], Note also that BICEP2 only observed in one wavelength, which made it difficult for them to prove the B-modes they saw were truly from gravitational waves. Ascertaining Eq.(28) properly, may help alleviate that problem. This value of Eq.(28) would have as its origins the near flat space physics given by Eq.(1) as its genesis with this to consider, as the start [5].

$$\begin{aligned}
\delta g_{rr} T_{rr} &\geq - \left| \frac{\hbar \cdot a^2(t) \cdot r^2}{V^{(4)}} \right| \Bigg|_{a \rightarrow 0} \rightarrow 0 \\
\delta g_{\theta\theta} T_{\theta\theta} &\geq - \left| \frac{\hbar \cdot a^2(t)}{V^{(4)}(1-k \cdot r^2)} \right| \Bigg|_{a \rightarrow 0} \rightarrow 0 \\
\delta g_{\phi\phi} T_{\phi\phi} &\geq - \left| \frac{\hbar \cdot a^2(t) \cdot \sin^2 \theta \cdot d\phi^2}{V^{(4)}} \right| \Bigg|_{a \rightarrow 0} \rightarrow 0
\end{aligned} \tag{29}$$

This Eq.(29) if confirmed experimentally is of potential decisive importance to the problem of discriminating between different cosmology models. Note in the case of Bicep 2 the Planck collaboration had that the frequency of dust signals was about the same as what was reported by Bicep 2 presumed gravitational waves.[20] Hence, the conclusion is inescapable. The value of the flatness calculation as of Eq.(1) and of getting a range of $\delta g_{ii} \sim a^2(t) \cdot \phi$, for say $f \leq 10KHz$ right may, with some additional fine tuning help us find a relic GW frequency range, so we avoid the BICEP 2 problem of getting signals from presumed relic GW producing conditions the same as what would be for dust, as has been stated clearly as the problem destroying the BICEP 2 findings as of 2014. This should be done with adherence to falsifiable measurements to be compared against [19, 20].

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