How to solve the velocity paradox in special relativity?

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Abstract

The present study examined the relativistic interpretation of the increased lifetime of high speed elementary particles, which is an important experimental evidence of time dilation. It is found that the principle of relativity cannot be upheld for both the distance and the velocity between the particle frame and the earth frame in special relativity. For the particle frame, the velocity obtained by directly using the principle of relativity is different from that computed from the distance and the time interval of the particles' proper lifetime, hence a velocity paradox. The relativistic interpretation needs to stipulate that the distance between the earth and the particles observed by the earth frame is larger than the same distance observed by the particle frame, which contradicts the principle of relativity. The aim of the present study is to present this velocity paradox to researchers and teachers in physics, in order to find a satisfactory solution.

Key words: Lorentz ether theory; special relativity; time dilation; muon; velocity paradox.

1 Introduction

Time dilation is an important result of special relativity (Lorentz 1904; Einstein 1905), which has been supported by the findings that the mean lifetime of unstable elementary particles moving at high velocity is much longer than those at rest or moving at low velocity. The first experimental evidence was the detection of a large number of muons at ground level. Muon is an unstable subatomic particle with a mean lifetime of about 2.2 μs (Mulan Collaboration 2007), so even at the speed of light muons would travel only around about 660 meters before their decay. Since naturally occurring muons on the earth are generally created in the upper atmosphere as consequences of collisions between cosmic ray protons and atomic nuclei. If there was no time dilation, very few muons would be detected at ground level. In early 1940s, however, Rossi and colleagues detected a large number of muons at ground level, which suggests the existence of time dilation for muons travelling at a speed close to that of light (Rossi and Hall 1941).

Later studies on muons and other particles provided more precise confirmation of increased lifetime due to their high velocity. Frisch and Smith (1963) compared the flux of muons at two sites with a height difference of 1907 m, Mount Washington and Cambridge, Massachusetts. Although it takes about 6.4 μs for muons at 0.994 *c* to traverse this distance, they detected approximately 563 muons per hour in Mount Washington and 412 muons per hour in Cambridge, which indicates a time dilation factor of 8.8 ± 0.8. This result is in good agreement with the predicted 8.4 ± 2. More experiments have been performed in particle accelerators with pions (Durbin et al 1952), kaons (Burrowes *et al.* 1959) and muons (Lundy 1962; Meyer *et al.* 1963; Eckhause *et al.* 1963), and all have demonstrated an increased lifetime due to high velocity.

The standard interpretation of the increased lifetime of high speed elementary particles is that the proper lifetime of the muons measured by the clocks at rest in the muon frame is still 2.2 μs, but the clocks on the earth record a longer lifetime due to time dilation. According to this interpretation, the mean lifetime of muons at rest on the earth is 2.2 μs as measured by the clocks on the earth, but it will be much larger than 2.2 μs when measured by clocks moving relative to the earth at a velocity close to that of light.

A careful analysis of this relativistic interpretation seems to suggest an inconsistency in the values of the distance and velocity between the earth and the particles as measured by the two reference frames. We might call this inconsistency the velocity paradox. The aim of the present study is to present this velocity paradox to researchers and teachers in physics in order to find a satisfactory solution within the relativistic framework. The rest of the paper is organized as follows: section 2 presents the velocity paradox; section 3 discusses the potential solutions to the paradox and concludes.

2 The principle of relativity and measurements of the same quantities by two frames

In the experiments on the increased lifetime of high speed particles, the important physical quantities are the distance between (one spot of) the earth and the particles, the time intervals measured by the clocks at rest on the earth and the clocks co-moving with the particles, and the velocity between the two frames. For the distance and the velocity, there are only two measurements: the distance and the velocity measured by the earth frame, and the distance and the velocity measured by the particle frame. For the time interval, there are four measurements: the time interval of events in the earth frame measured by the earth frame, the time interval of events in the earth frame measured by the particle frame, the time interval of events in the particle frame measured by the particle frame, and the time interval of events in the particle frame measured by the earth frame.

What is the distance between the earth and the muons from the perspective of observers stationary on the earth and observers co-moving with the muons? According to the principle of relativity, observers A and B who are stationary respectively in two inertial frames of reference with relative velocity v should measure an identical distance between them,

$$d_{BA,A} = d_{AB,B} . \qquad (1)$$

In equation (1), $d_{BA,A}$ is the distance between A and B measured by A and $d_{AB,B}$ the distance between A and B measured by B.

What are the time intervals measured by the clocks on the earth and the clocks co-moving with the muons? When there is no privileged frame, according to the principle of relativity, observers in two frames A and B with perfectly identical clocks should have the same readings of time for events involving objects at rest only in their own reference frames,

$$\Delta t_{A,A} = \Delta t_{B,B} \qquad (2)$$

In the above equation, $\Delta t_{A,A}$ means the time interval in frame A measured by the clocks in frame A, $\Delta t_{B,B}$ means the time interval in frame B measured by the clocks in frame B.

Because of time dilation, the time interval of events in frame B measured by clocks in frame A is longer than that measured by clocks in frame B, and vice verse,

$$\Delta t_{B,A} = \frac{\Delta t_{B,B}}{\sqrt{1-v^2/c^2}},\ \Delta t_{A,B} = \frac{\Delta t_{A,A}}{\sqrt{1-v^2/c^2}} \tag{3}$$

In the above equation, $\Delta t_{B,A}$ means the time interval in frame B measured by the clocks in frame A, $\Delta t_{A,B}$ means the time interval in frame A measured by the clocks in frame B.

Using these distance and time interval expressions, we obtain the velocity of the earth measured in the muons' frame,

$$v_{earth-muon,muon} = \frac{d_{earth-muon,muon}}{\Delta t_{muon,muon}} = \frac{1907}{2.2\times 10^{-6}} = 8.668 \times 10^8 \tag{4}$$

This velocity is nearly three times of the speed of light *c*, which violates the constancy of the speed of light. However, the principle of relativity should apply to the measurement of velocity as well, so we have

$$v_{earth-muon,muon} = v_{muon-earth,earth} \tag{5}$$

In equation (5), $v_{muon-earth,earth}$ is the velocity of the muons relative to the earth measured by the earth frame. If the velocity of muons measured by the observers on the earth is 0.994 *c*, the velocity of the earth measured by the observers co-moving with the muons should also be 0.994 *c*. Equation (5) is the standard assumption of special relativity, and its contradiction with equation (4) has been overlooked so far. Based on application of the principle of relativity to the distance and the velocity between the two reference frames and the time interval of events in the two reference frame, we have different values of the same velocity, hence a velocity paradox.

Some physicists might use the constancy of the speed of light to dismiss equation (4) and conclude that equation (5) is the correct description of the velocity measured by the observers co-moving with the muons. Then we have

$$d_{earth-muon,muon} = d_{muon-earth,earth}\sqrt{1 - v^2/c^2} \qquad (6)$$

The velocity measured by the muon frame is

$$v_{earth-muon,muon} = \frac{d_{earth-muon,muon}}{\Delta t_{muon,muon}} = \frac{d_{muon-earth,earth}\sqrt{1-v^2/c^2}}{\Delta t_{muon,earth}\sqrt{1-v^2/c^2}} = v_{muon-earth,earth}$$

Equation (6) appears to contradict equation (1) and solve the paradox, but in fact within the framework of special relativity the paradox cannot be solved without violating the principle of relativity. The reason is simple: because there is no privileged frame in special relativity, if equation (6) is correct, the principle of relativity will impose the following

$$d_{muon-earth,earth} = d_{earth-muon,muon}\sqrt{1 - v^2/c^2} \qquad (7)$$

Combining equations (6) and (7), we should still have equation (1)

$$d_{muon-earth,earth} = d_{earth-muon,muon} \qquad (8)$$

Therefore, we still have a velocity paradox here, the velocity obtained by applying the principle of relativity to the distance and time interval is different from that obtained by directly applying the principle of relativity to the velocity.

3 Discussions

The usual "solution" for the velocity paradox in the preceding section is to postulate length contraction for the distance between the earth and the muons measured by the muon frame. Instead of 1907 m, the distance contracts to shorter than 660 m. As explained earlier, length contraction being observed in only one reference frame violates the principle of relativity. The principle of relativity does not allow $d_{muon-earth,earth} \neq d_{earth-muon,muon}$.

Another possible solution is to argue that the time interval measured by the co-moving observers is also around 6.4 μs, and the mean lifetime of the muons has simply increased as measured by their own clocks. If this explanation holds, the increased lifetime of high speed elementary particles cannot be used as evidence of time dilation.

This velocity paradox would not arise in the theory of Lorentz and Poincaré, because the ether background provides a mechanism for asymmetric outcomes. When the observers on the earth measured a distance of 1907 m, the observers co-moving with the muons could still measure the same distance as less than 660 m because they have a large velocity relative to the ether frame. In special relativity, the distance (and the velocity) between two reference frames measured by them has to be equal to conform with the principle of relativity, because there is no physical background to support differences. The equality between the two frames in terms of both the velocity and the distance leads to the velocity paradox in special relativity. As all the experiments conducted so far cannot distinguish between Lorentz ether theory and Einstein's special relativity (Zhang 1997), the velocity paradox might suggest that Lorentz ether theory is superior to special relativity in explaining the results on the increased lifetime of unstable high speed elementary particles.

In conclusion, the standard relativistic interpretation of the increased lifetime of high speed unstable elementary particles provides two routes to obtain the velocity observed by the particle frame, which give different values for the same velocity and hence lead to a velocity paradox. The standard interpretation so far appears to make the earth frame more privileged than the particle frame to avoid this paradox, by asserting that the distance between the earth and the particles measured by the particle frame should undergo time dilation, while the same distance measured by the earth frame should not. Such an interpretation violates the principle of relativity and therefore it cannot be correct. If the distance measured by the particle frame is equal to that measured by the earth frame, the velocity measured by the particle frame will be larger than the speed of light, violating the constancy of the speed of light. This velocity paradox deserves the attention of the physics community and a satisfactory solution to this paradox will deepen our understanding of special relativity.

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