A new interpretation of the twin paradox

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Abstract

The present study has examined the prevalent explanations of the twin paradox and found that they are incorrect because they failed to follow the spirit and the logic of special relativity in their application of the Minkowski space-time diagrams. Applying Minkowski diagrams strictly according to special relativity reveals that the overall effect of the frame changes or accelerations is to return the clock of the travelling twin to the same reading as the other twin's clock in the earth frame. Therefore, within the framework of special relativity, there is no age difference between the twins when they meet again.

Keyword: time dilation; twin paradox, special relativity; Lorentz transformation; Minkowski diagram.

1. Introduction

Since the twin paradox was first proposed by Langevin in 1911, a large number of papers have been written to show that there is no real paradox in it and the travelling twin is younger when they meet again. Langevin's version of the twin paradox, in which one twin brother stays on the earth and the other travels outwards at a speed close to that of light and returns, has been explained by the asymmetry in their kinetics. There are two schools of thought among relativistic physicists on the mechanism through which the asymmetry causes differential aging of the twins: the acceleration school and the frame jumping school. The acceleration school thinks that acceleration experienced by the traveling twin causes the age difference (Langevin 1911; Einstein 1918). The frame jumping school considers that frame jumping triggered by acceleration rather than acceleration per se is the cause of differential aging of the twins (Boughn 1989; Debs and Redhead 1996; Gruber and Price 1998).

Both schools find weakness in the explanation of the other school. The acceleration based explanation has the difficulty to account for how same accelerations can lead to different time dilations. For example, using the same acceleration to turn the spacecraft back from outward velocity 0.99995*c* to -0.99995*c* in 1 "earth day" (or instantly), the traveling twin can either travel outward for 100 "earth years" or 1000 "earth years". Then, how can the same acceleration in the 100 years case cause a delay of 99 years, while in the case of 1000 years a delay of 990 years? The frame jumping based explanation has to assume implicitly that the clock reading on board of the spacecraft at the turning-back point is a dilated time rather than a proper time, which seems to be a Lorentzian assumption rather than an Einsteinian assumption (Ma 2014).

The aim of the present study is to investigate why the existing explanations are not entirely convincing and whether there is a relativistic explanation of the twin paradox more rigorous than the existing ones. The present study has reexamined the previous explanations and found that those explanations have not rigorously followed the spirit and the logic of special relativity. According to the standard interpretation of relativistic effects, these effects arise during the process of measurement rather than from materialistic changes. Correct application of the Minkowski space-time diagrams will show that there is no real paradox in the twin paradox and within the framework of special relativity there is no real age difference between the twins when they meet again.

2. The frame-jumping explanation

The Minkowski diagram has been considered as the tool to help people obtain the correct solutions to relativistic phenomena. For the twin paradox, the frame jumping explanation uses a Minkowski diagram to illustrate how a change of frame by the travelling twin results in the age difference between the twins when they meet again. We illustrate the frame-jumping explanation with a case of twin sisters Alice and Betty. If Alice stays on the earth for 10 years while Betty travels outwards at 0.99*c*, and then another 10 year for Betty's return journey to the earth. During Betty's away journey the twins both find the other person aging less but they would not be able to meet to verify their ages. To simplify the analysis, we assume that Betty can accelerate instantly from $v = 0$ to $v = 0.99c$ and decelerate instantly from 0.99*c* to 0.

In a Minkowski diagram (Fig.1), Betty travels out on line *OP* and in her frame Alice's event *A* is simultaneous with event *P*, so Alice is aging more slowly than her. When Betty changes direction of her spaceship, she jumps into a new frame of reference and now she regards *P* as simultaneous with Alice's event *B*, which corresponds to Alice aging incredibly fast during this instant.

Although Betty still finds Alice aging more slowly than her after the change of frame, which will not be sufficient to cancel out the age added to Alice during the turning around moment. Betty appears to have written off a large part of her age and become younger relative to Alice. In the Minkowski diagram the part that has been written off is the part on the *t*-axis between points *A* and *B* (Boughn 1989; Debs and Redhead 1996; Gruber and Price 1998).

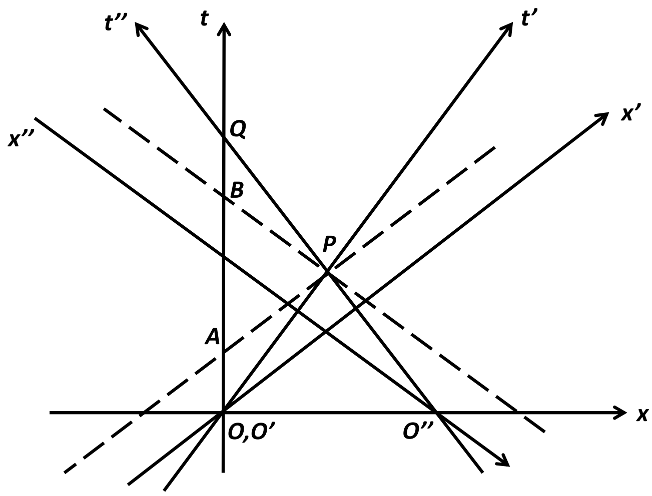

Fig.1. The Minkowski diagram with Alice, the twin sister on the earth, as the "stationary" observer. The traveler Betty departs at *O* (*O′*), turns back at *P* and returns to the earth at *Q*. At *P* Betty jumps from frame *O′* to frame *O″*.

In Alice's frame, Betty has been traveling at 0.99*c*, so that time dilation makes Betty age more slowly according to the Lorentz time dilation formula

$$t'_{Betty} = t_{Alice}\sqrt{1-v^2/c^2}\,. \qquad (1)$$

In Eq. (1), t'_{Betty} is Betty's time interval observed by Alice, and t_{Alice} is Alice's time interval measured by Alice herself. The consequence of the frame jumping by Betty becomes the mechanism that makes the outcomes observed by Alice and Betty compatible.

There is one defect in this frame-jumping explanation, that is, the treatment of the line of simultaneity at the turning-back point and the arrival at the earth is not consistent. At the turning-back point, the switch of direction makes Betty's line of simultaneity changing from *AP* to *BP* (Fig.1); i.e. previously the clocks at *A* and *P* indicated the same time, but now the clocks at *B* and *P* indicate the same time. However, when Betty arrives at the earth, she has the same velocity as that of Alice (both $v = 0$), in the same reference frame and on the same spot; but the frame-jumping explanation claims that Betty's and Alice's clocks indicate different times. This is equivalent to claiming that after turning around at *P*, *P* is simultaneous to *B*, but *P* is not simultaneous to *P*. This cannot be correct. The turning around by Betty's spacecraft and the landing of her spacecraft on the earth are decelerations (accelerations), they should have qualitatively the same effects on Betty's clock and her age.

3. A new interpretation of the twin paradox

In Alice's frame, Betty's age at point *P* corresponds to t_A when she travels at 0.99*c*, but in Betty's own frame Betty's age might not be t_A. To find out what Betty's age at point *P* is in her own frame, we divide Betty's journey into away and return stages and at the end of each stage the velocity between Alice and Betty is zero. Obviously in each stage Alice and Betty have no change of frame except at the end and the beginning. Then we can analyze their age relationship with Minkowski diagrams for each stage.

To Alice, it takes 10 years for the spacecraft to reach the turning-back point and another 10 years to come back to the earth. The point simultaneous with the turning back point *P* on the t-axis in Alice's frame is point E, halfway between *O* and *Q*. At the beginning of the away stage, Betty accelerates instantly to 0.99*c* and then travels away from Alice. According to special relativity, they both find the other person aging less, but their proper time is the same. This result can be obtained by using two Minkowski diagrams (Fig.2A and B) as well as the Lorentz transformation.

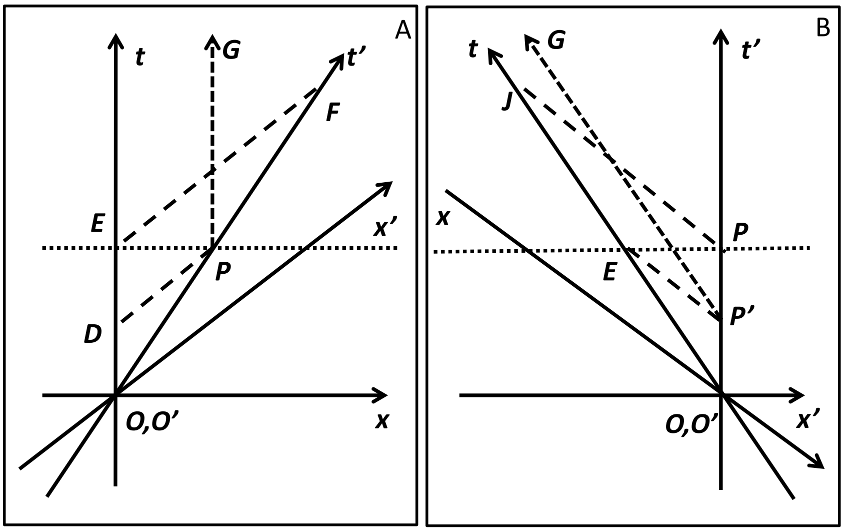

Fig.2 Minkowski diagrams for Betty's outward journey. A. Alice's frame is the observing frame. Betty travels along the t' axis of her own frame and reaches turning back point P which is on the same line of simultaneity as D. B. Betty's frame is the observing frame. Alice travels along the t axis of her own frame and reaches half-way point E which is on the same line of simultaneity as P'.

In Fig.2A, the line of simultaneity at P in Betty's frame before her deceleration to $v = 0$ is DP, while the line of simultaneity of E in Betty's frame is EF; therefore, Alice finds that Betty ages more slowly and Betty is much younger than her at P before Betty's deceleration. When Betty decelerates to $v = 0$ at P, Betty and Alice are in the same reference frame in which E and P are on the same line of simultaneity; therefore, they have the same age at the end of Betty's outward journey. In Fig.2A, PG represents Betty's world line if she keeps $v = 0$ after arriving at P.

Some researchers might find it difficult to accept that Alice and Betty have the same age when Betty decelerates to $v = 0$ at point P. In physics textbooks we have all seen the explanation on why relativistic effects such as time dilation arise during measurement due to the velocity between the two reference frames. But the textbooks have not spelt out what happens if the velocity between the two frames decreases to zero. The Minkowski diagram readily provides the answer, and Alice and Betty have the same age when Betty decelerates to $v = 0$ at point P.

In Fig.2B, the line of simultaneity at E in Alice's frame before Betty's deceleration is $E\,P'$, while the line of simultaneity at P before Betty's deceleration is

JP; therefore, Betty finds that Alice ages more slowly and Alice is much younger at *E* before Betty's deceleration. When Betty decelerates to $v = 0$, she jumps back to Alice's frame and her position in the Minkowski diagram jumps from *P* to *P′*. The line of simultaneity at *E* in Alice's (and Betty's) frame is *EP′*, so Alice and Betty will find that they have the same age again. In Fig. 2B, *P′G* represents Betty's world line if she keeps $v = 0$ after arriving at *P′*.

Some researchers may find it difficult to understand that Betty's deceleration makes her jumping from *P* to *P′* in Fig.2B. Many studies have shown that if a spacecraft at *R* accelerates instantly to *v*, its position in the Minkoski diagram will jump to *R′* (Fig.3A). The frame *t′O′x′* is moving relative to *tOx* at *v*, and $t' = 0$ and $x' = 0$ when $t = 0$ (and $x = 0$). Observers at rest in frame *tOx* will find that *R′* and *I* are at the same spatial location in frame *t′O′x′*, but appear at different times.

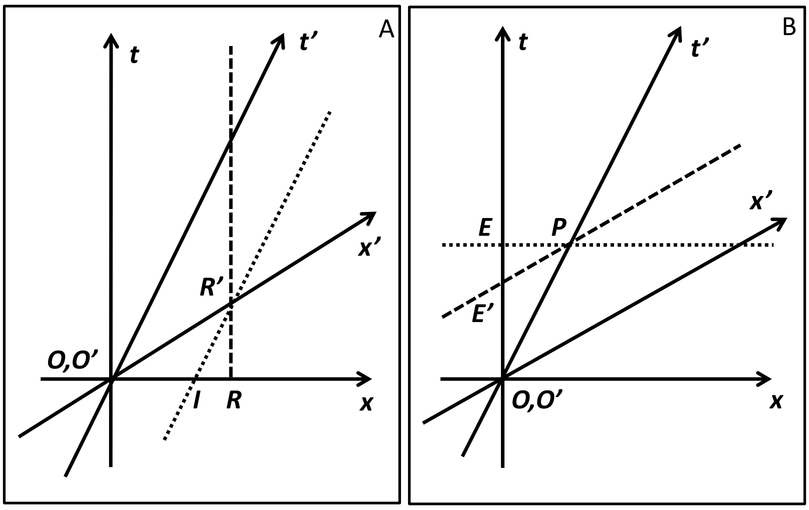

Fig.3 The Minkowski diagrams demonstrating the effects of instant acceleration on the coordinates of a traveler on *x*-axis and *t*-axis in the Minkowski diagrams. A. The traveler is at *R* on the *x*-axis before its instant acceleration and at *R′* after her instant acceleration. B. The traveler is at *E* on the *t*-axis before its instant acceleration and at *E′* after her instant acceleration.

Up to mow, hardly any studies have given a description on what happens when a spacecraft at rest at point *E* in frame *tOx* accelerates instantly to *v* (Fig.3B). As the spacecraft accelerates instantly to *v*, it is moving relative to frame *tOx* and undergoing time dilation (O′E′ represents a shorter time period). Therefore, the position of the spacecraft in the Minkowski diagram will jump from E to E′, which is on the line of simultaneity of P in frame $t'O'x'$. At the end of the outward journey, Betty decelerates instantly to $v = 0$; now Betty has a velocity $-v$ in frame $t'O'x'$, the same as that of Alice. Betty's position in the Minkowski diagram jumps from P to P', and her line of simultaneity at this instant changes from JP to EP'.

At the beginning of Betty's return journeys, she accelerates to $0.99c$ and we can draw the Minkowski diagrams as there are three reference frames O, O' and O'' with $x = x' = x'' = 0$, when $t = t' = t'' = 0$. The velocity between O and O' is v, and that between O and O'' is $-v$. Since neither Alice nor Betty will ever be stationary in frame O' during the return journey, we only need to examine the relationship between Alice and Betty in frames O and O''. When the return journey starts and Betty accelerates from $v = 0$ to $0.99c$ instantly at point P, she jumps from frame O into frame O'' at point P. Alice moves along the t axis of her own frame from E to Q, and Betty travels from P to Q (Fig.4A and B).

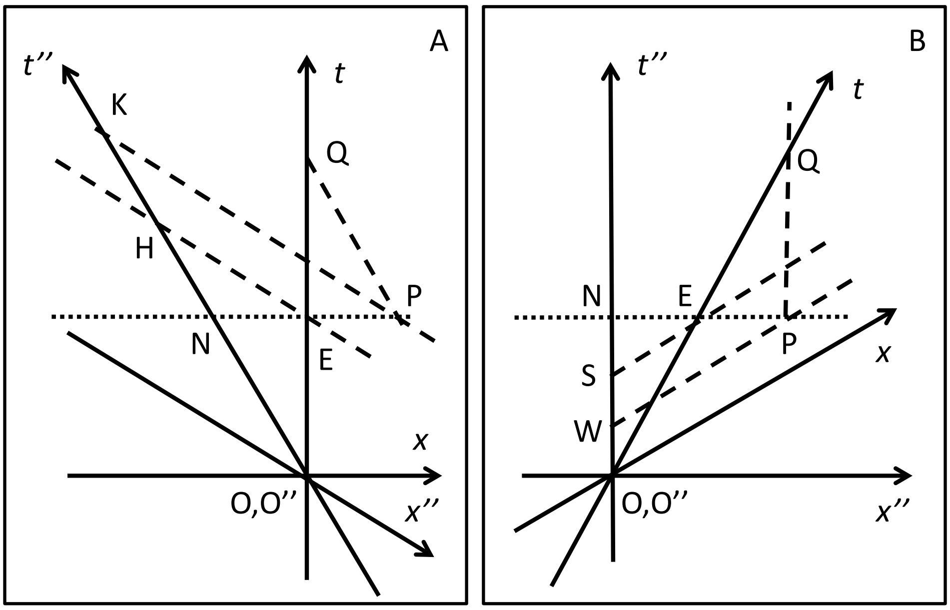

Fig.4 The Minkowski diagrams for Betty's return journey including the time of outward journey. A. Alice's frame is the observing frame. Betty travels along PQ which is parallel with the t'' axis of her own frame. B. Betty's frame is the observing frame. Alice travels along the t axis of her own frame and meets Betty at Q.

In Betty's frame, the line of simultaneity at *P* is *KP*, which is above *HE*, the line of simultaneity at *E* where Alice is at the beginning of Betty's return journey. So at the moment of Betty's acceleration, Alice finds that Betty ages incredibly fast and becomes much older than her. After Betty's acceleration, Alice finds that Betty ages more slowly than her, so their age difference decreases as they approach *Q*. At *Q*, Betty decelerates to $v = 0$, no matter whether Betty's slower aging process has let Alice's age catch up with her age, they will have the same age when they are both at rest at *Q* in the earth frame (Fig.4A).

The Minkowski diagram can also be drawn with frame O'' as the observing (stationary) frame (Fig.4B). Betty moves from *P* to *Q* in parallel to t'' axis and Alice travels from *E* to *Q*. In Alice's frame, the line of simultaneity at *E* is *SE*, which is above the line of simultaneity *WP*. So at the moment of Betty's acceleration, Betty finds that Alice ages incredibly fast and becomes much older than her. However, after the acceleration, Betty finds that Alice ages more slowly than her. Their age difference becomes smaller as they approach *Q*. When they arrive at *Q*, no matter whether Betty's age has caught up with that of Alice due to Alice's slower aging process, Betty's deceleration to $v = 0$ at point *Q* will make them have the same age.

4. Discussion

The cause of mistakes in the previous explanations is a failure to recognize that Alice and Betty have the same age when Betty decelerates to zero velocity at *P*. Instead of following Einstein's special relativity, the previous explanations actually have followed Lorentz ether theory and assumed implicitly or unwittingly that changes in the travelling twin's clock and age as observed by the staying twin are real changes as in Lorentz ether theory. The existing explanations are obviously self-contradictory in their treatment of treatment of the line of simultaneity at the turning-back point and the arrival at the earth. At the turning-back point, the deceleration (switch of direction) makes Betty's time simultaneous with the time as a distant location in her new frame, but the deceleration (Betty's arrival at the earth) could not make her time simultaneous with the time at the same location in her new frame.

Nearly all relativity physicists strongly believe that the travelling twin should be younger than the staying twin. In order to show that the travelling twin becomes younger according to the Lorentz time dilation formula, the explanation of the acceleration school has to ensure that the travelling twin is younger according to the Lorentz time dilation formula no matter how many years she has travelled and no matter at what velocity she has travelled. However, no experiment has found that acceleration can affect time dilation (Sherwin 1960; Farley et al 1968; Bailey et al 1972, 1977, 1979).

The frame jumping school of explanation has a similar problem. If the frame jumping is simply a change of frame, all the effects between the twins are still symmetric and the paradox cannot be solved. The travelling twin views the staying twin in exactly the same manner as the staying twin views the travelling twin except the jumping operations. Without giving frame change some direct effects, the travelling twin is not different from the staying twin.

Some researchers think that the twin paradox can be tackled algebraically with the Lorentz transformation. When the Lorentz transformation is used to explain the twin paradox, it is generally assumed that at the turning back point the traveller's clock reading observed by the traveller herself is

$$t'_1 = t_1\sqrt{1-v^2/c^2}\ . \qquad (2)$$

In Eq. (2), t_1 is the reading of the clocks on the earth observed by people on the earth. This assumption violates the principle of relativity within the framework of special relativity, because Eq. (2) makes the earth frame more privileged than the traveler's frame. Eq. (2) is an assumption based on Lorentz ether theory, not on special relativity.

Some researchers use radio signals sent by observers on the earth and received by the traveller as a measure for time elapsed (Bohm 1965), and they believe that this could give the result that the traveller is younger. Since there is no medium for the propagation of electromagnetic waves according to special relativity, by the principle of relativity, the observers on the earth would be younger when radio signals are sent by the traveller and received by the twin on the earth. As in special relativity the frames of the traveller and the twin on the earth are equal, if instant acceleration does not affect the signal emission and reception, the twins should have the same age. Therefore, the radio signal approach cannot logically show that the travelling twin is younger.

Some people may be concerned with the present conclusion that the twins have the same age after one twin's space travel, because the Hafele-Keating experiment has shown time dilation with clocks transported on airplane around the earth (Hafele and Keating 1972a,1972b). The Hafele-Keating experiment has a very different design and theoretical framework. It is not a direct test of the time difference between the base station clocks and the travelling clocks; instead, the theoretical prediction is based on their velocities relative to the frame of the center of the earth. This is like that the twins in the twin paradox have a common third-observer reference frame. If we compare the age difference of the twins relative to a third observer reference frame, there is no paradox. The experimental verification of the Hafele-Keating's theoretical calculation involves many different factors, the Sagnac effect and general relativity effects among many other things. Therefore, the result of Hafele-Keating experiment has no direct relationship with the solution of the twin paradox.

The present result has important theoretical and practical implications. The twin paradox has puzzled the physics community for over one hundred years, although many relativity physicists believe that they have solved it all along. The present study shows that they have been wrong all along. The present study first raises and solves the question on what happens when the traveller decelerates to zero velocity. When the traveller decelerates to zero velocity, she has the same age as her twin sister on the earth, which implies that the relativistic effects are truly measurement effects.

The present result has also addressed the issue of whether mankind can exploit time dilation effect for space travel. Before the present study, relativity physicists believed that time dilation might be exploited for space travel. For example, an astronaut aged 20 years old travels at 0.995c to a planet moving around a star 50 light years from the earth, it appears to people on the earth that she arrives at the planet at the age 25 and will be able to return (after 50 more years) at the age 30. However, according to the present result, people on the earth will find that when she decelerates to zero velocity for landing on the planet or for her return journey, she ages in an instant to 70 years old. If she lives for 100 years, she could not come back to the earth alive. The present result shows that time dilation cannot be exploited for space travel.

In conclusion, within the framework of special relativity the twins in the twin paradox have the same age when the travelling twin returns, so there is no real paradox. If acceleration affects time dilation through other mechanisms, such as general relativity effects and the Sagnac effect, it may create some age difference between the twins, but this is not an issue in terms of how to solve the twin paradox within special relativity. The result of the present study also implies that time dilation cannot be exploited for space travel.

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