Proof of Fermat’s last theorem (Part III of III) $a^n + b^n = c^n$ (n > 1 and odd)

Objet:
- Another form of Fermat’s last theorem: I prove that the Fermat’s last theorem consist in finding 3 integers (x, y, and z) such as
  \[(x + z)^n + (y + z)^n = (x + y + z)^n\]
- From the Pythagorean triple we obtain a square equals the sum of three squares
  If $c^2 = a^2 + b^2$, and where $d$ is the complement of $c$ to $(a + b)$ was $\begin{align*}
    (c-d)^2 &= (a-d)^2 + (b-d)^2 + d^2.
  \end{align*}$
- From each even integer we obtain at least a Pythagorean triple
- The surface of the Pythagorean triangle

Any number $s = \frac{w^3 - w}{4}$ is the surface of a Pythagorean triangle

\[
w^2 + \left( \frac{w^2 - 1}{2} \right)^2 = \left( \frac{w^2 + 1}{2} \right)^2
\]

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Another form of Fermat's last theorem:

\[
a^n + b^n = c^n
\]
\[
c = a + b - d
\]
\[
(a)^n + (b)^n = (c)^n
\]
\[
(a)^n + (b)^n = (a + b - d)^n
\]
\[
(a - d + d)^n + (b - d + d)^n = (a - d + b - d + d)^n
\]
\[
(a - d + d)^n + (b - d + d)^n = (a - d + b - d + d)^n
\]

If we take:
\[
x = a - d
\]
\[
y = b - d
\]
\[
z = d
\]

Fermat's last theorem consist in finding 3 integers (x, y, and z) such as
\[
(x + z)^n + (y + z)^n = (x + y + z)^n
\]
Using the new form of Fermat's last theorem
From the Pythagorean triple we obtain a square equals the sum of three squares
\[(x + z)^2 + (y + z)^2 = (x + y + z)^2\]
\[(x + z)^2 + (y + z)^2 = x^2 + 2xy + y^2 + 2yz + 2xz\]
\[(x + z)^2 + (y + z)^2 = x^2 + 2xy + y^2 + 2yz + 2xz\]
\[(x + z)^2 + (y + z)^2 = (x + y)^2 - y^2 + (y + z)^2 - z^2 + (x + z)^2 - x^2\]
\[(x + z)^2 + (y + z)^2 = (x + y)^2 - y^2 + (y + z)^2 - z^2 + (x + z)^2 - x^2\]
\[0 = (x + y)^2 - y^2 - z^2 - x^2\]
\[(x + y)^2 = y^2 + z^2 + x^2\]
This means that: \[(c - d)^2 = (a - d)^2 + d^2 + (b - d)^2\]

If \(c^2 = a^2 + b^2\), and where \(d\) is the complement of \(c\) to \((a + b)\) was
\[(c-d)^2 = (a-d)^2 + (b-d)^2 + d^2.\]
A square equals the sum of three squares
From each even integer we obtain at least a Pythagorean triple. For every even integer the list of Pythagorean triple is limited.

\[ a^2 + b^2 = c^2 \]
\[ (x + z)^2 + (y + z)^2 = (x + y + z)^2 \]
\[ x^2 + 2xz + z^2 + y^2 + 2yz + z^2 = x^2 + z^2 + y^2 + 2xz + 2yz + 2xy \]

After simplification we get
\[ z^2 = 2xy \quad (z \text{ is even, since } z = d) \]
\[ \frac{z^2}{2} = xy \]

Take couples xy dividers such as \( xy = \frac{z^2}{2} \), it is sufficient to calculate
\[ (x + z)^2 + (y + z)^2 = (x + y + z)^2 \]

Of each pair of dividers we obtain a Pythagorean triple
Each integer has at least a pair of dividers 1 and itself.

✓ To find all Pythagorean triples
✓ Take an even integer \( z \)
✓ Find \( x \) and \( y \) as \( xy = \frac{z^2}{2} \)

We have \( (x + z)^2 + (y + z)^2 = (x + y + z)^2 \)
The surface of the Pythagorean triangle

\[(x + z)^2 + (y + z)^2 = (x + y + z)^2\]

\[2xy = z^2\]

\[xy = \frac{z^2}{2}\]

\[xy\] is a number and every number \(a\) is the form \(a \times 1\)

\[xy = 1 \times xy\]

\[X = 1 \text{ et } y = \frac{z^2}{2}\]

\[S = \frac{(x + z)(y + z)}{2}\]

\[S = \frac{(x + z)(y + z)}{2} = \frac{(1 + z)(\frac{z^2}{2} + z)}{2}\]

\[2s = (1 + z)(\frac{z^2}{2} + z)\]

\[2s = \frac{z^2}{2} + \frac{z^3}{2} + z + z^2\]

\[2s = \frac{z^3}{2} + \frac{z^3}{2} + \frac{2z}{2} + \frac{2z^2}{2}\]

\[4s = z^2 + z^3 + 2z + 2z^2\]

\[4s = z^3 + 2z + 3z^2 + z - z + 1 - 1\]

\[4s = z^3 + 3z + 3z^2 + 1 - z - 1\]

\[4s = z^3 + 3z + 3z^2 + 1 - (z + 1)\]

\[4s = (z + 1)^3 - (z + 1)\]

\[S = \frac{(z + 1)^3 - (z + 1)}{4}\]

\[S = \frac{w^3 - w}{4}\]

Any number \(s = \frac{w^3 - w}{4}\) is the surface of a Pythagorean triangle

\[w^2 + \left(\frac{w^2 - 1}{2}\right)^2 = \left(\frac{w^2 + 1}{2}\right)^2\]