

Proof of Fermat's last theorem (Part III of III)

$$a^n + b^n = c^n \quad (n > 1 \text{ and odd})$$

Objet:

- Another form of Fermat's last theorem : I prove that the Fermat's last theorem consist in finding 3 integers (x, y, and z) such as $(x + z)^n + (y + z)^n = (x + y + z)^n$
- From the Pythagorean triple we obtain a square equals the sum of three squares
If $c^2 = a^2 + b^2$, and where d is the complement of c to $(a + b)$ was $(c-d)^2 = (a-d)^2 + (b-d)^2 + d^2$.
- From each even integer we obtain at least a Pythagorean triple
- *The surface of the Pythagorean triangle*

Any number $s = \frac{w^3 - w}{4}$ is the surface of a Pythagorean triangle

$$w^2 + \left(\frac{w^2 - 1}{2}\right)^2 = \left(\frac{w^2 + 1}{2}\right)^2$$

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Another form of Fermat's last theorem:

$$a^n + b^n = c^n$$

$$c = a + b - d$$

$$(a)^n + (b)^n = (c)^n$$

$$(a)^n + (b)^n = (a + b - d)^n$$

$$(a - d + d)^n + (b - d + d)^n = (a - d + b - d + d)^n$$

$$(a - d + d)^n + (b - d + d)^n = (a - d + b - d + d)^n$$

If we take :

$$x = a - d$$

$$y = b - d$$

$$z = d$$

Fermat's last theorem consist in finding 3 integers (x, y, and z) such as

$$(x + z)^n + (y + z)^n = (x + y + z)^n$$

Using the new form of Fermat's last theorem

From the Pythagorean triple we obtain a square equals the sum of three squares

$$(x + z)^2 + (y + z)^2 = (x + y + z)^2$$

$$(x + z)^2 + (y + z)^2 = x^2 + 2xy + y^2 + 2yz + +2xz$$

$$(x + z)^2 + (y + z)^2 = x^2 + 2xy + y^2 + 2yz + +2xz$$

$$(x + z)^2 + (y + z)^2 = (x + y)^2 - y^2 + (y + z)^2 - z^2 + (x + z)^2 - x^2$$

$$(x + z)^2 + (y + z)^2 = (x + y)^2 - y^2 + (y + z)^2 - z^2 + (x + z)^2 - x^2$$

$$0 = (x + y)^2 - y^2 - z^2 - x^2$$

$$(x + y)^2 = y^2 + z^2 + x^2$$

This means that: $(c - d)^2 = (a - d)^2 + d^2 + (b - d)^2$

*If $c^2 = a^2 + b^2$, and where d is
the complement of c to $(a + b)$ was
 $(c-d)^2 = (a-d)^2 + (b-d)^2 + d^2$.*

A square equals the sum of three squares

From each even integer we obtain at least a Pythagorean triple.
For every even integer the list of Pythagorean triple is limited.

$$a^2 + b^2 = c^2$$

$$(x + z)^2 + (y + z)^2 = (x + y + z)^2$$

$$x^2 + 2xz + z^2 + y^2 + 2yz + z^2 = x^2 + z^2 + y^2 + 2xz + 2yz + 2xy$$

After simplification we get

$$z^2 = 2xy \quad (z \text{ is even, since } z = d)$$

$$\frac{z^2}{2} = xy$$

Take couples xy dividers such as $xy = \frac{z^2}{2}$, it is sufficient to calculate

$$(x + z)^2 + (y + z)^2 = (x + y + z)^2$$

Of each pair of dividers we obtain a Pythagorean triple

Each integer has at least a pair of dividers 1 and itself.

✓ To find all Pythagorean triples

✓ Take an even integer z

✓ Find x and y as $xy = \frac{z^2}{2}$

We have $(x + z)^2 + (y + z)^2 = (x + y + z)^2$

The surface of the Pythagorean triangle

$$(x + z)^2 + (y + z)^2 = (x + y + z)^2$$

$$2xy = z^2$$

$$xy = \frac{z^2}{2}$$

xy is a number and every number a is the form $a * 1$

$$xy = 1 * xy$$

$$x = 1 \text{ et } y = \frac{z^2}{2}$$

$$S = \frac{(x + z)(y + z)}{2}$$

$$S = \frac{(x + z)(y + z)}{2} = \frac{(1 + z)\left(\frac{z^2}{2} + z\right)}{2}$$

$$2s = (1 + z)\left(\frac{z^2}{2} + z\right)$$

$$2s = \frac{z^2}{2} + \frac{z^3}{2} + z + z^2$$

$$2s = \frac{z^2}{2} + \frac{z^3}{2} + \frac{2z}{2} + \frac{2z^2}{2}$$

$$4s = z^2 + z^3 + 2z + 2z^2$$

$$4s = z^3 + 2z + 3z^2 + z - z + 1 - 1$$

$$4s = z^3 + 3z + 3z^2 + 1 - z - 1$$

$$4s = z^3 + 3z + 3z^2 + 1 - (z + 1)$$

$$4s = (z + 1)^3 - (z + 1)$$

$$S = \frac{(z + 1)^3 - (z + 1)}{4}$$

$$S = \frac{w^3 - w}{4}$$

Any number $s = \frac{w^3 - w}{4}$ is the surface of a Pythagorean triangle

$$w^2 + \left(\frac{w^2 - 1}{2}\right)^2 = \left(\frac{w^2 + 1}{2}\right)^2$$