Robert Dicke's Momentous Error - A Comment on Rev.Mod.Phys. 29 (1957), p. 363

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Abstract

It is shown that the paper 'Gravitation without a principle of equivalence' by American astrophysicist Robert Dicke (1916-1997) contains a simple, but consequential, technical mistake. The purpose of this comment however is not to blame Dicke, but to bring to mind the intriguing idea exposed in his article. The cosmology proposed by Dicke would have been in full agreement with Dirac's Large Number Hypothesis, had Dicke not gone astray at that decisive step. Instead of igniting the dispute with Dirac that followed (R. Dicke, Nature 192 (1961), p. 440; P. A. M. Dirac, Nature 192 (1961) p.441), the two researchers could have joined forces in creating an alternative cosmology that incorporated Mach's principle.

Dicke's variable speed of light version of general relativity. Despite the somehow misleading title 'gravitation without a principle of equivalence', Dicke essentially provides an alternative formulation of general relativity. Instead of a curved space with constant c, it is based on a flat space with variable speed of light. Dicke assumed that the speed of light was lower in the vicinity of masses, and in an analogy with optics, he introduced an index of refraction ([1], eq. 5)

$$\epsilon = \frac{c_0}{c} = 1 + \frac{2GM}{rc^2}. \tag{1}$$

that accounts for the correct light deflection at the Sun, 1.75 arc seconds. Since it has been shown elsewhere [2, 3] that the variable speed of light formulation of general relativity is in agreement with the classical tests, I will focus here on the cosmological implications of Dicke's model. The relation to Einstein's variable speed of light attempt [4, 5] is discussed elswhere [6]. With respect to the above equation (1), Dicke made the following intriguing comment:

> 'The small term on the r.h.s. of the equation is obviously related to the presence of the Sun. But what about the other term, 1? could it have its origin in the remainder of the matter in the universe?'

While Dicke's variable speed of light theory was equivalent to general relativity so far, this additional assumption, though natural, incorporates Mach's principle: gravity would be determined by all other masses in the universe. This formulation leads to a different cosmology.

Since, in this model, the presence of masses reduces the speed of light, the increase of the cosmic horizon (at every instant light arrives from masses that were previously invisible) necessarily causes the speed of light to deacrease with cosmological time scales. Since $c = \lambda f$ must hold at every instant, the change in c causes time and length scales to change accordingly.

This change affects all time and length measurements; to describe it in an appropriate technical framework, Dicke assumed a flat (absolute) space and an absolute time t, while the visible time t' (and lengths l') is measured by time and length scales that undergo a temporal variability. The only consistent way to achieve this is to assume $\dot{R} = c$ for the horizon R, that is, the cosmic expansion rate is just the speed of light. This leads to the following dependencies listed by Dicke on p. 366-367:

Quantity	Symbol	evolution t^γ	present epoch
horizon	R	$t^{\frac{1}{2}}$	10^{26}
speed of light	c	$t^{-\frac{1}{2}}$	10^{-26}
wavelengths	λ	$t^{-\frac{1}{4}}$	10^{-13}
frequencies	f	$t^{-\frac{1}{4}}$	10^{-13}
velocities	v	$t^{-\frac{1}{2}}$	10^{-26}
accelerations	a	$t^{-\frac{3}{4}}$	10^{-39}

Table 1.

This alternative cosmology has a series of surprising properties. Dicke commented that "the ratio of the gravitational to electromagnetical interaction between two elementary particles varies asymptotically inversely as the age of the universe. This agrees with Dirac's hypothesis ..."

Dirac's Large Number Hypothesis. Dicke noted here a relation to Dirac's Large Numner Hypotheses, a coincidence that was noted by Dirac in 1938, and I shall again focus the discussion on this particluar cosmological implication of Dicke's theory. Only one part of Dirac's hypothesis is fairly well known. Dirac wondered about the ratio of electric and graviational forces (about 10^{40}) and suggested that it is related to the age of the universe in 'atomic units'. He defined the atomic unit by the time light needs to pass a proton. Since the age of the universe t_u is related to the horizon by $R_u \approx ct_u$, the latter ratio can also be expressed as the size of the universe divided by the size of the proton[1] Incidentally, the two numbers are in the same order of magnitude.[2] In summary, the first part of Dirac's hypothesis is

$$\frac{F_e}{F_g} = \frac{e^2}{4\pi\epsilon_0 G m_p m_e} \approx 10^{40} \approx \frac{R_u}{r_p} := \tau \qquad (2)$$

[1]Since Rutherfords experiments in 1914, this was a well-defined number.

[2]Dirac never claimed that the coincidence was precise, because he acknowledged that an underlying theory was still missing. Such a theory, however, could well include other natural numbers such as the fine structure constant or the electron-proton mass ratio, and make the coincidence precise. The conceptual value of Dirac's idea consists of the calculability of these constants of nature as a matter of principle.

Dirac called this ratio the 'epoch' τ.[3]

This coincidence has received considerable attention and has motivated a series of observational tests that investigated whether the gravitational constant G changes over cosmological times. Until now, there is no evidence for such a change [7]. It is however important to realize that Dirac's prediction of such a change was an additional claim. Even if there is no visible change in G, the Large Number Hypotheses can reflect a profound feature of Nature of which the consequences are not yet fully understood.

It is much less well known that there is another observation by Dirac that strongly supports his first hypothesis: the number of protons in the universe is approximately 10^{80}, the square of the above ratio, or

$$\frac{M_u}{m_p} \approx \tau^2, \tag{3}$$

It is interesting that Dirac's thoughts can be combined with Dicke's assumption that the '1' in his formula has a cosmological meaning. He suggested that

$$\sum \frac{G}{c^2}\frac{m_i}{r_i} = 1 \tag{4}$$

A synthesis. However, Dirac's hypotheses go even further. Eqn.(4) would be satisfied as well if the mass of the proton were one billion times smaller and the number of protons was correspondingly larger, but eqn.(3) would not hold any longer. Thus Dicke was evidently motivated to fulfill (3) in his approach as well, and tried to calculate the number of elementary particles. In his model, the number of particles per unit volume (of absolute space) was constant. Since $R \sim \tau^{\frac{1}{2}}$, Dicke concluded that the number of particles varied as $\tau^{\frac{3}{2}}$ (p. 374). In a subsequent paragraph, he tried to justify the obvious difference to $N \sim \tau^2$ as suggested by Dirac. However, Dicke's conclusion was clearly wrong, due to a neglect of a feature of his own model. He had confounded the absolute time t introduced by him with the apparent time t'.

On p. 366, Dicke had given a list of quantities in which time and length scales showed an explicit dependence on time. If the speed of light varied with $c \sim t^{-\frac{1}{2}}$, then frequencies and wavelengths had to vary as $f \sim t^{-\frac{1}{4}}$ and $\lambda \sim t^{-\frac{1}{4}}$ because $c = f\lambda$ has to hold in every moment.[4] Since all observations, in particular time and length measurements, are performed with respect to $\sigma = 1/f$ and λ, the horizon R' appeared as $\frac{R}{\lambda} \sim t^{\frac{3}{4}}$ and the number of particles $N \sim t^{\frac{3}{2}}$ appeared proportional to R'^2 (which is proportional to R^3, mind the difference between 'absolute' and measured units). To summarize, the above table should be completed as follows:

[3] I introduced a change of notation in order to avoid a misunderstanding with the above index of refraction.

[4] The exponents must have the same value in order to be consistent with the classical tests of general relativity as expressed in Dicke's model.

Quantity	Symbol	evolution t^γ	present epoch
abstract time	t	t^1	10^{52}
horizon	R	$t^{\frac{1}{2}}$	10^{26}
speed of light	c	$t^{-\frac{1}{2}}$	10^{-26}
wavelengths	λ	$t^{-\frac{1}{4}}$	10^{-13}
frequencies	f	$t^{-\frac{1}{4}}$	10^{-13}
velocities	v	$t^{-\frac{1}{2}}$	10^{-26}
accelerations	a	$t^{-\frac{3}{4}}$	10^{-39}
perceived Horizon R'	$\frac{R}{\lambda}$	$t^{\frac{3}{4}}$	10^{39}
perceived epoch t'	$\frac{t}{\sigma}$	$t^{\frac{3}{4}}$	10^{39}
particles	N	$t^{\frac{3}{2}}$	10^{78}

Table 2.

To make the argument crystal clear, let's consider the state of the universe at the time $t = 10.000$ in atomic units.[5] The cosmic horizon, due to the square-root dependecy, had the size of $R = 100$, while the speed of light had fallen to $c = \frac{1}{100}$ of its initial value. Consequently, for wavelengths and frequencies $\lambda = \frac{1}{10}$ and $f = \frac{1}{10}$ holds, while the time scale $\sigma = 1/f = 10$ has grown by the corresponding amount. As a consequence, the horizon, seen with contracting length scales, seems to have the value $\frac{R}{\lambda} = 1.000$, as well as the perceived time $t' = \frac{t}{\sigma} = 1.000$, simulating a cosmos with a constant expansion rate. The number of visible particles is however proprtional to the third power of R, that is $N = 1.000.000$, a number that appears to be the R^2. This is Dirac's observation.

Outlook. Had Dicke, in his argument on p. 374 not overlooked the consequences of his own theory, he had obtained a spectacular agreement with Dirac's observations. It appears that Dicke was uncomfortable in his disagreement with Dirac; otherwise he would not have felt the necessity to justify his result [8]. Dirac [9], on the other hand, seemed not to have gone into the details of Dicke's article that showed a strong conceptual similarity to his own. Although Dirac had proposed a similar temporal evolution of the cosmic horizon, he did not consider a variation of the speed of light c as did Dicke.

In this short note, I have exposed just one obvious aspect of Dicke's idea that certainly deserves attention: full agreement with the Large Number Hypothesis. There are other profound consequences regarding the interpretation of the cosological red shift, which will be addressed in future paper.

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[5] $t = 1$ being the time when the universe was the size of a proton.

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