Abstract Let the model for the mass ratio of the proton to the electron begin with three objects: a ball of radius one; a line interval of length 4π; and, a gravitational force field $\left(\vec{F}_g\right)$, having a singularity. From these three objects we construct a ball of radius $R = (4\pi - 1/\pi)$ with a volume ejected of $(4\pi/3)\cdot[(1/\pi^2)(4\pi - 1/\pi)]$, due to rotational relativistic effects. If we assume that the electron is a ball of radius one, then the mass ratio of the proton to the electron is approximated by

$$M_p/M_e = (4\pi)(4\pi - 1/\pi)(4\pi - 2/\pi) = 1836.15 \tag{A}$$

This model also gives insight to the experimentally observed "shifting shape" of the proton. Finally, the model explains how the approximation given in Equation (A) is the Greatest Lower Bound (GLB) on the mass ratio of the proton to the electron.

The Model It is not unreasonable to assume that an electron is a ball. A ball in the sense of being a solid sphere. For convenience, set the radius of the electron to equal one. (That is, $r_e = 1$.) This model consists of three objects: a ball of radius one; a line interval of length 4π; and, a gravitational force field $\left(\vec{F}_g\right)$, having a singularity. The line segment is attached, tangent to the ball, at one endpoint.

Figure 1: Ball and Stick to Scale

Let $\mathcal{O}$ be the attachment point of the line interval to the ball. Let $\mathcal{R} = 4\pi$ and let $\mathcal{B}$ denote a ball centered at $\mathcal{O}$ with radius $\mathcal{R}$. The line segment and unit ball are displayed in Figure 1: *Ball and Stick to Scale.*

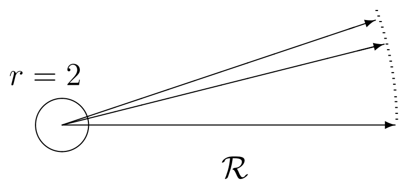

Figure 2: Creating the Ball with $\mathcal{R} = 4\pi$

Once and for all, let $\mathcal{O}$ be the origin of a coordinate system. Now consider the region ($\mathcal{B}$) swept out by the rapid, random motion of the line interval of length $\mathcal{R}$ about $\mathcal{O}$. This is depicted in Figure 2: *Creating the Ball with* $\mathcal{R} = 4\pi$. The ball with unit radius, attached to the 4π line interval, sweeps out a ball centered at the origin $\mathcal{O}$ which has radius $r = 2$.

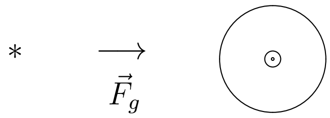

Figure 3: Singularity; Balls, Radii 4π, 2, and π^{-1}

The Inversion of the Spheres The gravitational singularity, denoted by the asterisk ($*$), and generating a force field $\left(\vec{F}_g\right)$, induces an inversion about the reference sphere centered at $\mathcal{O}$ and radius $r = 2$. We observe that in this inversion of the spheres that the entire region outside ball $\mathcal{B}$ is mapped into the ball centered at $\mathcal{O}$ with radius π^{-1}. Let b denote the ball having center at $\mathcal{O}$ and radius π^{-1}. (See Figure 3: *Singularity; Balls, Radii* 4π*, 2, and* π^{-1}.)

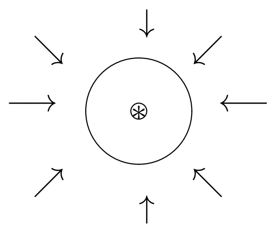

Figure 4: Singularity At Center Ball $\mathcal{B}$

The gravitational attraction of the singularity centered at $\mathcal{O}$ collapses the ball b to a near point mass. This contraction of $\mathcal{R}$ gives rise to the reduced radius $R = (4\pi - \pi^{-1})$ and the corresponding ball B centered at $\mathcal{O}$ with radius R. In fact, it would appear to draw $\mathcal{B}$ to a single point; however, the gravitational attraction of the singularity induces the *centrifugal force* in the ball B; moreover, singularity cannot induce unlimited angular velocity of rotation. Special relativity says $v = \|\vec{v}\| = \|\vec{r} \times \vec{\omega}\| < c$, where c is the speed of light in free space, v is the tangential velocity, $\vec{r}$ is the position vector, and $\vec{\omega}$ is the angular velocity. This bounds the angular rotation $\omega = \|\vec{\omega}\|$.

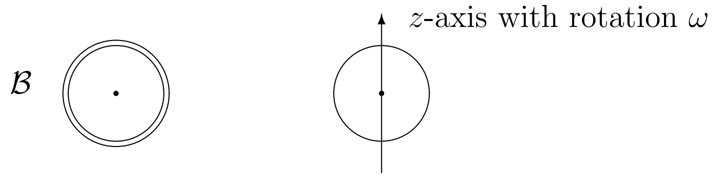

Figure 5: Absorbing the Ball b

The gravitational force field $\left(\vec{F}_g\right)$ has a singularity which, when located in the ball b absorbs the ball b. As $\mathcal{B}$ begins to collapse under the influence of $\vec{F}_g$, the gravitational field induces rotation (ω). Since nothing can travel faster than c, the speed of light in free space, the angular velocity ω in $\mathcal{B}$ is bounded. Let the z-axis be the axis of rotation. Compare this to the *accretion disk* of a black hole in astronomy. Also notice the parabolic shape of rotating liquid in a cylinder.

Not Drawn to Scale

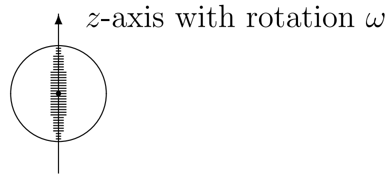

Figure 6: Rotating the Reduced Ball B

The Claim Claim that the singularity in the gravitational field collapses $\mathcal{B}$ to a ball B with radius $(4\pi - \pi^{-1})$ and ejects a quantity of volume equal to $(4\pi/3)\left[\pi^{-2} \cdot (4\pi - \pi^{-1})\right]$ by centrifugal force. This volume amount defines the ratio

$$\frac{(4\pi/3)(4\pi - \pi^{-1})^3 - (4\pi/3)\pi^{-2}(4\pi - \pi^{-1})}{(4\pi/3) \cdot 1} = 1836.15$$

$$\left(4\pi - \frac{1}{\pi}\right)^3 - \frac{1}{\pi^2}\left(4\pi - \frac{1}{\pi}\right) = 1836.15 \tag{1}$$

$$(4\pi)\left(4\pi - \frac{1}{\pi}\right)\left(4\pi - \frac{2}{\pi}\right) = \left(4\pi - \frac{1}{\pi}\right)^3 - \frac{1}{\pi^2}\left(4\pi - \frac{1}{\pi}\right) \tag{2}$$

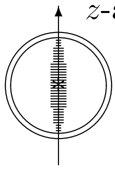

Figure 7: Singularity $(*)$ Center of $\mathcal{B}$

Remark The model consists of a ball of radius $(4\pi - \pi^{-1})$ from which a mass of volume $(4\pi/3) \cdot [\pi^{-2}(4\pi - \pi^{-1})]$ is removed. A candidate for this removed mass is the prolate ellipsoid with axes $\{(4\pi - \pi^{-1}), \pi^{-1}, \pi^{-1}\}$. Other candidates include the spherical sector and the spherical wedge. We will give an argument for the prolate ellipsoid. The spherical sector and spherical wedge for the removed mass also offer a surface area for their curved surfaces of $(4\pi) \cdot (\pi^{-2}) = 4/\pi$. One possibility is that the proton is a "shape shifter," transitioning from one candidate to another. The effect is that the proton surface changes form, as observed by different scattering patterns.

Spherical Sector It is defined to be union of a spherical cap and the cone formed by the apex of the sphere and the base of the cap. If the radius of the sphere is denoted by R and the height of the cap by h, the volume of the spherical sector is

$$V_s = \frac{2\pi R^2 h}{3}$$

The surface area of the curved surface

$$A_s = 2\pi R h$$

If the volume of the spherical sector (V_s) and the radius R are known, then the area (A_s) can be calculated by

$$A_s = \frac{3V_s}{R} \tag{3}$$

Prolate Ellipsoid Let $R > r > 0$. A prolate ellipsoid is a surface obtained by rotating an ellipse about its major axis, R. The minor axes are $\{r, r\}$. The volume of a prolate ellipsoid is $V_e = (4\pi/3)Rr^2$.

Spherical Wedge A spherical wedge (or ungula) is a portion of a ball bounded by two plane semi-disks. Let R be the radius of the ball and α

be the dihedral angle. The volume of a spherical wedge is

$$V_w = \frac{\alpha}{2\pi} \cdot \frac{4}{3}\pi R^3 = \frac{2}{3}\alpha R^3.$$

The surface area of the lune corresponding to the same wedge is given by

$$A_w = \frac{\alpha}{2\pi} \cdot 4\pi R^2 = 2\alpha R^2 = \frac{3}{R} V_w$$

An Exercise Claim $V_s = V_w \Longleftrightarrow A_s = A_w$.

An Exercise Let $\mathcal{B}$ be a closed ball with center at the origin $\mathcal{O}$ and radius $\mathcal{R}$. Rotate the ball $\mathcal{B}$ about the z-axis. Assume that the surface of the ball $\mathcal{B}$ is uniformly compressed inwards by a nearby gravitation singularity $\vec{F}_g$ and that the centrifugal force $\vec{F}_c$ creates a vacuum about the z-axis by compression of the interior of $\mathcal{B}$. Show that the shape of the vacuum created by rotation is a prolate ellipsoid. (See Figure 6: *Rotating the Reduced Ball B.*)

A Solution Let $\mathcal{B}$ be a ball centered at the origin $\mathcal{O}$ with radius $\mathcal{R}$. Let the z-axis be the axis of rotation and let the centrifugal force along the x-axis be denoted by $\vec{F}_c$. We have the formula

$$\vec{F}_c = m x \omega^2 \,\hat{\mathbf{i}}$$

where m is a particle of mass and ω is the angular velocity of the rotating (liquid) interior of $\mathcal{B}$. Since nothing can travel faster than c, the speed of light in free space, the angular velocity ω in $\mathcal{B}$ is bounded.

$$m = \frac{m_0}{\sqrt{1-\beta^2}} \qquad \beta = \frac{v}{c} \qquad \beta^2 = \frac{\omega^2 x^2}{c^2}$$

Let $\vec{F}_g$ be the gravitation vector. We define $g = \left\|\vec{F}_g\right\|$ and $m_0\vec{F}_g = m_0 G_x \hat{\mathbf{i}} + m_0 G_z \hat{\mathbf{k}}$. $\vec{F} = \vec{F}_c - m_0 G_x \hat{\mathbf{i}}$. The angle α is the angle made between $\vec{F}$ and $\vec{G}_z$.

$$\tan(\alpha) = \frac{m x \omega^2 - m_0 G_x}{-m_0 G_z} = -\frac{F}{G_z} \tag{4}$$

Ansatzes We make the *Ansatzes* that $G_x/G_z = (c^2/g)\delta(x)$,

$$\int_0^x (G_{\tilde{x}}/G_z)\, d\tilde{x} = \int_0^x (c^2/g)\delta(\tilde{x}) d\tilde{x} = c^2/g$$

and that $G_z = -g$. Observe that m_0 appears in both the numerator and denominator of Equation (5) and therefore cancel each other.

$$\frac{dz}{dx} = \tan(\alpha) = \frac{x\omega^2}{-g \cdot \sqrt{1 - (\omega^2 x^2)/c^2}} + \frac{G_x}{G_z} \tag{5}$$

$$z(x) = \int_0^x \frac{\tilde{x}\omega^2 \, d\tilde{x}}{-g \cdot \sqrt{1 - (\omega^2 \tilde{x}^2)/c^2}} + \int_0^x \frac{G_{\tilde{x}}}{G_z} \, d\tilde{x} \tag{6}$$

$$z(x) = \int_0^x \frac{\tilde{x}\omega^2 \, d\tilde{x}}{-g \cdot \sqrt{1 - (\omega^2 \tilde{x}^2)/c^2}} + \frac{c^2}{g} \tag{7}$$

$$z(x) = \frac{c^2}{g} \left[\sqrt{1 - \frac{x^2\omega^2}{c^2}} - 1 \right] + \frac{c^2}{g} \tag{8}$$

The above relation defines the region of an ellipse. This is a relation for the planar ellipse. It generalizes to a prolate ellipsoid. Recall that $v = \mathcal{R}\omega < c$, where c is the speed of light in free space, ω is the angular velocity, v is the tangential velocity, and $\mathcal{R}$ is the radius vector. Also, $0 \leq v = \|\vec{\omega} \times \hat{\mathbf{x}}\| < c$.

Algebra We work out and present the quadratic equation for the prolate ellipsoid.

$$\begin{aligned}
z &= \frac{c^2}{g} \left[\sqrt{1 - \frac{x^2\omega^2}{c^2}} \right] \\
\frac{g}{c^2}\, z &= \left[\sqrt{1 - \frac{x^2\omega^2}{c^2}} \right] \\
\left(\frac{g}{c^2}\, z \right)^2 &= 1 - \frac{x^2\omega^2}{c^2} \\
\left(\frac{g^2}{c^4} \right) z^2 + \left(\frac{\omega^2}{c^2} \right) x^2 &= 1 \\
\frac{z^2}{c^4/g^2} + \frac{x^2}{c^2/\omega^2} &= 1
\end{aligned}$$

More generally,

$$\frac{z^2}{c^4/g^2} + \frac{x^2}{c^2/\omega^2} + \frac{y^2}{c^2/\omega^2} = 1$$