ALL YOU NEED TO KNOW ABOUT EUCLIDEAN AND EUCLIDEAN TYPE INNER PRODUCT SCHEME

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Abstract

In this research monograph, a novel type of Colloquial Definition of *Euclidean Inner Product* and *Outer Product* is advented. Based on this definition, the author consequently presents a Proof for the formula for the *Euclidean Inner Product*.

Theory I

Colloquial Definition of Euclidean Inner Product and Outer Product

Consider two vectors $\vec{A} = \sum_{i=1}^{n} a_i \hat{e}_i$ and $\vec{B} = \sum_{i=1}^{n} b_i \hat{e}_i$

We propose an Inner Product Of the kind detailed as below.

$$\vec{A} \cdot \vec{B} = \sum_{i=1}^{n} \left\{ b_i^2 F_1 + a_i^2 F_2 \right\}$$

$$\begin{aligned} &where\ F_1 = 1, F_2 = 0\ when\ a_i > b_i \\ &and\ F_2 = 1, F_1 = 0\ when\ a_i < b_i \end{aligned}$$

And we also propose an *Outer Product* of the kind detailed below.

Consider two vectors $\vec{A} = \sum_{i=1}^{n} a_i \hat{e}_i$ and $\vec{B} = \sum_{i=1}^{n} b_i \hat{e}_i$

Then their accurate Outer Product is given by

$$\vec{A} \cdot \vec{B} = \sum_{i=1}^{n} \left\{ \left(a_i b_i - b_i^2 \right) F_1 + \left(a_i b_i - a_i^2 \right) F_2 \right\}$$

$$\begin{aligned} &where\ F_1 = 1, F_2 = 0\ when\ a_i > b_i \\ &and\ F_2 = 1, F_1 = 0\ when\ a_i < b_i \end{aligned}$$

Theory II

Proof for the formula of *Euclidean Inner Product*

The colloquial definition of Inner Product mentioned in the above section (Theory I) motivates us now to propose a Proof for the formula of *Euclidean Inner Product* computed as shown below:

<table>
<tr><td colspan="18">If
$a_i < b_i$
$\vec{A}\cdot\vec{B}_{FirstTerm} = \sum_{i=1}^{n}\{a_i^2\}$
Which is the area gotten by squaring the lower dimension
Then, the left over area is given by
$\left(\sum_{i=1}^{n}\{(b_i-a_i)a_i\}\right)$</td></tr>
<tr><td colspan="10">If $(b_i-a_i)<a_i$
$\vec{A}\cdot\vec{B}_{SecondTerm} = \sum_{i=1}^{n}\{(b_i-a_i)^2\}$
Now, the Left over area is given by
$\left(\sum_{i=1}^{n}\{[(a_i-(b_i-a_i))](b_i-a_i)\}\right)$</td><td colspan="8">If $a_i<(b_i-a_i)$
$\vec{A}\cdot\vec{B} = \sum_{i=1}^{n}\{(a_i)^2\}$
Now, the Left over area is given by
$\left(\sum_{i=1}^{n}\{[((b_i-a_i)-a_i)](a_i)\}\right)$</td></tr>
<tr><td colspan="5">If
$(a_i-(b_i-a_i))<(b_i-a_i)$
$\vec{A}\cdot\vec{B}_{ThirdTerm} = \sum_{i=1}^{n}\{(a_i-(b_i-a_i))^2\}$
Now the left over area is given by
$\left(\sum_{i=1}^{n}\{(b_i-a_i)-(a_i-(b_i-a_i))\}(a_i-(b_i-a_i))\right)$</td><td colspan="5">If
$((b_i-a_i)-a_i)<(a_i-(b_i-a_i))$
$\vec{A}\cdot\vec{B}_{ThirdTerm} = \sum_{i=1}^{n}\{((b_i-a_i)-a_i)^2\}$
Now the left over area is given by
$\left(\sum_{i=1}^{n}\{(a_i-(b_i-a_i))-((b_i-a_i)-a_i)\}((b_i-a_i)-a_i)\right)$</td><td colspan="4">If
$((b_i-a_i)-a_i)<a_i$
$\vec{A}\cdot\vec{B}_{ThirdTerm} = \sum_{i=1}^{n}\{((b_i-a_i)-a_i)^2\}$
Now the left over area is given by
$\left(\sum_{i=1}^{n}\{a_i-((b_i-a_i)-a_i)\}((b_i-a_i)-a_i)\right)$</td><td colspan="4">If
$a_i<((b_i-a_i)-a_i)$
$\vec{A}\cdot\vec{B}_{ThirdTerm} = \sum_{i=1}^{n}\{a_i^{2}\}$
Now the left over area is given by
$\left(\sum_{i=1}^{n}\{((b_i-a_i)-a_i)-a_i\}(a_i)\right)$</td></tr>
<tr><td colspan="4">If
$\{(b_i-a_i)-(a_i-(b_i-a_i))\}<(a_i-(b_i-a_i))$
$\vec{A}\cdot\vec{B}_{FourthTerm} = \ldots\ldots$
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$\{(a_i-(b_i-a_i))-((b_i-a_i)-a_i)\}<((b_i-a_i)-a_i$
$\vec{A}\cdot\vec{B}_{FourthTerm} = \ldots\ldots$
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$\vec{A}\cdot\vec{B}_{FourthTerm} = \ldots\ldots$
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........................</td><td colspan="4">If
$\{((b_i-a_i)-a_i)-a_i\}<(a_i)$
$\vec{A}\cdot\vec{B}_{FourthTerm} = \ldots\ldots$
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Therefore, we can write the net Holistic Inner Product for the explicitly computed first case (along a certain branch) as

$$\vec{A}\cdot\vec{B}=\sum_{i=1}^{n}\left\{a_i^2\right\}+\sum_{i=1}^{n}\left\{(b_i-a_i)^2\right\}+\sum_{i=1}^{n}\left\{(a_i-(b_i-a_i))^2\right\}+.......$$

In this fashion, we can compute the net *Holistic Inner Product* along the appropriate branch as dictated by the numerical values of

a_i

And

b_i

We can note that this infinite sum is also equal to

$$\vec{A}\cdot\vec{B}=\sum_{i=1}^{n}\left\{a_i\,b_i\right\}$$

Therefore, this also forms the ***Proof*** of

$$\vec{A}\cdot\vec{B}=\sum_{i=1}^{n}\left\{a_i\,b_i\right\}$$

THEORY II.I

Euclidean Inner Product of two N-Dimensional Matrices A and B

In this research section, the *Euclidean Inner Product of two N-Dimensional Matrices A and B* is slated by the author.

One can note that, one can find the Euclidean Inner Product of two N-Dimensional Matrices A and B using the following definition [1]

$$A \cdot B = \sum_{i_1}\sum_{i_2}\cdots\cdots\sum_{i_{n-1}}\sum_{i_n} A_{i_1 i_2 i_3 \ldots\ldots i_n} B_{i_1 i_2 i_3 \ldots\ldots i_n}$$

where $A_{i_1 i_2 i_3 \ldots\ldots i_n}$ and $B_{i_1 i_2 i_3 \ldots\ldots i_n}$ are the elements of the two N-Dimensional Matrices A and B.

THEORY III

Geometrical Inner Product And The Fundamental Postulate Of Modern Quantum Theory

In this research section, a novel type of *Geometrical Inner Product And The Fundamental Postulate Of Modern Quantum Theory* Is Advented.

Consider two normalized vectors $\hat{A} = \frac{1}{\sqrt{\sum_{i=1}^{n} a_i^2}} \sum_{i=1}^{n} a_i \hat{e}_i$ and $\hat{B} = \frac{1}{\sqrt{\sum_{i=1}^{n} b_i^2}} \sum_{i=1}^{n} b_i \hat{e}_i$

We propose a Geometrical Inner Product Of the kind detailed as below.

$$\hat{A} \cdot \hat{B} = \left\{ 1 - \left| Sin\left(\frac{\theta}{2}\right) \right| \right\}$$

Example:

Now, we can write the Fundamental Postulate Of The Modern Quantum Theory as the Similarity between any two normalized states $\hat{A}$ and $\hat{B}$ is given by $\hat{A} \cdot \hat{B} = \left\{ 1 - \left| Sin\left(\frac{\theta}{2}\right) \right| \right\}$.

THEORY III.I

A Novel Type Of Geometric Inner Product And The Fundamental Postulate Of Quantum Theory (Version 2)

In this research section the author has presented '*A Novel Type Of Geometric Inner Product And The Fundamental Postulate Of Quantum Theory (Version 2)*

Consider two normalized vectors $\hat{A} = \frac{1}{\sqrt{\sum_{i=1}^{n} a_i^2}} \sum_{i=1}^{n} a_i \hat{e}_i$ and $\hat{B} = \frac{1}{\sqrt{\sum_{i=1}^{n} b_i^2}} \sum_{i=1}^{n} b_i \hat{e}_i$

We propose a Geometric Inner Product Of the kind detailed as below.

$$\hat{A} \cdot \hat{B} = \left\{ 1 - \left| Sin\left(\frac{\theta}{2}\right) \right| \right\}$$

where theta is the smaller angle between the two vectors considered.

The motivation to present such a Geometric Inner Product is described below.

Firstly, we consider a circle with the center of it as the origin for the three orthogonal Cartesian axes, and considering any two points on it and joining these points to the origin we form two position vectors. Let these be denoted by the vectors

$\hat{A}$ and $\hat{B}$

Since these vectors are normalized, the only aspect that determines their similarity is the extent of their nearness. This extent of nearness is the distance between them (between the two points (arrowheads of the two position vectors) on the circle).

This distance is given by

$$2Sin\left(\frac{\theta}{2}\right)$$

Where theta is the smaller angle between the two position vectors.

However, since , we have to take care of the condition that when they are nearest (i.e., the same), the similarity is maximum, i.e., their dot product should be 1. And when they are totally apart, it must be 0. Therefore, to achieve this we rewrite the dot product as

$$\hat{A} \cdot \hat{B} = \left\{ 1 - \left| Sin\left(\frac{\theta}{2}\right) \right| \right\}$$

Wherein, we have divided the extent of nearness distance

$2Sin\left(\frac{\theta}{2}\right)$

is divided by 2, a modulus operator applied (to take care of theta more than 180 degrees) and the whole term is subtracted from 1.

> ***Note:***
>
> When we have to find the *Inner Product* of two non normalized and different vectors, we consider the larger magnitude vector upto the smaller magnitude vector and can find their extent of similarity using $\hat{A}\cdot\hat{B}=\left\{1-\left|Sin\left(\frac{\theta}{2}\right)\right|\right\}$ and we can note that $|\overline{B}|=k|\overline{A}|$ where $k>1$. Therefore, we can say that the two vectors are as much similar as given by $\hat{A}\cdot\hat{B}=\left\{1-\left|Sin\left(\frac{\theta}{2}\right)\right|\right\}$ and that $|\overline{B}|=k|\overline{A}|$ where $k>1$.

Now, we can simply state the ***Fundamental Postulate Of Quantum Theory*** which states that the similarity between any two states is given by their dot product, in this case, $\hat{A}\cdot\hat{B}=\left\{1-\left|Sin\left(\frac{\theta}{2}\right)\right|\right\}$.

THEORY IV, V

Curved Path Geometric Inner Product And Outer Product and In N-Dimensional Space and the consequential Fundamental Postulate Of Quantum Theory

In this research section, a novel type of *Curved Path Geometric Inner Product And Outer Product and In N-Dimensional Space and the consequential Fundamental Postulate Of Quantum Theory* is advented.

Theory

Consider two normalized vectors $\hat{A}=\frac{1}{\sqrt{\sum_{i=1}^{n}a_i^2}}\sum_{i=1}^{n}a_i\hat{e}_i$ and $\hat{B}=\frac{1}{\sqrt{\sum_{i=1}^{n}b_i^2}}\sum_{i=1}^{n}b_i\hat{e}_i$

We propose a Geometric Inner Product Of the kind detailed as below.

$\hat{A}\cdot\hat{B}=\left|\frac{\theta}{\pi}-1\right|$ for $0\leq\theta\leq 2\pi$

where theta (in radians) is the smaller angle between the two vectors considered.

The motivation to present such a Geometric Inner Product is described below.

Firstly, we consider a circle with the center of it as the origin for the three orthogonal Cartesian axes, and considering any two points on it and joining these points to the origin we form two position vectors. Let these be denoted by the vectors

$\hat{A}$ and $\hat{B}$

Since these vectors are normalized, the only aspect that determines their similarity is the extent of their nearness. This extent of nearness is the ***curved distance*** between them (between the two points (arrowheads of the two position vectors) on the circle).

This distance is given by

θ

where theta is the smaller angle between the two position vectors.

However, since , we have to take care of the condition that when they are nearest (i.e., the same), the similarity is maximum, i.e., their dot product should be 1. And when they are totally apart, it must be 0. Therefore, to achieve this we rewrite the dot product as

$\hat{A}\cdot\hat{B}=\left|\frac{\theta}{\pi}-1\right|$ for $0\le\theta\le 2\pi$

Outer Product

The Outer Product will therefore be

$\hat{A}\cdot\hat{B}=\left|\left(\frac{\pi-\theta}{\pi}\right)-1\right|$ for $0\le\theta\le 2\pi$

Curved Inner Product In N-Dimensional Space

Consider two normalized vectors $\hat{A}=\frac{1}{\sqrt{\sum_{i=1}^{n}a_i^2}}\sum_{i=1}^{n}a_i\hat{e}_i$ and $\hat{B}=\frac{1}{\sqrt{\sum_{i=1}^{n}b_i^2}}\sum_{i=1}^{n}b_i\hat{e}_i$

We propose a Geometric Inner Product Of the kind detailed as below.

$\hat{A}_n\cdot\hat{B}_n=\left|\frac{\theta_n}{\pi_n}-1\right|$ for $0\le\theta_n\le(2\pi)_n$

where theta***n*** (in radians) is the hyper angle (in ***n*** dimensional space) between the two vectors considered in ***n*** dimensional space. And

$(2\pi)_n$

Is the total hyper-steradianical type angle in ***n*** dimensional space.

The motivation to present such a Geometric Inner Product is described below.

Firstly, we consider a ***hyper sphere*** (in ***n*** dimensional space with unit radius) with the center of it as the origin for the ***n*** orthogonal axes, and considering any two points on it and joining these points to the origin we form two position vectors. Let these be denoted by the vectors

$\hat{A}_n$ *and* $\hat{B}_n$

Since these vectors are normalized, the only aspect that determines their similarity is the extent of their nearness. This extent of nearness is the ***curved hyperdistance*** between them (between the two points (arrowheads of the two position vectors) on the ***hyper sphere*** (in ***n*** dimensional space with unit radius)) .

This distance is denoted by

θ_n

where theta***n*** is the smaller hyper angle (in ***n*** dimensional space) between the two position vectors.
However, since , we have to take care of the condition that when they are nearest (i.e., the same), the similarity is maximum, i.e., their dot product should be 1. And when they are totally apart, it must be 0. Therefore, to achieve this we rewrite the dot product as

$\hat{A}_n \cdot \hat{B}_n = \left| \frac{\theta_n}{\pi_n} - 1 \right|$ for $0 \le \theta_n \le (2\pi)_n$

and

$(2\pi)_n$

Is the total hyper-steradianical type angle in ***n*** dimensional space.

Outer Product

The Outer Product will therefore be

$\hat{A}_n \cdot \hat{B}_n = \left| \left(\frac{\pi_n - \theta}{\pi_n} \right) - 1 \right|$ for $0 \le \theta_n \le (2\pi)_n$

where

π_n

Is the appropriate (with respect to the inertia of context) hyper-steradianical type angle in ***n*** dimensional space.

Fundamental Postulate Of Quantum Theory

Such outer product is useful in characterizing NP problems. And also in characterizing large primes.

Now, we can simply state the ***Fundamental Postulate Of Quantum Theory*** which states that the similarity between any two states is given by their dot product, in this case, $\hat{A}_n \cdot \hat{B}_n = \left| \frac{\theta_n}{\pi_n} - 1 \right|$ for $0 \le \theta_n \le (2\pi)_n$ where the description of the variables used already detailed in the previous paragraphs.

THEORY VI

The Universal Quantum Basis

In this research section, the author presents a novel concept of Universal Quantum Basis
For The Inner Product, the Universal Quantum Basis can be given by

$$UQB = \sum_{i=1}^{n} \left(\hat{A}_n \cdot \hat{B}_n \right) \hat{e}_i$$ i.e., for $$0 \le \theta_n \le (2\pi)_n$$

$$UQB = \sum_{i=1}^{n} \left| \frac{\theta_i}{\pi_i} - 1 \right| \hat{e}_i$$ i.e., for $$0 \le \theta_n \le (2\pi)_n$$

Similarly, a seasoned reader of our research monographs can derive the Universal Quantum Basis for the case of Outer Product.

References

1. *http://arxiv.org/abs/1009.3809v1 'One, Two, Three and N Dimensional String Search Algorithms'*
2. *Modern Quantum Mechanics* by J.J. Sakurai.
3. *Linear Algebra* by Hoffman , Kunze.

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