Modern Physics Letters A

KALUZA-KLEIN NATURE OF ENTROPY FUNCTION

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In the present study, we mainly investigate the nature of entropy function in non-flat Kaluza-Klein universe. We prove that the first and generalized second laws of gravitational thermodynamics are valid on the dynamical apparent horizon.

Keywords: Kaluza-Klein; dark energy; thermodynamics.

PACS Nos.: 04.50.+h; 95.30.Tg; 98.80.Cq.

1. Introduction

Recently, Planck collaboration [1] have observed that the matter in our universe is dominated by two mysterious components: 68.3 percent dark energy and 26.8 percent dark matter. It is hypothesized that the Universe is flat, homogeneous and isotropic over large scale and the dark components put the Universe into the phase of accelerated expansion [1–5].

Many cosmologists commonly think that extra-dimension may be a useful candidate to explain the dark part of universe. Although there are many suggestions such as Tachyon [6], K-essence [7], quintom [8], phantom [9], quintessence [10], Chaplygin gas [11], Polytropic gas [12], modified gravity [13,14] and reconstruction in modified theories [15] for the explanation of dark universe, the dark nature of our universe is completely unknown [16]. The cosmological constant has been assumed to be the best and simplest instrument to investigate the dark energy and dark matter. Actually, it is the earliest theoretical candidate, but it gives some difficulties like cosmic-coincidence puzzle and fine-tuning [17]. The current value of cosmological constant is about $10^{-55}cm^{-2}$, but the corresponding value in particle physics is 10^{120} times grater than this factor [18, 19]. This is the difficulty known as the fine-tuning. On the other hand, the cosmic-coincidence problem comes forward due to the comparison of dark matter and dark energy in the present expanding universe [18]. The recent observations have indicated that multi-dimensional theories can give satisfying answers for such problems.

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The scheme of study is the following. In the next section, we take the 5D Friedmann-Robertson-Walker spacetime as representation of our universe as a first step, then we introduce the selected dark energy scenario, calculate the density of extended holographic dark energy and formulate the corresponding generalized Friedmann and continuity equations. In the third section, we present the analysis of the validity of the first and generalized second laws of gravitational thermodynamics to investigate the Kaluza-Klein nature of universal entropy function. The last section is devoted to final remarks.

2. Dark Energy Scenario in Kaluza-Klein Universe

Kaluza [20] and Klein [21] assumed an extra dimension in general relativity to unify the gravity and electromagnetism into one theory. In the present study, we use the five-dimensional (5D) Kaluza-Klein model and investigate the nature of universal entropy function. Here, the following Kaluza-Klein type metric [22] is considered

$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2) + (1-kr^2)d\psi^2\right], \tag{1}$$

where $a(t)$ is the scale factor and k denotes the curvature parameter. Note that the values of curvature parameter 0, -1 and $+1$ corresponds to the flat, closed and open spacetime models, respectively. We also assume the Universe to be filled with dark energy and dark matter and the five-dimensional energy-momentum tensor of a perfect dark fluid is represented by [23]

$$T_{\mu\nu} = (\rho + P)U_\mu U_\nu - g_{\mu\nu}P, \qquad (\mu, \nu = 0, 1, 2, 3, 5), \tag{2}$$

where ρ, P and U_μ are the energy density, the pressure of dark fluid and the 5-velocity vector, respectively. Note that $\rho = \rho_m + \rho_e$ and $P = P_m + P_e$ where the subscripts m and e denote the dark matter and dark energy, respectively. Here, we also have $U_\mu U^\mu = 1$.

The Einstein's field equations are defined by

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi G T_{\mu\nu}, \tag{3}$$

where $R_{\mu\nu}$, $g_{\mu\nu}$, R and G are the Ricci tensor, the metric tensor, the curvature scalar and the gravitational cosntant, respectively. Using equations (2) and (3) it can be written

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi G\left[(\rho + P)U_\mu U_\nu - g_{\mu\nu}P\right]. \tag{4}$$

On the other hand, we consider that the dark fluid is a mixture of dark matter and dark energy, thence it can be defined that $P = P_m + P_e$ and $\rho = \rho_m + \rho_e$. This field equation with the line-element (1) yields two independent equations

$$H^2 + \frac{k}{a^2} = \frac{4\pi G}{3}\rho, \tag{5}$$

$$2H^2 + \dot{H} + \frac{k}{a^2} = -\frac{8\pi G}{3}P, \tag{6}$$

where $H = \frac{\dot{a}}{a}$ is the Hubble parameter. Next, the continuity equation gives

$$\dot{\rho} + 4H(\rho + P) = 0. \tag{7}$$

Besides, considering the Gamma law equation

$$P = (\gamma - 1)\rho, \tag{8}$$

one can write the continuity equation in another nice form:

$$\dot{\rho} + 4H\gamma\rho = 0, \tag{9}$$

where γ is the state parameter. In this dark scenario, using the fractional densities,

$$\Omega_m = \frac{4\pi G\rho_m}{3H^2}, \qquad \Omega_e = \frac{4\pi G\rho_e}{3H^2}, \qquad \Omega_k = -\frac{k}{H^2a^2}, \tag{10}$$

we may rewrite the Friedmann equation (5) in a very elegant form:

$$\sum_{i=1}^{3}\Omega_i = 1, \tag{11}$$

where $i = m, e, k$.

Now, we obtain an expression for the extended holographic dark energy model in the Kaluza-Klein theory. On this purpose, first, we consider the mass-radius relation of (N+1)-dimensional Schwarzschild black hole [24] given by

$$M = \frac{(N-1)A_{N-1}R_H^{N-2}}{16\pi G}, \tag{12}$$

where A_{N-1} is the area of unit N-sphere, R_h represents the black hole horizon scale and G stands for the gravitational constant in $(N+1)$ dimensions. The gravitational constant is related with M_{N+1} which denotes $(N+1)$-dimensional Planck mass and usual 4D Planck mass M_{pl} as [25]

$$8\pi G = M_{N+1}^{1-N} = M_{pl}^{-2}V_{N-3}, \tag{13}$$

where V_{N-3} describes the volume of extra-dimensional space. Hence, the mass relation (12) becomes

$$M = \frac{(N-1)A_{N-1}R_H^{N-2}M_{pl}^2}{2V_{N-3}}. \tag{14}$$

The extended holographic dark energy density in terms of the quantities given above is defined as [26]

$$L^3\rho_e \sim \frac{(N-1)A_{N-1}L^{N-2}M_{pl}^2}{2V_{N-3}}, \tag{15}$$

which implies

$$\rho_e = \frac{\beta^2(N-1)A_{N-1}L^{N-5}M_{pl}^2}{2V_{N-3}}, \tag{16}$$

where β is a constant parameter. In the Kaluza-Klein theory, choosing $N = 4$ gives

$$\rho_e = \frac{3\beta^2 A_3}{2L}, \tag{17}$$

where L is the infrared cutoff of the Universe. Using $A_3 = 2\pi^2 L^3$ (the area of 4-sphere), we obtain

$$\rho_e = 3\beta^2\pi^2 L^2. \tag{18}$$

The dynamical apparent horizon in the Friedmann-Robertson-Walker (FRW) universe is given by [27, 28]

$$R_h = \left[H^2 + \frac{k}{a^2}\right]^{-\frac{1}{2}}. \tag{19}$$

For the flat FRW spacetime $k = 0$, it becomes the Hubble horizon $R_h = \frac{1}{H}$. The infrared cutoff in terms of dynamical apparent horizon is defined as $L = 1/R_h$ and it yields

$$\rho_e = 3\beta^2\pi^2 \left[H^2 + \frac{k}{a^2}\right], \tag{20}$$

which describes the extended holographic dark energy model. Assuming $k \to 0$ gives us the original well-known holographic dark energy model:

$$\lim_{k\to 0} \rho_e = \rho_h = 3\beta^2\pi^2 H^2. \tag{21}$$

3. Thermodynamics in Kaluza-Klein Theory

Here, we check the validity of laws of universal thermodynamics on the dynamical apparent horizon to discuss the nature of cosmological entropy function.

At this step, we use the Gibb's law of thermodynamics [29]

$$T_h dS_I = PdV + dE_I, \tag{22}$$

where S_I, P, E_I and T_h are the internal entropy, the pressure, the internal energy and the temperature of system, respectively. Here, we also assume that the system is in equilibrium which implies all components of this system have the same temperature [30]. The internal energy in the system is given by

$$E_I = \rho V, \tag{23}$$

where V is the extra-dimensional volume and defined as

$$V = \frac{1}{2}\pi^2 L^4. \tag{24}$$

We know that all fluids in our universe acquire the same temperature after establishing of equilibrium [31], otherwise the energy flow would deform the geometry [29, 32]. The temperature of dynamical apparent horizon T_h is related to its radius R_h [32–34]

$$T_h = (2\pi R_h)^{-1}. \tag{25}$$

We also consider the dynamical apparent horizon as the infrared cutoff $R_h = L$. Next, the entropy of the dynamical apparent horizon is written as [35]

$$S_h = \frac{A}{4G}, \tag{26}$$

where $A = 2\pi^2 L^3$ is the area of 4-sphere. Taking time derivative of equation (26) gives

$$\frac{dS_h}{dt} = \frac{3\pi^2}{2G} L^2 \dot{L} = \frac{3\pi^2}{2G} R_h^2 \dot{R}_h = \frac{3H\pi^2}{2G} R_h^5 \left(\frac{k}{a^2} - \dot{H} \right). \tag{27}$$

After multiplying both sides of this relation with a factor $T_h = \frac{1}{2\pi R_h}$, we obtain

$$T_h dS_h = \frac{3H\pi}{4G} R_h^4 \left(\frac{k}{a^2} - \dot{H} \right) dt. \tag{28}$$

The first law of gravitational thermodynamics on the dynamical apparent horizon is defined as

$$-dE_I = T_h dS_h. \tag{29}$$

Besides, the measure of energy crossing on the dynamical apparent horizon is described by using the following relation [36]

$$-dE_I = 2\pi^2 R_h^4 H T_{\mu\nu} U^\mu U^\nu dt = 2\pi^2 R_h^4 H(\rho + P) dt = -\frac{3\pi}{4} H \dot{H} L^4 dt. \tag{30}$$

Inserting L in this relation yields

$$dE_I = \frac{3H\pi}{4G} R_h^4 \left(\dot{H} - \frac{k}{a^2} \right) dt. \tag{31}$$

Hence, we explore that the first law of gravitational thermodynamics valid on the dynamical apparent horizon for all kinds of energies as it is independent of dark energy. In literature, Sharif and Saleem investigated the first law of thermodynamics for the flat FRW universe in Kaluza-Klein theory [23], and they found that

$$-dE_I = \frac{1}{\pi} T_h dS_h, \tag{32}$$

which means

$$-dE_I \neq T_h dS_h. \tag{33}$$

Namely the first law is not valid in the Kaluza-Klein universe with a non-flat FRW. Using equations (28) and (31) with the limiting conditions $k = 0$ and $G = 1$ to reduce our model into the one used in Ref. [23], it can be easily get

$$T_h dS_h = -\frac{3H\pi}{4} R_h^4 \dot{H} dt = -\frac{3\pi}{4} \frac{\dot{H}}{H^3} dt, \tag{34}$$

and

$$dE_I = \frac{3H\pi}{4} R_h^4 \dot{H} dt = \frac{3\pi}{4} \frac{\dot{H}}{H^3} dt. \tag{35}$$

Above results give us $-dE_I = T_h dS_h$ which means that the first law is valid in the flat Kaluza-Klein universe. In other words, our results are not agree with those obtained by Sharif and Saleem [23] and give us the expected outcome.

Furthermore, for the generalized second law of thermodynamics to be satisfied for the dynamical apparent horizon, we investigate derivative of the total entropy $S_t = S_I + S_h$. Considering the Gibb's law and using energy relation, we can write

$$T_h dS_I = (\rho + P)(\dot{V} - 4HV)dt. \tag{36}$$

Inserting the definition of volume of 4-sphere gives

$$T_h dS_I = 2\pi^2 R_h^3 (\dot{R}_h - HR_h)(\rho + P)dt. \tag{37}$$

Making use of equations (5) and (6), we obtain

$$\dot{H} = \frac{k}{a^2} - \frac{8\pi G}{3}(\rho + P), \tag{38}$$

and this result yields

$$T_h dS_I = \frac{3\pi}{4G} R_h^3 (\dot{R}_h - HR_h)\left(\frac{k}{a^2} - \dot{H}\right) dt, \tag{39}$$

Next, collecting equations (28) and (39) gives

$$T_h d(S_I + S_h) = \frac{3\pi}{4G}\left(\frac{k}{a^2} - \dot{H}\right) R_h^3 \dot{R}_h dt. \tag{40}$$

Hence, we can write

$$\dot{S}_I + \dot{S}_h = \frac{3\pi^2}{2G}\left(\frac{k}{a^2} - \dot{H}\right) R_h^4 \dot{R}_h. \tag{41}$$

Equation (41) shows that the generalized second law of gravitational thermodynamics is always satisfied throughout the history of universe. Furthermore, we take again the limiting conditions $k = 0$ and $G = 1$ to reduce our model into the one used in Ref. [23] and obtain

$$\dot{S}_I + \dot{S}_h = \frac{3\pi^2}{2}\frac{\dot{H}^2}{H^6} \geq 0, \tag{42}$$

and, on the other hand, Sharif and Saleem [23] found that

$$\dot{S}_I + \dot{S}_h = \frac{3\pi^2}{8}\left[4\frac{\dot{H}^2}{H^6} - 3\frac{\dot{H}}{H^4}\right] dt \geq 0. \tag{43}$$

Unlike the equation (43) obtained in Ref. [23], our result eqn. (42) describes the validity of the generalized second law correctly. Furthermore, taking into account the de Sitter scale factor $a(t) \sim e^{Ht}$ with $H = constant$ in equation (42), we find $\dot{S}_I + \dot{S}_L = 0$ which corresponds to a reversible adiabatic expansion of the Universe. Furthermore, in another specific case, $\dot{S}_I + \dot{S}_L$ may tend to infinity. Such condition may happen for very large time, thence the entropy will behave very interesting and reach its maximum value. Besides, this enigmatic behavior transforms all the usable energy in universe into an unusable form. This is the heat death scenario of a system and one of the predicted fates of universe. At this stage, all the thermodynamic free energy in our Universe will be derogated and the motion of life cannot sustain any more [37].

4. Final Remarks

We have considered the five dimensional Kaluza-Klein spacetime in the thermal equilibrium state and assumed that the Universe is filled with dark energy and dark matter. Making use of these assumptions the validity of first and generalized second laws of gravitational thermodynamics on the apparent horizon with the Hawking temperature have been discussed to investigate the nature of universal entropy function. We have calculated separately the variation of entropy function for each dark fluid contents and for the apparent horizon itself. These cosmological laws have been turned out to be independent of the fifth dimension and selected dark energy model. According to these investigations, we have discussed also two special conditions: (i) the reversible adiabatic expansion and (ii) the heat death scenario. In addition, we have extended the results obtained by Sharif and Saleem for the flat FRW spacetime [23] to those ones performed for the non-flat five dimensional FRW universe. It has been shown that our results are not agree with those obtained in Ref. [23]. We also want to mention here that our results are consistent with the general relativity [38] and previous studies [39–44] published in literature.

Acknowledgement

We would like to thank the referee for giving such constructive comments and important suggestions which substantially helped improving the quality of the paper. This study was supported by the Research Fund of Mersin University in Turkey with project number: 2015-AP4-1231.

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