Refining the Unruh Metric Tensor Uncertainty Principle for a lower bound to Graviton mass as a Compliment to the NLED modification of GR giving an upper bound to a graviton mass

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This paper is to address what a fluctuation of a metric tensor becomes in early universe conditions. This metric fluctuation in conjunction with Barbour's work on emergent time, allows for a lower bound to a graviton mass, and this lower bound mass for the graviton is contrasted with the NLED (non linear electrodynamic) calculations which may lead to an upper bound to a graviton mass. We show that the supposition of flat space uncertainty in energy is not supportable in initial phases of GR

Keywords: Emergent time, heavy gravity, metric tensor perturbations, HUP

1. Introduction

This article starts with updating what was done in [1] , which is symbolized by, if the scale factor is very small, metric variance

$$
\begin{aligned}
&\left\langle \left(\delta g_{uv}\right)^2 \left(\hat{T}_{uv}\right)^2 \right\rangle \geq \frac{\hbar^2}{V_{Volume}^2} \\
&\xrightarrow[uv\to tt]{} \left\langle \left(\delta g_{tt}\right)^2 \left(\hat{T}_{tt}\right)^2 \right\rangle \geq \frac{\hbar^2}{V_{Volume}^2} \qquad (1) \\
&\& \quad \delta g_{rr} \sim \delta g_{\theta\theta} \sim \delta g_{\phi\phi} \sim 0^+
\end{aligned}
$$

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Then, if $\Delta l \cdot \Delta p \geq \frac{\hbar}{2}$ We will be using the approximation given by Unruth [2,3], of a generalization we will write as

$$
\begin{aligned}
(\Delta l)_{ij} &= \frac{\delta g_{ij}}{g_{ij}} \cdot \frac{l}{2} \\
(\Delta p)_{ij} &= \Delta T_{ij} \cdot \delta t \cdot \Delta A
\end{aligned}
\tag{2}
$$

Then if $a^2(t) \sim 10^{-110}, r \equiv l_P \sim 10^{-35} meters$ the surviving version of Eq. (2) is, then, if $\Delta T_{tt} \sim \Delta\rho$ and we use the Friedman-Walker metric, that then

$$
\begin{aligned}
& V^{(4)} = \delta t \cdot \Delta A \cdot r \\
& \delta g_{tt} \cdot \Delta T_{tt} \cdot \delta t \cdot \Delta A \cdot \frac{r}{2} \geq \frac{\hbar}{2} \\
& \Leftrightarrow \delta g_{tt} \cdot \Delta T_{tt} \geq \frac{\hbar}{V^{(4)}}
\end{aligned}
\tag{3}
$$

This Eq. (3) is such that we can extract, up to a point the HUP principle for uncertainty in time and energy, with one very large caveat added, namely if we use the fluid approximation of space-time $T_{ii} = diag(\rho, -p, -p, -p)$ Then

$$
\Delta T_{tt} \sim \Delta\rho \sim \frac{\Delta E}{V^{(3)}}
\tag{4}
$$

Then, if $\delta g_{tt} \sim a^2(t) \cdot \phi << 1$ from Giovannin [4] then as well as [2,3]

$$
\begin{aligned}
& \delta t \Delta E \geq \frac{\hbar}{\delta g_{tt}} \neq \frac{\hbar}{2} \\
& Unless \quad \delta g_{tt} \sim O(1)
\end{aligned}
\tag{5}
$$

So, Eq. (5) is, due to the surviving metric very different from the traditional HUP, and now it is time to bring up the main point of this document, that of graviton mass, and an argument as to a lower bound to the graviton mass which is a direct consequence of Eq.(4) above and which is addressed next. Finally, afterwards we will allude to arguments as to NLED (non linear electrodynamics) [5] which may allow for upper bound to heavy gravitons.

2. A lower bound to Graviton mass and why it matters

We will be looking at the early universe treatment of the write up by Barbour [6] as to emergent time, which in the case of Gravitons, and Planck distance would lead

$$(\delta t)^2_{emergent} = \frac{\sum_i m_i l_i \cdot l_i}{2 \cdot (E - V)} \rightarrow \frac{m_{graviton} l_P \cdot l_P}{2 \cdot (E - V)} \quad (6)$$

Then, we come up with the following, namely if $(\delta t)^2_{emergent} = \delta t^2$ in Eq.(3),

$$m_{graviton} \geq \frac{2\hbar^2}{(\delta g_{tt})^2 l_P^2} \cdot \frac{(E - V)}{\Delta T_{tt}^2} \quad (7)$$

Key to Eq. (7) will be identification of the kinetic energy which is written as $E - V$. This identification will be the key point raised in this manuscript. Note that [11 raises the distinct possibility of an initial state, just before the 'big bang' of a kinetic energy dominated 'pre inflationary' universe. I.e. in terms of an inflaton $\dot{\phi}^2 >> (P.E \sim V)$ [7] . The key finding which is in [7] is, that, if the kinetic energy is dominated by the 'inflaton' that $K.E. \sim (E - V) \sim \dot{\phi}^2 \propto a^{-6}$, and that further refinements will lead to a Kinetic energy proportional to $\rho_w \propto a^{-3(1-w)}$ with the proviso that w <-1, in effect, what we are saying is that during the period of the pre 'Planckian regime' we can seriously consider an initial density proportional to Kinetic energy, and call this K.E. as proportional to [7]. Our preliminary estimates have been that $m_{graviton} \geq 10^{-70} grams$ and that next we will be trying to come up with argument for the bound $10^{-70} grams \geq m_{graviton} \geq 10^{-70} grams$ We argue that the $m_{graviton} \geq 10^{-70} grams$ is significant and is related to information flow, from a prior to the present universe, a point we will elaborate upon, next.

3. Information flow, Gravitons, and also upper bounds to Graviton mass

Here we can view the possibility of considering the following, namely [8] is extended by [9] so we can we make the following identification?

$$N = N_{graviton}\Big|_{r_H} = \frac{c^3}{G \cdot \hbar} \cdot \frac{1}{\Lambda} \approx \frac{1}{\Lambda} \quad (8)$$

Should the N above, be related to entropy, and Eq. (8) This supposition has to be balanced against the following identification, namely, as given by T. Padmanabhan[10,11]

$$\Lambda_{Einstein-Const.Padmanabhan} = 1/l_{Planck}^{2} \cdot \left(E/E_{Planck}\right)^{6} \qquad (9)$$

But should the energy in the numerator in Eq. (9) be given as say by Eq. (1) above, then there would have been defacto quintessence, i.e. variation in the "Einstein constant" , which would have a large impact upon mass of the graviton, with a sharp decrease in g_* being consistent with an evolution to the ultra light value of the Graviton , with initial frequencies of the order of say for wavelength values initially the size of an atom,

$$\omega_{initial}\Big|_{r_H \sim atomic-size} \sim 10^{21} Hz \qquad (10)$$

The final value of the frequency would be of a magnitude smaller than one Hertz, so as to have value of the mass of the graviton would be then of the order of 10^-62 grams, due to Eq.(9) approaching [8]

$$\Lambda_{Einstein-Const.} = 1/l_{Radius-Universe}^{2} \qquad (11)$$

Leading to the upper bound of the Graviton mass of about 10^-62 grams [8,9]in the present era

$$m_{graviton} = \frac{\hbar}{c} \cdot \sqrt{\frac{(2\Lambda)}{3}} \approx \sqrt{\frac{(2\Lambda)}{3}} \qquad (12)$$

Here, the evolution of the frequency, in question would be governed by NLED, of the sort initially started by, if $a_{\min} \sim 10^{-55}$ and Eq.(13) below given by having $a_0 \sim 1$, and the frequencies as to the evolution of space time adjusted so that we will see , eventually , an evolution given by the bracket in the below mentioned Eq. (14), i.e. [ ] to the 1/4th power approaches 1 in the present era, so then the frequency of radiation drops significantly below 1 Hertz, in the present era, with the initial configuration for when $a_{\min} \sim 10^{-55}$ given as a starting point. The idea would then to have the initial frequency, initially about , or higher than the value given in Eq. (10) drop to below 1 Hertz, which would pre sage the magnitude of

Eq. (13) dropping to 1 in the present era, commensurate with Eq. (9) becoming as of Eq. (11), with the present low level of the cosmological constant today, leading to an upper bound of the graviton mass of the order of 10^-62 grams. It all starts off via use of Eq.(13) below.[5]

$$\alpha_0 = \sqrt{\frac{4\pi G}{3\mu_0 c}} B_0$$
$$\hat{\lambda}(defined) = \Lambda c^2 / 3 \tag{13}$$

$$a_{\min} = a_0 \cdot \left[\frac{\alpha_0}{2\hat{\lambda}(defined)} \left(\sqrt{\alpha_0^2 + 32\hat{\lambda}(defined) \cdot \mu_0 \omega \cdot B_0^2} - \alpha_0 \right) \right]^{1/4}$$

Information flow, in this situation would be proportional to entropy, with entropy in the "present era" commensurate with S (entropy) ~ N [12,] , with N given by Eq. (8) , and of the magnitude of 10 ^ 122, which is in line with [13]

4. Conclusion. A lot of work ahead

The bracketing of graviton mass is significant. We look forward to giving precise delineation of details as **Appendix A** below as to initial entropy , in more detail. This is an upper bound to initial entropy given below, starting off, and we would expect the entropy to be lower again, still but not zero. Additional details are in [14]. One of the things to investigate as to an inter relationship between Gravity and magnetic fields would be the relation given in [5], that of, and also as to its role in Eq.(13) in the initial to present era.

$$B > \frac{1}{2 \cdot \sqrt{10\mu_0 \cdot \omega}} \tag{14}$$

Appendix A. Initial entropy, from first principles.

We are making use of the Padmanabhan publication of [10, 11] where we will make use of

$$\rho_\Lambda \approx \frac{GE_{system}{}^6}{c^8 \hbar^4} \Leftrightarrow \Lambda \approx \frac{1}{l_{Planck}^2} \cdot \left(E_{system} / E_{Planck} \right)^6 \tag{A1}$$

Then, if E_{system} is for the energy of the Universe as a bridge between Pre Planckian, to Planckian physics regimes we could write, then

$$E_{system} \propto n_{gravitons} \cdot m_{graviton}$$

$$\Lambda \approx \frac{1}{l^2_{Radius-Universe-today}} \qquad \text{(A2)}$$

$$\Leftrightarrow m_{graviton} \sim 10^{-62} grams \Rightarrow n_{gravitons} \sim 10^{37}$$

$$\Rightarrow S_{initial(graviton)} \sim 10^{37} at - Planck - time$$

The value of entropy, $S_{initial(graviton)} \sim 10^{37}$ should be contrasted with [13]

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