The Baryonic Tully-Fisher relation combined with the elementary particle Dark Matter halo hypothesis leads to a universal Dark Matter gravitational acceleration constant for galaxies.

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Abstract

In this paper I combine the elementary particle Dark matter halo hypothesis with the Baryonic Tully-Fisher relation. It results in a universal Dark Matter galaxy gravitational centripetal acceleration and connects the galaxy specific Dark Matter radius uniquely to the galaxy rotation curve's final velocity. This allows the precise operational definition of the galaxy specific Dark Matter density function and mass function.

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I. A NEW DARK MATTER MODEL

In a previous paper I introduces a new Dark Matter model [1]. In the present paper I investigate the connection of my model with the Baryonic Tully-Fisher relation. First I will briefly reintroduce my elementary particle Dark Matter halo model.

It is common knowledge that a Dark Matter mass function linear in r can explain the flatness of galaxy rotation curves at large r. This is the emperical starting point of our Dark Matter model. Given a rest mass m_0 at $r = 0$, it will have an additional spherical Dark Matter halo containing an extra mass, with Dark Matter properties only, in the sphere with radius r as

$$m_{\text{DM}} = \frac{r}{r_{\text{DM}}} m_0 \tag{1}$$

in which the Dark Matter radius r_{DM} should have a value somewhere in between 10 kpc and 20 kpc, so approximately once or twice the radius of an average luminous galaxy. We haven't determined yet if this Dark Matter radius has a unique value for every galaxy or perhaps even a universal value, in which case it would be a constant of nature. As everything indicates that Dark Matter only interacts as being a source mass of gravity, this extra mass m_{DM} only comes into play when the rest mass m_0 acts as a source of gravity. So m_{DM} doesn't contribute to the inertial mass of m_0, nor does it contribute to its gravitational charge when acted upon by a force of gravity.

The total gravitational source mass m_g of an elementary particle m_0 contained within a sphere with radius r will then be given by

$$m_g = m_0 + m_{\text{DM}} = m_0 + \frac{r}{r_{\text{DM}}} m_0 = m_0 \left(1 + \frac{r}{r_{\text{DM}}}\right). \tag{2}$$

The total mass of an elementary particle at rest inside a galactic sphere of radius r_{DM} wil be twice the original rest mass at $r = 0$. Disturbances in the elementary Dark Matter halo due to changes in position and momentum of m_0 at $r = 0$ will travel with matter wave velocity through the halo, with

$$v_{wave} = \frac{c^2}{v_{particle}} \tag{3}$$

If the elementary particle has velocity zero, the disturbances at the source can travel instantaneously through the entire halo, producing a Newtonian instantaneous force field of gravity. The DM halo functions as a medium for the stochastic or subquantum thermodynamic de Broglie matter waves. It is the halo that vibrates when matter waves packages pass by. This directly implies that the halo has local Born-interpretation probability-density properties, the Dark Matter halo is a storage place of stochastic (sub)quantum information.

Particles that will be kicked out of an original $r = 0, p = 0$ position and momentum and acquire a velocity approaching c will have a matter wave velocity approaching c from above and there will be a considerable delay regarding the Dark Matter halo adjustment to the new situation of the source particle. Considerable has to be interpreted as on galactic travel and time scales with $r_{\rm DM} \approx 50.000 ly$. In such circumstances, large retardation effects should be expected, diluting the conspiracy between dark-matter and baryons on cosmic scales.

The identification of the Dark Matter halo with the de Broglie matter wave medium is necessary in order to assures our proposal to be in full accordance with Special Relativity and pre-spin Wave Quantum Mechanics. In our view we connect the de Broglie's later thoughts about the matter wave field as a subquantum thermodynamic medium to Verlinde's latest ideas of gravity as emergent from quantum information [2],[3].

The idea of gravity as an entropic force caused by changes in the information associated with the positions of material bodies of Verlinde, and de Broglie's ideas of the matter wave field as an subquantum thermodynamic turmoil connects with the observations regarding Dark Matter. From a recent paper by Koopmans et.al. we quote:

> In both spiral and elliptical galaxies with prominent baryonic components, there appears to be a conspiracy between dark-matter and baryons, leading to a nearly universal total mass distribution out to the largest measured radii that is very close to isothermal (i.e. $\rho \sim r^{-2}$), with only a small intrinsic scatter between systems. [4]

This is a key motivation for our proposed axiom. The observation in the quote indicates towards some kind of a source like connection between baryons and their Dark Matter halo. The elementary particle Dark Matter halo mass content has been derived from a mass density that is inversely proportional to $4\pi r^2$. So the mass density of the halo drops of or dilutes at the surface of an ever larger sphere in the same way that all classical central sources do. We therefore define a Dark Matter halo mass density as

$$\rho_{\rm DM} = \frac{m_0}{4\pi r^2 r_{\rm DM}} \tag{4}$$

and then the spherically symmetric gravitational source mass m_g inside a sphere of radius

r is given by

$$m_g = \int_V \rho_{\rm DM} dV = \int_r \rho_{\rm DM} 4\pi r^2 dr = \int_r \frac{m_0}{r_{\rm DM}} dr = \frac{m_0}{r_{\rm DM}} r + m_0 \tag{5}$$

with the last factor as the obvious constant of integration, given the starting point of our model that we have $m_g = m_0$ at $r = 0$.

The density and mass functions in our proposal of the elementary particle's halo are chosen to match the values necessary to arrive at the constant velocity rotation curve of galaxies. It is the astrophysical experimental input, especially the $\rho \sim r^{-2}$ Dark Matter density distribution observation, see [4], that is turned into axiomatic definitions regarding the properties of elementary rest masses.

II. THE VIRIAL THEOREM AND THE GALAXY ROTATION CURVES

Given the definition of the gravitational potential as

$$\phi = -\frac{GM}{r} \tag{6}$$

with gravitatinal source mass M as

$$M = M_0 + \frac{rM_0}{r_{\rm DM}} \tag{7}$$

we get a gravitational potential at r as

$$\phi = -\frac{GM_0}{r} - \frac{GM_0}{r_{\rm DM}} = \phi_0 + \phi_{\rm DM} \tag{8}$$

For the resulting force of gravity on a classical mass m we get the Newtonian result

$$\boldsymbol{F} = -m\nabla\phi = -m\nabla\phi_0 + -m\nabla\phi_{\rm DM} = -m\nabla\phi_0 = \frac{GM_0 m}{r^2}\hat{r}. \tag{9}$$

This is due to the fact that the new mass factor varies linear over r and thus results in a additional potential term that is constant. Our Dark Matter halo acts as a gauge term in the source that produces a constant term $\phi_{\rm DM}$ in the potential and thus has zero effect on the force.

But the extra term is effecting the gravitational energy of a satellite mass m in the field of a source mass M. This gravitational energy is given by

$$U_g = m\phi = m\phi_0 + m\phi_{\rm DM} = -\frac{GM_0 m}{r} - \frac{GM_0 m}{r_{\rm DM}}. \tag{10}$$

Now we assume that the virial theorem is still valid, actually we assume that it is more fundamental than the force relation from which it has been originally derived, giving $2U_k = -U_g$, so $v^2 = -\phi$ for orbiting satellites and

$$v^2 = -\phi = \frac{GM_0}{r} + \frac{GM_0}{r_{\text{DM}}}. \tag{11}$$

If we let $r \to \infty$ then

$$v_f^2 = \frac{GM_0}{r_{\text{DM}}}, \tag{12}$$

which is a constant, the galaxy rotational velocity curves' final constant value.

This result allows us to give an estimate of r_{DM} by applying this to the Milky Way galaxy. We get

$$r_{\text{DM}} = \frac{GM_0}{v^2} \approx \frac{6,67.10^{-11} \cdot 1,99 \cdot 10^{30} \cdot 1,4.10^{11}}{(230.10^3)^2} = 3,5.10^{20}m = 11,4kpc. \tag{13}$$

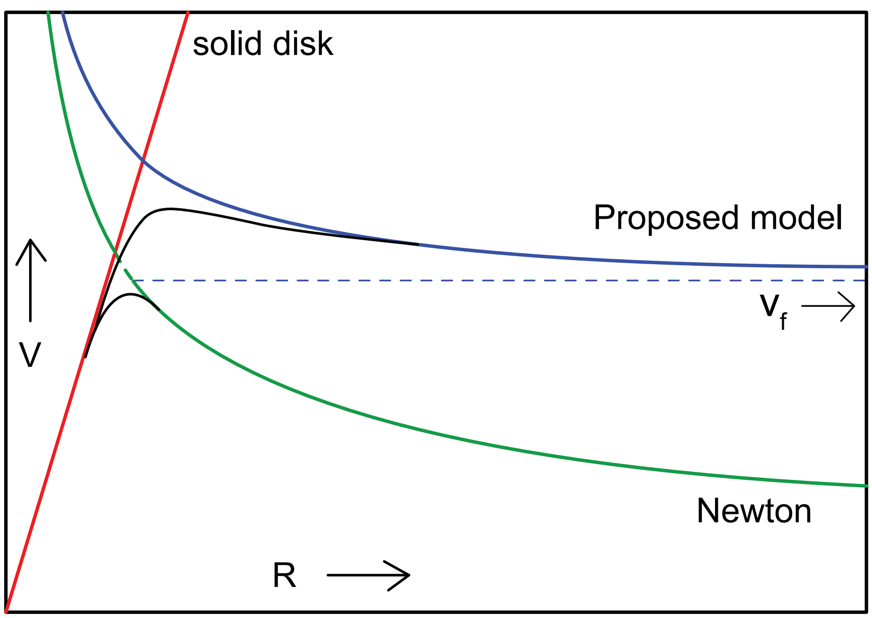

FIG. 1. Velocity versus radius potentials of Newtonian versus our model's expectation.

Actual galaxy velocity rotation curves vary considerably from our model with its point like mass distribution. Real galaxies have disk like or spherical like mass distributions which cause deviations from our single particle model. But given the luminous mass distribution, the associated galaxy Dark Matter halo as a summation over all the elementary particle Dark Matter halo's should be computable. Every galaxy has it's own specific luminous mass distribution and then also a particular Dark Matter halo. These calculations should eventually lead to a determination of the value of the galaxy specific Dark Matter radius r_{DM}, the Dark Matter galaxy constant of my proposal.

III. THE DARK MATTER CONSTANT AND THE BARYONIC TULLY-FISHER RELATION

The question whether the Dark Matter radius r_{DM} is a galaxy specific constant or a universal constant can be answered using the Baryonic Tully-Fisher relation, BTF. The fundamental relation underpinning the Tully-Fisher relation between galaxy luminosity and rotational velocity is one between final rotation velocity v_f and total baryonic disk mass M_d. In the 2005 paper of McGaugh the baryonic version of the LT relation has the form

$$M_d = 50 v_f^4, \tag{14}$$

see [5]. In this form, M_d is expressed in solar mass $M_\odot = 1,99 \cdot 10^{30} kg$ units and the final velocity of the galactic rotation velocity curve v_f is expressed in km/s. If we express the galactic mass in kg and the velocity in m/s we get the total baryonic mass, final velocity relations as

$$M_b = 1,0 \cdot 10^{20} v_f^4. \tag{15}$$

In order to appreciate and interpret the result correctly we start again with Eqn.(10)

$$U_g = U_b + U_{\text{DM}} = m\phi_b + m\phi_{\text{DM}} = -\frac{GM_b m}{r} - \frac{GM_b m}{r_{\text{DM}}}. \tag{16}$$

With $r \gg r_{\text{DM}}$ this approaches

$$U_g \approx U_{\text{DM}} = m\phi_{\text{DM}} = -\frac{GM_b m}{r_{\text{DM}}}. \tag{17}$$

and with the virial theorem we get for $r \gg r_{\text{DM}}$ the Dark Matter virial theorem

$$v_f^2 = \frac{GM_b}{r_{\text{DM}}} \tag{18}$$

which combines with Eqn.(15) to

$$v_f^2 = 6,64 \cdot 10^9 \frac{v_f^4}{r_{\text{DM}}} \tag{19}$$

and this leads to

$$\frac{v_f^2}{r_{\text{DM}}} = 1,5 \cdot 10^{-10} \frac{m}{s^2} = a_{\text{DM}} \tag{20}$$

This is the acceleration, minus a factor 6 or 2π, Verlinde referred to in connection to cH_0, as a universal acceleration somehow related to Dark Matter. In our model this acceleration is the universal centripetal Dark Matter gravitational acceleration impacted upon gravitational charges like stars in galactic spiral arms. Universal has to be interpreted as "as far as the Baryonic Tully-Fisher relation holds". And it is an energy related centripetal acceleration because it was derived from the energy relations, in a situation where the derivative of the potential, the real force, should be near zero.

IV. GALAXY SPECIFIC DARK MATTER DENSITY FORMULA

The result allows us to determine the galaxy specific r_{DM} for every galaxy with known v_f as

$$r_{\text{DM}} = \frac{v_f^2}{a_{\text{DM}}} = \frac{v_f^2}{1,5 \cdot 10^{-10} \frac{m}{s^2}} \tag{21}$$

leading to

$$m_g = m_0 + m_{\text{DM}} = m_0 \left(1 + \frac{a_{\text{DM}}}{v_f^2} r \right) . \tag{22}$$

and

$$\rho_{\text{DM}} = \frac{m_0 a_{\text{DM}}}{4\pi r^2 v_f^2} \tag{23}$$

with m_0 as the individual baryonic particles in the specific galaxies. In these formulations, our original Dark Matter radius r_{DM} has disappeared due to the insertion of the BTF relation and only known parameters of Dark Matter astrophysics appear in them.

If we define a Dark Matter halo radius specific centripetal acceleration as

$$a_{c_{\text{DM}}}(r) = \frac{v_f^2}{r} \tag{24}$$

we can write the previous results as

$$m_g = m_0 \left(1 + \frac{a_{\text{DM}}}{a_{c_{\text{DM}}}} \right) . \tag{25}$$

and

$$\rho_{\text{DM}} = \rho \frac{a_{\text{DM}}}{a_{c_{\text{DM}}}} \tag{26}$$

as long as it remains clear that the virial theorem on the level of the energies was used and that these accelerations are only semi-coriolis like and not originating from Newton's force law.

These results in turn should allow the astrophysicists to determine the mass distribution of the Dark Matter halo's of galaxies with greater accuracy, leading to better understanding of the rotation curves in between the core and v_f and of the properties of Dark Matter in our universe. Using the density formula, our model's area of verification/falsification should be extendable to galactic Einstein lensing.

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