Armenian Transformation Equations For Relativity

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100th Anniversary of the Special Relativity Theory

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Abstract

In our article, we derive a new transformation of relativity for inertial systems using the following guidelines:

1. We use **vector notations** to obtain new transformation equations in a general form.

2. In the process of deriving the general new transformation equations in vector form, **we keep the term** $\vec{v} \times \vec{r}$ **as well**.

3. We add **one more postulate** to the existing two, to resolve ambiguity problems in the orientation of the inertial coordinate systems axes.

4. Newly obtained transformation equations **need to satisfy the linear transformation fundamental laws**.

5. Addition of velocities we calculate in two different ways: by linear superposition and by differentiation, and they need to coincide each other. If not, then **we force them to match** for obtaining the final relation between coefficients.

After using the above mentioned general guidelines, we obtain straight and inverse transformation equations named the *Armenian transformation equations* and they are expressed in two notations: by *absolute relative velocity* $\vec{v}$ and by *local relative velocity* $\langle\vec{v}\rangle$.

Armenian transformation equations are the replacement for the Lorentz transformation equations.

Contents

Introduction to the Armenian Transformation Equations for Relativity

The Lorentz transformation equations, as we know them, in two dimensional time-space (t, x) or in four-dimensional time-space $(t, \vec{r})$, when the inertial systems move at a constant relative velocity $\vec{v}$ along one of the chosen axis, are linear orthogonal transformations. In these cases Lorentz transformations are a group and satisfy the fundamental linear transformation rules: $L(v)L(u') = L(u)$. Where the resultant transformation is a Lorentz transformation as well with the resultant velocity $u = (v + u')/\left(1 + \frac{vu'}{c^2}\right)$. In general, when the relative velocity $\vec{v}$ of the inertial systems S and S' have an arbitrary direction, then the Lorentz transformation is not a group and are therefore less discussed as a case. Only a few brave authors [1], [6], [7], [9] and [10] mention and discuss this general case (axes of the inertial systems they take parallel to each other as usual). The main linear transformation law $L(\vec{v})L(\vec{u}') = L(\vec{u})$ fails for the general Lorentz transformation. Since, however, a resultant transformation must be a Lorentz transformation as well, physicist need therefore to add an extra *artificial transformation* called the *Thomas precession,* to compensate for the error. This is the Achilles heel in the Lorentz transformation equations and more precisely in all special and general theory of relativity. ***Therefore it is an imperative, that the Lorentz transformation equations be replaced by new ones, which must be consistent with linear transformation fundamental laws and have a common sense in respect to reality.*** Here we shall derive these *New transformation equations*, using **pure mathematical logic without any limitations** and the following three postulates:

1. *All physical laws have the same mathematical(tensor) form in all inertial systems.*

2. *There exists a* ***boundary velocity****, denoted as* c*, between micro and macro worlds, which is the same in all inertial systems.*

3. *The simplest transformation equations of the moving particle between two inertial systems* S *and* S' *we have only when relative velocity, measured in two inertial systems, satisfy the relation* $\vec{v}' = -\vec{v}$.

These first two postulates are **almost the same** as the Special Relativity Theory postulates, but the third postulate is quite new and necessary for receiving the simplest transformation equations **without ambiguity problems** in orientation of the inertial systems axes.

All authors that I know, derive the Lorentz transformation equations using two Cartesian coordinates (t, x) or as a general way using four Cartesian coordinates (t, x, y, z). Nobody (that I know of) uses **vector notations** to derive general transformation equations for relativity. Many authors **artificially construct** general Lorentz transformation equations in vector form using special Lorentz transformation equations and therefore those generalized results cannot be correct. The laws of logic tell us, that we need to go from the general case to the special case. That's why we derive our *New transformation equations* using the **most general considerations** and adapting **vector notation**. The great merit of the vectors in the theoretical and applied problems is that equations describing physical phenomena can be formulated *without reference to any particular coordinate system*, without worry that coordinate systems axes are parallel to each other or not. However, in actually carrying out the calculations we need to find a *suitable coordinate system (our third postulate)* where equations can have the simplest form. Therefore to receive the correct transformation we need to use only vector notations and focus on it entirely. Using this new promising approach and one additional postulate we derive **truly correct transformation equations in the most general and simplest form**.

The other question can arise - why are we calling our newly received transformation equations the *Armenian Transformation Equations*? The answer is very simple. *This research was done for more than 20 years in Armenia by an Armenian and the manuscripts were written in Armenian.* ***This research is purely from the mind of an Armenian and from the Holy land of Armenia****, therefore we can rightfully call these newly derived transformation equations the* ***Armenian Transformation Equations*** *and the theory the* ***Armenian Relativity Theory*** or ***ART***.

Part 1 - Pure Mathematical Approach

In this *part* we will use a purely mathematical approach to derive transformation equations for relative movement, without using physical realities. In *part 2* we will exercise the received *New transformation equations* using a physical approach to find the relation between coefficients and eventually to obtain the final form of the *Armenian transformation*.

1.1 - Three Dimensional Species Transformation

We need to use a 3 dimensional approach which will best describe our existence. Let us describe an event of the moving particle in the inertial system S by $(t, \vec{r})$ and in the uniform moving inertial system S' by $(t', \vec{r}')$, where t and t' are the *local times* and $\vec{r}$ and $\vec{r}'$ are the radius vectors of the moving particle. Relations between these radius vectors with respect to two inertial systems in general can be represented in the following way.

$$\begin{cases} \vec{r}' = \alpha_1 \vec{r} + \alpha_2 \vec{v} + \alpha_3 \dfrac{1}{c}(\vec{v} \times \vec{r}) \\ \vec{r} = \alpha_1' \vec{r}' + \alpha_2' \vec{v}' + \alpha_3' \dfrac{1}{c}(\vec{v}' \times \vec{r}') \end{cases} \quad 1.1\text{-}1$$

Three vectors $\vec{v}$, $\vec{r}$ and $\vec{v} \times \vec{r}$ or $\vec{v}'$, $\vec{r}'$ and $\vec{v}' \times \vec{r}'$ are not coplanar, therefore the resolution of radius vectors $\vec{r}'$ and $\vec{r}$ in those three directions are unique.

Transformations of the *local time* t' and t in general can be constructed from infinite scalar terms as showing below:

$$\begin{cases} t' = \beta_1 t + \beta_2 \dfrac{1}{c^2}(\vec{v}\vec{r}) + \beta_3 t^2 + \beta_4 v^2 + \beta_5 \vec{r}^2 + \beta_6 \dfrac{1}{c^4}(\vec{v}\vec{r})^2 + \ldots \\ t = \beta_1' t' + \beta_2' \dfrac{1}{c^2}(\vec{v}'\vec{r}') + \beta_3' t'^2 + \beta_4' v'^2 + \beta_5' \vec{r}'^2 + \beta_6' \dfrac{1}{c^4}(\vec{v}'\vec{r}')^2 + \ldots \end{cases} \quad 1.1\text{-}2$$

Since the *local time* and space in the inertial systems are homogeneous and isotropic, therefore time-space transformation must be linear in respect to the *local time* and radius vector, and that can only happen when the first power of time and radius vectors are involved. Besides that, the other terms that contain the power of v^2 vanish as well, because we assume that the inertial systems origin coincide ($\vec{r}' = \vec{r} = 0$) at $t' = t = 0$. Therefore in the *local time* transformation equations $(1.1-2)$ we keep only the first two terms.

To calibrate the transformation equations coefficients, we use a universal constant - boundary velocity c, which by the **second principle of relativity** has the same value for all the inertial systems.

Finally, as a 3 dimensional species, we need to solve the following system of equations.

$$\begin{cases} t' = \beta_1 t + \beta_2 \dfrac{1}{c^2}(\vec{v}\vec{r}) \\ \vec{r}' = \alpha_1 \vec{r} + \alpha_2 \vec{v} + \alpha_3 \dfrac{1}{c}(\vec{v} \times \vec{r}) \end{cases} \Leftrightarrow \begin{cases} t = \beta_1' t' + \beta_2' \dfrac{1}{c^2}(\vec{v}'\vec{r}') \\ \vec{r} = \alpha_1' \vec{r}' + \alpha_2' \vec{v}' + \alpha_3' \dfrac{1}{c}(\vec{v}' \times \vec{r}') \end{cases} \quad 1.1\text{-}3$$

According to the **first principle of relativity**, all laws of physics, including transformation laws, must be the same in all inertial systems. Therefore all numerical coefficients in the transformations $(1.1-3)$ must be equal to each other.

$$\begin{cases} \beta_1' = \beta_1 \\ \beta_2' = \beta_2 \end{cases} \quad \text{and} \quad \begin{cases} \alpha_1' = \alpha_1 \\ \alpha_3' = \alpha_3 \end{cases} \quad 1.1\text{-}4$$

Coefficients α_2 and α_2' are **not numerical quantities** and the **first principle of relativity** doesn't apply for them and therefore they are not equal to each other. We need to find a transformation law for them as well. These time-like coefficients we will call *real time* and for the simplicity and physical-meaning purposes we denote them as:

$$\begin{cases} \alpha_2' = -\sigma' \\ \alpha_2 = -\sigma \end{cases} \quad 1.1\text{-}5$$

Finally, according to the ***third principle of relativity***, we can obtain the simplest transformation equations when:

$$\vec{v}' = -\vec{v} \quad 1.1\text{-}6$$

Substituting primed values from $(1.1-4)$, $(1.1-5)$ and $(1.1-6)$ into the transformation equations $(1.1-3)$ we get:

- Straight transformation

$$\begin{cases} t' = \beta_1 t + \beta_2 \dfrac{1}{c^2}\vec{v}\vec{r} \\ \vec{r}' = \alpha_1 \vec{r} - \vec{v}\sigma + \alpha_3 \dfrac{1}{c}\vec{v} \times \vec{r} \end{cases} \quad 1.1\text{-}7$$

- Inverse transformation

$$\begin{cases} t = \beta_1 t' - \beta_2 \dfrac{1}{c^2}\vec{v}\vec{r}' \\ \vec{r} = \alpha_1 \vec{r}' + \vec{v}\sigma' - \alpha_3 \dfrac{1}{c}\vec{v} \times \vec{r}' \end{cases} \quad 1.1\text{-}8$$

We need to be very careful, because in these transformation equations the *real times* σ and σ' are not the physical *local times* of the inertial systems. In general, we need to assume, that they can be a function from both *local times* t and t' and therefore we need to find those relations as well. All other coefficients can be constants or they need to be dependent on relative velocity $\vec{v}$ only.

To find the limitations of this coefficients, we need to approximate the *New transformation equations* $(1.1-7)$ and $(1.1-8)$ when $v \ll c$, and then they need to coincide with the Galilean transformation equations as shown below.

Approximation of $(1.1-7)$ ⟺ Galilean Transformation

$$\begin{cases} t' \approx \beta_1 t \\ \vec{r}' \approx \alpha_1 \vec{r} - \vec{v}\sigma \end{cases} \quad \Leftrightarrow \quad \begin{cases} t' = t \\ \vec{r}' = \vec{r} - \vec{v}t \end{cases} \qquad 1.1\text{-}9$$

And

Approximation of $(1.1-8)$ ⟺ Galilean Transformation

$$\begin{cases} t \approx \beta_1 t' \\ \vec{r} \approx \alpha_1 \vec{r}' + \vec{v}\sigma' \end{cases} \quad \Leftrightarrow \quad \begin{cases} t = t' \\ \vec{r} = \vec{r}' + \vec{v}t' \end{cases} \qquad 1.1\text{-}10$$

Transformation coefficients according to $(1.1-9)$ and $(1.1-10)$ need to satisfy the following limitations.

$$\begin{cases} \alpha_1 > 0 \\ \beta_1 > 0 \end{cases} \quad \text{and} \quad \begin{cases} \lim\limits_{v\to 0} \alpha_1 = 1 \\ \lim\limits_{v\to 0} \beta_1 = 1 \end{cases} \quad \text{and} \quad \begin{cases} \lim\limits_{v\to 0} \sigma' = t' \\ \lim\limits_{v\to 0} \sigma = t \end{cases} \qquad 1.1\text{-}11$$

One more thing is clear, if we are looking for a **new time-space transformation**, different from the Galilean transformation, then on top of the coefficient limitations $(1.1-11)$, they must also satisfy the following condition:

$$\alpha_1 \neq 1 \quad \text{and} \quad \beta_1 \neq 1 \qquad 1.1\text{-}12$$

To derive the *New transformation equations*, first we need to use only, as we have mentioned already, *pure mathematics without limitations* and then, if necessary, to use realms of the physical realities.

1.2 - Derivation of the Armenian Transformation Equations

1. Inserting values t' and $\vec{r}'$ from $(1.1-7)$ into the expression of t in $(1.1-8)$

$$t = \beta_1 t' - \beta_2 \frac{1}{c^2}\vec{v}\vec{r}' = \beta_1\left(\beta_1 t + \beta_2 \frac{1}{c^2}\vec{v}\vec{r}\right) - \beta_2 \frac{1}{c^2}\vec{v}\left(\alpha_1\vec{r} - \vec{v}\sigma + \alpha_3 \frac{1}{c}\vec{v}\times\vec{r}\right) =$$
$$= \beta_1^2 t + \beta_1\beta_2\frac{1}{c^2}\vec{v}\vec{r} - \alpha_1\beta_2\frac{1}{c^2}\vec{v}\vec{r} + \beta_2\frac{v^2}{c^2}\sigma = \beta_1^2 t + \beta_2\frac{v^2}{c^2}\sigma - (\alpha_1 - \beta_1)\beta_2\frac{1}{c^2}\vec{v}\vec{r}$$

From the above calculation for the *real time* σ we get:

$$\beta_2\frac{v^2}{c^2}\sigma = (1-\beta_1^2)t + (\alpha_1 - \beta_1)\beta_2\frac{1}{c^2}\vec{v}\vec{r} \qquad 1.2\text{-}1$$

2. Inserting values t and $\vec{r}$ from $(1.1-8)$ into the expression of t' in $(1.1-7)$

$$t' = \beta_1 t + \beta_2 \frac{1}{c^2}\vec{v}\vec{r} = \beta_1\left(\beta_1 t' - \beta_2 \frac{1}{c^2}\vec{v}\vec{r}'\right) + \beta_2 \frac{1}{c^2}\vec{v}\left(\alpha_1\vec{r}' + \vec{v}\sigma' - \alpha_3 \frac{1}{c}\vec{v}\times\vec{r}'\right) =$$
$$= \beta_1^2 t' - \beta_1\beta_2\frac{1}{c^2}\vec{v}\vec{r}' + \alpha_1\beta_2\frac{1}{c^2}\vec{v}\vec{r}' + \beta_2\frac{v^2}{c^2}\sigma' = \beta_1^2 t' + \beta_2\frac{v^2}{c^2}\sigma' + (\alpha_1 - \beta_1)\beta_2\frac{1}{c^2}\vec{v}\vec{r}'$$

From the above calculation for the *real time* σ' we get:

$$\beta_2\frac{v^2}{c^2}\sigma' = (1-\beta_1^2)t' - (\alpha_1 - \beta_1)\beta_2\frac{1}{c^2}\vec{v}\vec{r}' \qquad 1.2\text{-}2$$

3. Inserting value $\vec{r}'$ from $(1.1-7)$ into the expression of $\vec{r}$ in $(1.1-8)$

$$\vec{r} = \alpha_1\vec{r}' + \vec{v}\sigma' - \alpha_3\frac{1}{c}\vec{v}\times\vec{r}' = \alpha_1\left(\alpha_1\vec{r} - \vec{v}\sigma + \alpha_3\frac{1}{c}\vec{v}\times\vec{r}\right) + \vec{v}\sigma' - \alpha_3\frac{1}{c}\vec{v}\times\left(\alpha_1\vec{r} - \vec{v}\sigma + \alpha_3\frac{1}{c}\vec{v}\times\vec{r}\right) =$$
$$= \alpha_1^2\vec{r} - \alpha_1\vec{v}\sigma + \alpha_1\alpha_3\frac{1}{c}\vec{v}\times\vec{r} + \vec{v}\sigma' - \alpha_1\alpha_3\frac{1}{c}\vec{v}\times\vec{r} - \alpha_3^2\frac{1}{c^2}\vec{v}\times(\vec{v}\times\vec{r}) = \alpha_1^2\vec{r} + (\sigma' - \alpha_1\sigma)\vec{v} - \alpha_3^2\frac{1}{c^2}(\vec{v}\vec{r})\vec{v} + \alpha_3^2\frac{v^2}{c^2}\vec{r}$$

From the above calculation we obtain a vector relation.

$$\left(\alpha_1^2 + \alpha_3^2\frac{v^2}{c^2} - 1\right)\vec{r} + \left(\sigma' - \alpha_1\sigma - \alpha_3^2\frac{1}{c^2}\vec{v}\vec{r}\right)\vec{v} = 0 \qquad 1.2\text{-}3$$

4. Inserting value $\vec{r}$ from $(1.1-8)$ into the expression of $\vec{r}'$ in $(1.1-7)$

$$\vec{r}' = \alpha_1\vec{r} - \vec{v}\sigma + \alpha_3\frac{1}{c}\vec{v}\times\vec{r} = \alpha_1\left(\alpha_1\vec{r}' + \vec{v}\sigma' - \alpha_3\frac{1}{c}\vec{v}\times\vec{r}'\right) - \vec{v}\sigma + \alpha_3\frac{1}{c}\vec{v}\times\left(\alpha_1\vec{r}' + \vec{v}\sigma' - \alpha_3\frac{1}{c}\vec{v}\times\vec{r}'\right) =$$

$$= \alpha_1^2\vec{r}' + \alpha_1\vec{v}\sigma' - \alpha_1\alpha_3\frac{1}{c}\vec{v}\times\vec{r}' - \vec{v}\sigma + \alpha_1\alpha_3\frac{1}{c}\vec{v}\times\vec{r}' - \alpha_3^2\frac{1}{c^2}\vec{v}\times(\vec{v}\times\vec{r}') = \alpha_1^2\vec{r}' - (\sigma - \alpha_1\sigma')\vec{v} - \alpha_3^2\frac{1}{c^2}(\vec{v}\vec{r}')\vec{v} + \alpha_3^2\frac{v^2}{c^2}\vec{r}'$$

From the above calculation we obtain a vector relation.

$$\left(\alpha_1^2 + \alpha_3^2\frac{v^2}{c^2} - 1\right)\vec{r}' - \left(\sigma - \alpha_1\sigma' + \alpha_3^2\frac{1}{c^2}\vec{v}\vec{r}'\right)\vec{v} = 0 \qquad 1.2\text{-}4$$

Equating $(1.2-3)$ and $(1.2-4)$ null vectors components to zero we obtain the following:

- Relation between coefficients $\alpha_1 > 0$ and α_3

$$\alpha_1^2 + \alpha_3^2\frac{v^2}{c^2} = 1 \qquad \text{or} \qquad \alpha_1 = \sqrt{1 - \alpha_3^2\frac{v^2}{c^2}} \qquad 1.2\text{-}5$$

- *real time* transformation equations

$$\begin{cases} \sigma' = \alpha_1\sigma + \alpha_3^2\frac{1}{c^2}\vec{v}\vec{r} \\ \sigma = \alpha_1\sigma' - \alpha_3^2\frac{1}{c^2}\vec{v}\vec{r}' \end{cases} \qquad 1.2\text{-}6$$

Equations $(1.2-1)$ and $(1.2-2)$ are expressing the relations between *real times* σ and σ' into *local times* and radius vectors in the same inertial system, and vice versa relation as well

$$\begin{cases} \beta_2\frac{v^2}{c^2}\sigma' = (1-\beta_1^2)t' - (\alpha_1 - \beta_1)\beta_2\frac{1}{c^2}\vec{v}\vec{r}' \\ \beta_2\frac{v^2}{c^2}\sigma = (1-\beta_1^2)t + (\alpha_1 - \beta_1)\beta_2\frac{1}{c^2}\vec{v}\vec{r} \end{cases} \quad \text{or} \quad \begin{cases} (1-\beta_1^2)t' = \beta_2\frac{v^2}{c^2}\left[\sigma' + (\alpha_1 - \beta_1)\frac{1}{v^2}\vec{v}\vec{r}'\right] \\ (1-\beta_1^2)t = \beta_2\frac{v^2}{c^2}\left[\sigma - (\alpha_1 - \beta_1)\frac{1}{v^2}\vec{v}\vec{r}\right] \end{cases} \qquad 1.2\text{-}7$$

Substituting values $\beta_2\frac{1}{c^2}\vec{v}\vec{r}'$ from the first equation of $(1.1-8)$ and $\beta_2\frac{1}{c^2}\vec{v}\vec{r}$ from the first equitation of $(1.1-7)$ into the first system of equations $(1.2-7)$ we obtain relations between *real times* σ and σ' into *local times* t and t' only.

$$\begin{cases} \beta_2\frac{v^2}{c^2}\sigma' = (1-\alpha_1\beta_1)t' + (\alpha_1 - \beta_1)t \\ \beta_2\frac{v^2}{c^2}\sigma = (1-\alpha_1\beta_1)t + (\alpha_1 - \beta_1)t' \end{cases} \quad \text{or} \quad \begin{cases} \sigma' = \dfrac{1-\alpha_1\beta_1}{\beta_2\frac{v^2}{c^2}}t' + \dfrac{\alpha_1 - \beta_1}{\beta_2\frac{v^2}{c^2}}t \\ \sigma = \dfrac{1-\alpha_1\beta_1}{\beta_2\frac{v^2}{c^2}}t + \dfrac{\alpha_1 - \beta_1}{\beta_2\frac{v^2}{c^2}}t' \end{cases} \qquad 1.2\text{-}8$$

Remark (1) *Not all received equations are independent. For example, equations* $(1.2-7)$ *can be derived from the equations* $(1.2-6)$ *using transformation equations* $(1.1-7)$ *and* $(1.1-8)$*, also relation* $(1.2-5)$*.*

For our convenience we need to denote

$$\begin{cases} \alpha_3 = -\alpha \\ \beta_2 = \beta \\ \alpha_1 = \sqrt{1 - \alpha^2\frac{v^2}{c^2}} = \gamma(v^2) \end{cases} \qquad 1.2\text{-}9$$

Remark (2) *When we have only one relative velocity, for simplicity purposes instead of using* $\gamma(v^2)$ *we use just* γ*. When we have multiple inertial systems we then use* $\gamma(v^2)$*,* $\gamma(u'^2)$*,* $\gamma(u^2)$*, and so on.*

If we substitute $\alpha_1 = \gamma$ and $\beta_2 = \beta$ in the second system of equations $(1.2-8)$ and then using the third system of $(1.1-11)$ we obtain the following limitations between coefficients:

$$\begin{cases} \lim\limits_{v\to 0} \dfrac{1-\gamma\beta_1}{\beta\frac{v^2}{c^2}} = 1 \\ \lim\limits_{v\to 0} \dfrac{\gamma - \beta_1}{\beta\frac{v^2}{c^2}} = 0 \end{cases} \qquad 1.2\text{-}10$$

1.3 - Expressing the Armenian Transformation Equations by Real Time

Inertial systems with *real times* σ, σ' which transformed according to $(1.2-6)$ and radius vectors $\vec{r}, \vec{r}'$ which transformed by $(1.1-7)$ and $(1.1-8)$, are the genuine components of the event. Also if we denote the real zero components of the event in a four-dimensional time-space as:

$$\begin{cases} h = c\sigma \\ h' = c\sigma' \end{cases} \qquad 1.3\text{-}1$$

Then the *New transformation equations* expressed by the *real time* or by real zero components become:

- Straight transformation

$$\begin{cases} \sigma' = \gamma\sigma + \alpha^2 \frac{1}{c^2}\vec{v}\vec{r} \\ \vec{r}' = \gamma\vec{r} - \vec{v}\sigma - \alpha\frac{1}{c}\vec{v}\times\vec{r} \end{cases} \quad \text{or} \quad \begin{cases} h' = \gamma h + \alpha^2 \frac{1}{c}\vec{v}\vec{r} \\ \vec{r}' = \gamma\vec{r} - \frac{1}{c}\vec{v}h - \alpha\frac{1}{c}\vec{v}\times\vec{r} \end{cases} \qquad 1.3\text{-}2$$

- Inverse transformation

$$\begin{cases} \sigma = \gamma\sigma' - \alpha^2 \frac{1}{c^2}\vec{v}\vec{r}' \\ \vec{r} = \gamma\vec{r}' + \vec{v}\sigma' + \alpha\frac{1}{c}\vec{v}\times\vec{r}' \end{cases} \quad \text{or} \quad \begin{cases} h = \gamma h' - \alpha^2 \frac{1}{c}\vec{v}\vec{r}' \\ \vec{r} = \gamma\vec{r}' + \frac{1}{c}\vec{v}h' + \alpha\frac{1}{c}\vec{v}\times\vec{r}' \end{cases} \qquad 1.3\text{-}3$$

- From transformations $(1.3-2)$ and $(1.3-3)$ we obtain an *invariant expression* calling it *real interval* and denoting it as s_R and we can express it by a *real time* or by a real zero component as shown below.

$$s_R^2 = c^2\sigma'^2 + \alpha^2 r'^2 = c^2\sigma^2 + \alpha^2 r^2 \quad \text{or} \quad s_R^2 = h'^2 + \alpha^2 r'^2 = h^2 + \alpha^2 r^2 \qquad 1.3\text{-}4$$

Definition (1) *Any set of components* $(a_0, \vec{a})$ *which transforms in a similar way to* $(1.3-2)$ *and* $(1.3-3)$*, also the distance between two events which* ***depend on time and on radius vector only*** *and expressed by* $(1.3-4)$*, will be called a* ***four-vector****.*

Remark (3) *From the above definition 1 it follows that the real interval* s_R *in* $(1.3-4)$ *can be dependent on time and radius vector of the event only, if the coefficient* α *is a constant.* ***This is the first crucial result****. Coefficient* α *is a truly universal time-space constant characterizing the medium of time-space or the moving particle.*

$$\alpha = Const \qquad 1.3\text{-}5$$

Another important consequence following from the *real interval* expression $(1.3-4)$ is that it is a *monotonic increasing value*, or more correctly, a *strictly monotonic increasing value*. Because, if there is no spatial movement at all, the *real time* is flowing nonstop and therefore the *real interval* s_R only increases. Therefore, we can implement and define the idea of *absolute time* or *universal time* for the event.

Definition (2) *Using a real interval expression* $(1.3-4)$ *we define the* ***absolute time of the event****, denoting it as* τ*, which is "flowing" by the boundary speed* c *and it is also invariant and strictly a monotonic increasing value. Absolute time* τ *and it's differential is shown below.*

$$\begin{cases} \tau = \frac{1}{c}s_R = \sqrt{\sigma'^2 + \alpha^2\frac{1}{c^2}\vec{r}'^2} = \sqrt{\sigma^2 + \alpha^2\frac{1}{c^2}\vec{r}^2} \\ d\tau = \frac{1}{c}ds_R = \sqrt{d\sigma'^2 + \alpha^2\frac{1}{c^2}d\vec{r}'^2} = \sqrt{d\sigma^2 + \alpha^2\frac{1}{c^2}d\vec{r}^2} \end{cases} \qquad 1.3\text{-}6$$

Absolute time τ doesn't have a singularity problem like the *Lorentz transformation* proper-time, but it has a real time characteristics. From $(1.3-6)$ we can obtain the expression for *real time* and it's differential:

$$\begin{cases} \sigma' = \tau\sqrt{1-\alpha^2\frac{1}{c^2}\left(\frac{\vec{r}'}{\tau}\right)^2} \\ \sigma = \tau\sqrt{1-\alpha^2\frac{1}{c^2}\left(\frac{\vec{r}}{\tau}\right)^2} \end{cases} \quad \text{or} \quad \begin{cases} d\sigma' = d\tau\sqrt{1-\alpha^2\frac{1}{c^2}\left(\frac{d\vec{r}'}{d\tau}\right)^2} \\ d\sigma = d\tau\sqrt{1-\alpha^2\frac{1}{c^2}\left(\frac{d\vec{r}}{d\tau}\right)^2} \end{cases} \qquad 1.3\text{-}7$$

1.4 - Expressing the Armenian Transformation Equations by Local Time

To express the *New transformation equations* by *local time* t' and t, we need to insert *real time* values σ' and σ from $(1.2-7)$ into radius vector expressions $(1.1-7)$ and $(1.1-8)$, also using notations $(1.2-9)$ we get.

- Straight transformation equations become:

$$\begin{cases} t' = \beta_1 t + \beta\frac{1}{c^2}\vec{v}\vec{r} \\ \vec{r}' = \gamma\vec{r} - \dfrac{1-\beta_1^2}{\beta\frac{v^2}{c^2}}\vec{v}t - (\gamma-\beta_1)\frac{1}{v^2}(\vec{v}\vec{r})\vec{v} - \alpha\frac{1}{c}\vec{v}\times\vec{r} \end{cases} \qquad 1.4\text{-}1$$

- Inverse transformation equations become:

$$\begin{cases} t = \beta_1 t' - \beta\frac{1}{c^2}\vec{v}\vec{r}' \\ \vec{r} = \gamma\vec{r}' + \dfrac{1-\beta_1^2}{\beta\frac{v^2}{c^2}}\vec{v}t' - (\gamma-\beta_1)\frac{1}{v^2}(\vec{v}\vec{r})\vec{v} + \alpha\frac{1}{c}\vec{v}\times\vec{r}' \end{cases} \qquad 1.4\text{-}2$$

Transformation equations expressed by *local time* $(1.4-1)$ and $(1.4-2)$ in case of approximation of $v \ll c$ need to coincide with the Galilean transformation equations, as we've already stated, and that can only happen, when in addition to the conditions $(1.1-11)$, $(1.1-12)$ and $(1.2-10)$ also exists the condition:

$$\frac{1-\beta_1^2}{\beta} > 0 \qquad 1.4\text{-}3$$

From the transformation equations $(1.4-1)$ or $(1.4-2)$ we get the invariant expression for the event, which we will call the *local interval* and denoted as s_L

$$s_L^2 = c^2t'^2 + \frac{\beta^2\frac{v^2}{c^2}}{1-\beta_1^2}r'^2 = c^2t^2 + \frac{\beta^2\frac{v^2}{c^2}}{1-\beta_1^2}r^2 \qquad 1.4\text{-}4$$

By four-vector definition in section 1.3, the interval must depend only on time and radius vector of the event, and will ***never be the function of the relative velocity***. Therefore, the coefficient in the formula of *local interval* $(1.4-4)$ need to satisfy the following condition:

$$\frac{\beta^2\frac{v^2}{c^2}}{1-\beta_1^2} = q = Const \qquad 1.4\text{-}5$$

Therefore the invariant *local interval* expression $(1.4-4)$ becomes:

$$s_L^2 = c^2t'^2 + qr'^2 = c^2t^2 + qr^2 \qquad 1.4\text{-}6$$

From the condition $(1.4-3)$ follows that the coefficient q in $(1.4-5)$ has a sign of β and in general,we assume, that it is a real number. Because at this moment we do not know the sign of the coefficient q, therefore we **cannot talk** about *absolute time* derived from s_L. Here we can talk only about proper-time, denoting it by τ_L. This proper-time and it's differential is:

$$\begin{cases} \tau_L = \frac{1}{c}s_L = \sqrt{t'^2 + q\frac{1}{c^2}r'^2} = \sqrt{t^2 + q\frac{1}{c^2}r^2} \\ d\tau_L = \frac{1}{c}ds_L = \sqrt{dt'^2 + q\frac{1}{c^2}dr'^2} = \sqrt{dt^2 + q\frac{1}{c^2}dr^2} \end{cases} \qquad 1.4\text{-}7$$

Part 2 - A Physical Approach

In this part, in addition to the pure mathematical methods, we will try to use some physical realities as well, to obtain the final form of the *New transformation equations*. In kinematics we are dealing with two basic physical quantities - distance and time. Quantity distance is very real, we can see it and we can touch it, which we can't say about the quantity of time. Time, as such, is a very mysterious quantity and in our article we are using many types of time. In the section 1.1 we already define *local time t* and *real time* σ. In the sections 1.3 and 1.4 we also define *absolute time* τ and proper time τ_L. Therefore the displacement rate of the radius vectors of the moving particle in respect to these different times generate different types of velocities, as shown three of them below, for arbitrary movement and for uniform movement separately.

$$\begin{cases} \textit{Local Velocity} \text{ or Newtonian Velocity} & \Rightarrow \quad \langle\vec{u}\rangle = \dfrac{d\vec{r}}{dt} \quad \text{or} \quad \langle\vec{u}\rangle = \dfrac{\vec{r}}{t} \\ \textit{Real Velocity} & \Rightarrow \quad \vec{u}_R = \dfrac{d\vec{r}}{d\sigma} \quad \text{or} \quad \vec{u}_R = \dfrac{\vec{r}}{\sigma} \\ \textit{Absolute Velocity} & \Rightarrow \quad \vec{u}_A = \dfrac{d\vec{r}}{d\tau} \quad \text{or} \quad \vec{u}_A = \dfrac{\vec{r}}{\tau} \end{cases} \qquad \text{2-A}$$

On top of those velocities in $(2-A)$, at the beginning of our article, we already used *relative velocity* $\vec{v}$ to derive *New transformation equations* and we need to identify that velocity as well. The type of relative velocity $\vec{v}$ is unknown for now, because we don't know in respect to which time we've made our calculation to find the displacement rate of the inertial systems. Therefore

$$\text{Type of the Relative Velocity} \quad \vec{v} \quad - \quad \textbf{\textit{Is Unknown}} \qquad \text{2-B}$$

2.1 - Movement of the Origin Inertial System S' Expressed by Local Time

In this section we will illustrate only, how the scientific community, from the birth of Special Relativity Theory until now, hasn't even imagined the existence of two types of velocities - *local velocity* and *absolute velocity*. Of course, in SRT there exists the idea of the Minkovsky velocity, which is not a real physical quantity, but just a handy instrument for mathematical calculation. Therefore, scientists don't distinguish these two velocities, have felled an "**unknown trap**". Existence of this **entrapment**, which brings us to a dead end, we like to show here.

Lets calculate the velocity of the origin of Inertial system S' with respect to the inertial system S expressed by *local time*. If we put $\vec{r}' = 0$ in $(1.4-2)$ and using velocity definitions $(2-A)$, then for uniform *local velocity* of the origin of the inertial system S' we get:

$$\text{If} \quad \vec{r}' = 0 \quad \text{then} \quad \begin{cases} t = \beta_1 t' \\ \vec{r} = \dfrac{1-\beta_1^2}{\beta\dfrac{v^2}{c^2}}\vec{v}t' \end{cases} \quad \text{therefore} \quad \langle\vec{v}\rangle = \frac{\vec{r}}{t} = \frac{1-\beta_1^2}{\beta_1\beta\dfrac{v^2}{c^2}}\vec{v} \qquad \text{2.1-1}$$

From $(2.1-1)$ we obtain a relation formula between *local and relative velocities* of the moving origin inertial system S'.

$$\langle\vec{v}\rangle = \frac{1-\beta_1^2}{\beta_1\beta\dfrac{v^2}{c^2}}\vec{v} \quad \text{or} \quad \vec{v} = \frac{\beta_1\beta\dfrac{v^2}{c^2}}{1-\beta_1^2}\langle\vec{v}\rangle \qquad \text{2.1-2}$$

In *relative velocity* formula $(2.1-2)$ we still need to find coefficients β_1 and β. To do that we have **two choices**:

$$1)\ \vec{v} = \langle\vec{v}\rangle \quad \text{or} \quad 2)\ \vec{v} \neq \langle\vec{v}\rangle \qquad \text{2.1-3}$$

2.1.1 - First Choice - Relative Velocity $\vec{v}$ is the Same as Local Velocity $\langle\vec{v}\rangle$.

$$\vec{v} = \langle\vec{v}\rangle \qquad \text{2.1-4}$$

This first choice seems very intuitive and hard to deny, which is easily done by the founding fathers of Special

Relativity Theory. But ***this is the trap*** which I am trying to describe here, and we need to avoid it. Before moving further let's expose how this trap works.

Inserting value $\vec{v}$ from $(2.1-4)$ into the *relative velocity* formula $(2.1-2)$ we get the following relation between coefficients:

$$1-\beta_1^2 = \beta_1\beta\frac{v^2}{c^2} \tag{2.1-5}$$

Solving $(2.1-5)$ in respect to β_1 and remembering $(1.1-11)$ we take only the positive root of β_1.

$$\beta_1 = -\frac{1}{2}\beta\frac{v^2}{c^2}+\sqrt{1+\left(\frac{1}{2}\beta\frac{v^2}{c^2}\right)^2} \tag{2.1-6}$$

Also inserting value $1-\beta_1^2$ from $(2.1-5)$ into $(1.4-5)$ we get the expression for q.

$$q = \frac{\beta}{\beta_1} \tag{2.1-7}$$

Because, according to $(1.1-11)$, β_1 is positive, then from $(2.1-7)$ it follows that β and q have the same sign which we already mentioned in the section 1.4. Substituting value β_1 from $(2.1-6)$ into $(2.1-7)$ we obtain the following equation.

$$\beta = q\left\{-\frac{1}{2}\beta\frac{v^2}{c^2}+\sqrt{1+\left(\frac{1}{2}\beta\frac{v^2}{c^2}\right)^2}\right\} \tag{2.1-8}$$

Solving $(2.1-8)$ with respect to β and substituting obtained β into $(2.1-7)$ we get the value β_1 as well.

$$\beta = \frac{q}{\sqrt{1+q\frac{v^2}{c^2}}} \quad \text{and} \quad \beta_1 = \frac{1}{\sqrt{1+q\frac{v^2}{c^2}}} \tag{2.1-9}$$

After all of this preparation we can finally prove the contradiction of the first choice $(2.1-4)$. For that purpose, we need to recall the second limitation of $(1.2-10)$ and compare coefficients β_1 from $(2.1-9)$ and γ from $(1.2-9)$. Here there is the possibility of only two assumptions.

$$\text{a) } \beta_1-\gamma = 0 \quad \text{or} \quad \text{b) } \beta_1-\gamma \neq 0 \tag{2.1-10}$$

a) If we assume

$$\beta_1 = \gamma \quad \text{then} \quad \frac{1}{\sqrt{1+q\frac{v^2}{c^2}}} = \sqrt{1-\alpha^2\frac{v^2}{c^2}} \tag{2.1-11}$$

Solving equation $(2.1-11)$ with respect to q we get the expression for q depending on the relative velocity $\vec{v}$

$$q = \frac{\alpha^2}{1-\alpha^2\frac{v^2}{c^2}} \tag{2.1-12}$$

Conclusion (1) *Expression* $(2.1-12)$ *is contradictory to the fact of* $(1.4-5)$*, which manifests that coefficient* q ***must be constant and doesn't depend on the relative velocity****. Therefore, the first assumption* ***is incorrect****.*

b) If we assume

$$\beta_1-\gamma \neq 0 \tag{2.1-13}$$

Then inserting value $1-\beta_1^2$ from $(2.1-5)$ into transformation equations $(1.4-1)$ and $(1.4-2)$ we obtain

- Straight transformation equations.

$$\begin{cases} t' = \beta_1 t+\beta\frac{1}{c^2}\vec{v}\vec{r} \\ \vec{r}' = \gamma\vec{r}-\beta_1\vec{v}t-(\gamma-\beta_1)\frac{1}{v^2}(\vec{v}\vec{r})\vec{v}-\alpha\frac{1}{c}\vec{v}\times\vec{r} \end{cases} \tag{2.1-14}$$

- Inverse transformation equations.

$$\begin{cases} t = \beta_1 t' - \beta \frac{1}{c^2} \vec{v}\vec{r}' \\ \vec{r} = \gamma \vec{r}' + \beta_1 \vec{v} t' - (\gamma - \beta_1) \frac{1}{v^2} (\vec{v}\vec{r}) \vec{v} + \alpha \frac{1}{c} \vec{v} \times \vec{r}' \end{cases} \qquad 2.1\text{-}15$$

Even in this "*entrapment choice"* which appears as though we are following a logical path, the received *New transformation equations* $(2.1-14)$ and $(2.1-15)$ are very different from the Lorentz transformation equations.

Conclusion $(\mathbf{2})$ *Transformation equations* $(2.1-14)$ *and* $(2.1-15)$ *are not looking bad. However, when we calculate successive transformations, it* ***does not satisfy the linear transformation fundamental laws*** *and therefore, according to the general guideline, we regard it as an* ***incorrect transformation****. Therefore, the first intuitive choice* $\vec{v} = \langle\vec{v}\rangle$ *has failed. In the section* 2.2 *we prove that* ***relative velocity*** $\vec{v}$ ***in fact is an absolute velocity***

2.1.2 - Second Choice - Relative Velocity $\vec{v}$ is **Not Equal** to the Local Velocity $\langle\vec{v}\rangle$.

$$\vec{v} \neq \langle\vec{v}\rangle \qquad 2.1\text{-}16$$

To analyze this second choice, we need first, to calculate the movement of the origin of the inertial system S' in respect to the inertial system S using transformation equations $(1.3-2)$ and $(1.3-3)$, which we will discuss in the next section.

2.2 - Movement of the Origin Inertial System S' Expressed by Real Time

Let us find now the velocity origin of the inertial system S' in respect to the inertial system S expressed by *real time* and then we can make important conclusions from that. For that purpose, if we put $\vec{r}' = 0$ into $(1.3-3)$ and also recalling velocity definitions $(2-A)$, then for uniform *real velocity* of the origin of the inertial system S' we get:

$$\text{If} \quad \vec{r}' = 0 \quad \text{then} \quad \begin{cases} \sigma = \gamma\sigma' \\ \vec{r} = \vec{v}\sigma' \end{cases} \quad \text{therefore} \quad \vec{v}_R = \frac{\vec{r}}{\sigma} = \frac{1}{\gamma}\vec{v} \qquad 2.2\text{-}1$$

And *absolute time* expression $(1.3-6)$ when $\vec{r}' = 0$ becomes:

$$\tau^2 = \sigma'^2 = \sigma^2 + \alpha^2 \frac{1}{c^2} \vec{r}^2 \qquad 2.2\text{-}2$$

Now, in the *real velocity* expression $(2.2-1)$, dividing the left fraction numerator and denominator to the *absolute time* τ and inserting also the value γ from $(1.2-9)$ into it, we get:

$$\frac{\frac{\vec{r}}{\tau}}{\frac{\sigma}{\tau}} = \frac{\vec{v}}{\sqrt{1 - \alpha^2 \frac{v^2}{c^2}}} \qquad 2.2\text{-}3$$

From the definition of velocities in $(2-A)$ following, that *absolute relative velocity* $\vec{v}_A$ of the origin inertial system S' is:

$$\frac{\vec{r}}{\tau} = \vec{v}_A \qquad 2.2\text{-}4$$

From $(2.2-2)$ we can calculate the expression for $\frac{\sigma}{\tau}$ and inserting the value $\frac{\vec{r}}{\tau}$ from $(2.2-4)$ into it, we get:

$$\frac{\sigma}{\tau} = \sqrt{1 - \alpha^2 \frac{1}{c^2} \left(\frac{\vec{r}}{\tau}\right)^2} = \sqrt{1 - \alpha^2 \frac{v_A^2}{c^2}} \qquad 2.2\text{-}5$$

Now substituting values $\frac{\vec{r}}{\tau}$ from $(2.2-4)$ and $\frac{\sigma}{\tau}$ from $(2.2-5)$ into $(2.2-3)$ we obtain a vector equation:

$$\frac{\vec{v}_A}{\sqrt{1 - \alpha^2 \frac{v_A^2}{c^2}}} = \frac{\vec{v}}{\sqrt{1 - \alpha^2 \frac{v^2}{c^2}}} \qquad 2.2\text{-}6$$

Vector equation $(2.2-6)$ will have only one solution, and that is:

$$\vec{v} = \vec{v}_A \qquad 2.2\text{-}7$$

Conclusion (3) ***This is the second crucial result****. From* $(2.2-7)$ *we obtain the final proof that relative velocity* $\vec{v}$ *which we used to derive our New transformation equations is in fact* ***not a local velocity*** *but an* ***absolute velocity****.*

Remark (4) *This is a very important point in the Armenian Theory of Relativity. Therefore, in the rest of this article we will omit index "A" denoting absolute velocity.*

Using fact $(2.2-7)$, definition of velocities $(2-A)$, also $(1.3-1)$ and system of equations $(1.3-7)$ we can define zero components of the *absolute velocity* of the moving particle in two inertial systems in the following way:

$$\begin{cases} u_0' = \dfrac{dh'}{d\tau} = c\dfrac{d\sigma'}{d\tau} = c\sqrt{1-\alpha^2\dfrac{1}{c^2}\left(\dfrac{d\vec{r}'}{d\tau}\right)^2} = c\sqrt{1-\alpha^2\dfrac{u'^2}{c^2}} = c\gamma(u'^2) \\ u_0 = \dfrac{dh}{d\tau} = c\dfrac{d\sigma}{d\tau} = c\sqrt{1-\alpha^2\dfrac{1}{c^2}\left(\dfrac{d\vec{r}}{d\tau}\right)^2} = c\sqrt{1-\alpha^2\dfrac{u^2}{c^2}} = c\gamma(u^2) \end{cases} \quad 2.2\text{-}8$$

Also, the same way, we can define a zero component of the *absolute relative velocity* $\vec{v}$ as:

$$v_0 = c\sqrt{1-\alpha^2\frac{v^2}{c^2}} = c\gamma(v^2) \quad 2.2\text{-}9$$

From $(2.2-8)$ and $(2.2-9)$ it follows that components of the *absolute velocity* of the moving particle or the *absolute relative velocity* satisfy the invariant relation.

$$u_0'^2 + \alpha^2 u'^2 = u_0^2 + \alpha^2 u^2 = v_0^2 + \alpha^2 v^2 = c^2 \quad 2.2\text{-}10$$

Remark (5) *In the subsection* 2.4.4 *we will prove that components of the absolute velocity of the moving particle form a four-vector.*

2.3 - Calculation of Local Velocities and the Important Consequences

Lets calculate *local velocities* of the moving particle in two inertial systems S and S' using the definition for velocities from $(2-A)$.

$$\begin{cases} \langle\vec{u}'\rangle = \dfrac{d\vec{r}'}{dt'} = \dfrac{d\vec{r}'}{d\tau}\dfrac{d\tau}{dt'} = \vec{u}'\dfrac{d\tau}{dt'} \\ \langle\vec{u}\rangle = \dfrac{d\vec{r}}{dt} = \dfrac{d\vec{r}}{d\tau}\dfrac{d\tau}{dt} = \vec{u}\dfrac{d\tau}{dt} \end{cases} \quad 2.3\text{-}1$$

To continue calculation $(2.3-1)$ we need to find differentials $\frac{d\tau}{dt'}$ and $\frac{d\tau}{dt}$. For that purpose substituting notations $(1.2-9)$ into $(1.2-7)$ and then differentiating *local times* t' and t by *absolute time* τ and inserting also values $\frac{d\sigma'}{d\tau}$ and $\frac{d\sigma}{d\tau}$ from $(2.2-8)$ into it we get:

$$\begin{cases} [1-\beta_1^2(\vec{v})]\dfrac{dt'}{d\tau} = \beta(\vec{v})\dfrac{v^2}{c^2}\left\{\gamma(u'^2) + [\gamma(v^2)-\beta_1(\vec{v})]\dfrac{1}{v^2}\vec{v}\vec{u}'\right\} \\ [1-\beta_1^2(\vec{v})]\dfrac{dt}{d\tau} = \beta(\vec{v})\dfrac{v^2}{c^2}\left\{\gamma(u^2) - [\gamma(v^2)-\beta_1(\vec{v})]\dfrac{1}{v^2}\vec{v}\vec{u}\right\} \end{cases} \quad 2.3\text{-}2$$

Substituting expressions $\frac{d\tau}{dt'}$ and $\frac{d\tau}{dt}$ from $(2.3-2)$ into $(2.3-1)$ we get the expression for *local velocities*.

$$\begin{cases} \langle\vec{u}'\rangle = \dfrac{1-\beta_1^2(\vec{v})}{\beta(\vec{v})\dfrac{v^2}{c^2}} \cdot \dfrac{\vec{u}'}{\gamma(u'^2) + [\gamma(v^2)-\beta_1(\vec{v})]\dfrac{1}{v^2}\vec{v}\vec{u}'} \\ \langle\vec{u}\rangle = \dfrac{1-\beta_1^2(\vec{v})}{\beta(\vec{v})\dfrac{v^2}{c^2}} \cdot \dfrac{\vec{u}}{\gamma(u^2) - [\gamma(v^2)-\beta_1(\vec{v})]\dfrac{1}{v^2}\vec{v}\vec{u}} \end{cases} \quad 2.3\text{-}3$$

Expressions in $(2.3-3)$ are not addition formulas for the velocities of the moving particle but it is a *local velocity* conversion formula in the same inertial system. Therefore *local velocity* of the particle needs to be dependent only on the *absolute velocity* of the particle in the same inertial system and must be **independent of the relative velocity** $\vec{v}$. But that only can happen when coefficients in $(2.3-3)$ satisfy the following conditions.

$$\begin{cases} \beta_1(\vec{v}) = \gamma(v^2) \\ \beta(\vec{v}) = Const = \beta \end{cases} \qquad 2.3\text{-}4$$

Inserting values from $(2.3-4)$ into the first equation of the limitation in $(1.2-10)$

$$\lim_{v\to 0} \frac{1-\gamma\beta_1}{\beta\frac{v^2}{c^2}} = \lim_{v\to 0} \frac{1-\gamma^2}{\beta\frac{v^2}{c^2}} = \lim_{v\to 0} \frac{\alpha^2}{\beta} = \frac{\alpha^2}{\beta} = 1 \qquad 2.3\text{-}5$$

Finally from $(2.3-5)$ we obtain a relation between two constant coefficients α and β.

$$\beta = \alpha^2 \qquad 2.3\text{-}6$$

Conclusion (4) *$(2.3-4)$ and $(2.3-6)$ are the **third crucial results**. Inserting value β_1 and β from $(2.3-4)$ and $(2.3-6)$ into $(1.4-5)$ we find a value for constant coefficient q as well. Finally we obtain all necessary relations between the New transformation equation coefficients and they are:*

$$\begin{cases} \beta = q = \alpha^2 = Const \\ \beta_1 = \gamma = \sqrt{1-\alpha^2\frac{v^2}{c^2}} \end{cases} \qquad 2.3\text{-}7$$

Substituting coefficients $(2.3-7)$ into $(1.4-1)$ and $(1.4-2)$ we obtain the final form of the *New transformation equations*, which we will call the *Armenian transformation equations* and they are:

$$\begin{cases} t' = \gamma(v^2)t + \alpha^2\frac{1}{c^2}\vec{v}\vec{r} \\ \vec{r}' = \gamma(v^2)\vec{r} - \vec{v}t - \alpha\frac{1}{c}\vec{v}\times\vec{r} \end{cases} \quad \Leftrightarrow \quad \begin{cases} t = \gamma(v^2)t' - \alpha^2\frac{1}{c^2}\vec{v}\vec{r}' \\ \vec{r} = \gamma(v^2)\vec{r}' + \vec{v}t' + \alpha\frac{1}{c}\vec{v}\times\vec{r}' \end{cases} \qquad 2.3\text{-}8$$

Using $(1.2-9)$ and inserting values of coefficients from $(2.3-7)$ into *real times* expressions $(1.2-8)$ we obtain

$$\begin{cases} \sigma' = t' \\ \sigma = t \end{cases} \qquad 2.3\text{-}9$$

Remark (6) *Relation $(2.3-9)$ manifest that **real time is the local time**. Therefore, from now on we are omitting all indexes "R" and "L" describing variables as real or local respectively. Particularly interval expressions $(1.3-4)$ and $(1.4-6)$ coincide to each other and we will call it Armenian interval of the event and denoted it by s.*

$$s^2 = c^2t'^2 + \alpha^2r'^2 = c^2t^2 + \alpha^2r^2 \qquad 2.3\text{-}10$$

Substituting coefficients from $(2.3-7)$ into $(2.3-2)$ we obtain expressions for the differentials $\frac{dt'}{d\tau}$ and $\frac{dt}{d\tau}$.

$$\frac{dt'}{d\tau} = \gamma(u'^2) = \sqrt{1-\alpha^2\frac{u'^2}{c^2}} \quad \text{and} \quad \frac{dt}{d\tau} = \gamma(u^2) = \sqrt{1-\alpha^2\frac{u^2}{c^2}} \qquad 2.3\text{-}11$$

Also substituting coefficients $(2.3-7)$ into $(2.3-3)$ we obtain a conversion formula for any arbitrary velocities.

$$\langle\vec{u}\rangle = \frac{1}{\gamma(u^2)}\vec{u} = \frac{\vec{u}}{\sqrt{1-\alpha^2\frac{u^2}{c^2}}} \quad \Rightarrow \quad \vec{u} = \frac{\langle\vec{u}\rangle}{\sqrt{1+\alpha^2\frac{\langle u\rangle^2}{c^2}}} \qquad 2.3\text{-}12$$

From $(2.3-12)$ it follows that for **any arbitrary velocity** we obtain a value for γ expressed by *absolute velocity* or by *local velocity*.

$$\gamma(u^2) = \sqrt{1-\alpha^2\frac{u^2}{c^2}} = \frac{1}{\sqrt{1+\alpha^2\frac{\langle u\rangle^2}{c^2}}} = \gamma(\langle u\rangle^2) \qquad 2.3\text{-}13$$

Using notation for γ from $(2.3-13)$ we rewrite the velocity conversion formula $(2.3-12)$ for **any arbitrary velocity**.

$$\langle\vec{u}\rangle = \frac{1}{\gamma(u^2)}\vec{u} \quad \Leftrightarrow \quad \vec{u} = \gamma(\langle u\rangle^2)\langle\vec{u}\rangle \qquad 2.3\text{-}14$$

From *local* and *absolute velocity* conversion formula $(2.3-12)$ follows their limits.

$$\begin{cases} 0 \le |\vec{u}| \le \dfrac{1}{\alpha}c \\ 0 \le |\langle\vec{u}\rangle| < \infty \end{cases} \qquad 2.3\text{-}15$$

2.4 - Superposition and Transformation of the Absolute and Local Velocities

In this section we need to derive an addition formula for *absolute and local velocities* of the moving particle with respect to two inertial systems S and S'. Following our general guideline, we will obtain it in ***two different ways*** - by linear superposition and by differentiation, and **they need to coincide each other**. This is the **important criteria**, which asserts, that our new received transformation equations passes the mathematical test to be **correct**. To prove this, we need do the following:

1. Use two successive transformations of the *Armenian transformation equations* $(2.3-8)$ to obtain relations between *absolute velocities* of the moving particle in two inertial systems, using a linear superposition law by the scheme:

$$\widehat{A}(\vec{v})\widehat{A}(\vec{u}') = \widehat{A}(\vec{u}) \qquad 2.4\text{-}1$$

Where operator $\widehat{A}$ is the *Armenian transformation matrix*

2. Use a *differentiation method* to obtain relations between *absolute velocities* of the moving particle in two inertial systems, using *Armenian transformation equations* $(2.3-8)$.

Lets now consider three inertial systems S, S' and S'', where the *absolute relative velocities* are:

$$\begin{cases} \vec{v} \ - \ \text{is } \textit{absolute relative velocity} \text{ of the inertial systems } S \text{ and } S' \\ \vec{u}' \ - \ \text{is } \textit{absolute relative velocity} \text{ of the inertial systems } S' \text{ and } S'' \\ \vec{u} \ - \ \text{is } \textit{absolute relative velocity} \text{ of the inertial systems } S \text{ and } S'' \end{cases} \qquad 2.4\text{-}2$$

2.4.1 - Successive Transformations to Derive Superposition Formula for Absolute Velocities

We need to use *Armenian transformation equations* $(2.3-8)$, where coefficients α, as we've already mentioned, is constant and therefore doesn't depend on *absolute relative velocities* of the inertial systems. Lets make two successive transformations.

Straight and inverse *Armenian transformation equations* between three inertial systems as you can see below.

Straight Armenian Transformation		*Inverse Armenian Transformation*	
$S' \Rightarrow S$		$S \Rightarrow S'$	
$\begin{cases} t' = \gamma(v^2)t + \alpha^2 \dfrac{1}{c^2}\vec{v}\vec{r} \\ \vec{r}' = \gamma(v^2)\vec{r} - \vec{v}t - \alpha\dfrac{1}{c}\vec{v}\times\vec{r} \end{cases}$	$\Leftrightarrow$	$\begin{cases} t = \gamma(v^2)t' - \alpha^2 \dfrac{1}{c^2}\vec{v}\vec{r}' \\ \vec{r} = \gamma(v^2)\vec{r}' + \vec{v}t' + \alpha\dfrac{1}{c}\vec{v}\times\vec{r}' \end{cases}$	2.4-3
Straight Armenian Transformation		*Inverse Armenian Transformation*	
$S'' \Rightarrow S'$		$S' \Rightarrow S''$	
$\begin{cases} t'' = \gamma(u'^2)t' + \alpha^2 \dfrac{1}{c^2}\vec{u}'\vec{r}' \\ \vec{r}'' = \gamma(u'^2)\vec{r}' - \vec{u}'t' - \alpha\dfrac{1}{c}\vec{u}'\times\vec{r}' \end{cases}$	$\Leftrightarrow$	$\begin{cases} t' = \gamma(u'^2)t'' - \alpha^2 \dfrac{1}{c^2}\vec{u}'\vec{r}'' \\ \vec{r}' = \gamma(u'^2)\vec{r}'' + \vec{u}'t'' + \alpha\dfrac{1}{c}\vec{u}'\times\vec{r}'' \end{cases}$	2.4-4

Straight Armenian Transformation $\quad\quad$ *Inverse Armenian Transformation*

$$S'' \Rightarrow S \qquad\qquad S \Rightarrow S''$$

$$\begin{cases} t'' = \gamma(u^2)t + \alpha^2\frac{1}{c^2}\vec{u}\vec{r} \\ \vec{r}'' = \gamma(u^2)\vec{r} - \vec{u}t - \alpha\frac{1}{c}\vec{u}\times\vec{r} \end{cases} \quad\Leftrightarrow\quad \begin{cases} t = \gamma(u^2)t'' - \alpha^2\frac{1}{c^2}\vec{u}\vec{r}'' \\ \vec{r} = \gamma(u^2)\vec{r}'' + \vec{u}t'' + \alpha\frac{1}{c}\vec{u}\times\vec{r}'' \end{cases} \tag{2.4-5}$$

Implementing two successive straight transformations(it doesn't matter if we use straight or inverse transformations) $(2.4-4)$ and $(2.4-5)$ by scheme $S'' \Rightarrow S' \Rightarrow S$. After eliminating S' we can get transformation $S'' \Rightarrow S$. We can calculate resultant velocity $\vec{u}$ in two ways - using *local time* transformation equations or radius vector transformation equations. In both ways we are getting the same result. Therefore it is reasonable to use here a more simple one, which is the *local time* transformation equations.

Substituting t' and $\vec{r}'$ from $(2.4-3)$ into the expression t'' in $(2.4-4)$ we obtain:

$$\begin{aligned} t'' &= \gamma(u'^2)t' + \alpha^2\frac{1}{c^2}\vec{u}'\vec{r}' = \gamma(u'^2)\left[\gamma(v^2)t + \alpha^2\frac{1}{c^2}\vec{v}\vec{r}\right] + \alpha^2\frac{1}{c^2}\vec{u}'\left[\gamma(v^2)\vec{r} - \vec{v}t - \alpha\frac{1}{c}\vec{v}\times\vec{r}\right] = \\ &= \gamma(v^2)\gamma(u'^2)t + \gamma(u'^2)\alpha^2\frac{1}{c^2}(\vec{v}\vec{r}) + \gamma(v^2)\alpha^2\frac{1}{c^2}(\vec{u}'\vec{r}) - \alpha^2\frac{1}{c^2}(\vec{v}\vec{u}')t - \alpha^3\frac{1}{c^3}\vec{u}'(\vec{v}\times\vec{r}) = \\ &= \left[\gamma(v^2)\gamma(u'^2) - \alpha^2\frac{1}{c^2}(\vec{v}\vec{u}')\right]t + \alpha^2\frac{1}{c^2}\left[\gamma(v^2)\vec{u}' + \gamma(u'^2)\vec{v} + \alpha\frac{1}{c}(\vec{v}\times\vec{u}')\right]\vec{r} \end{aligned}$$

Now equating the above result to the time t'' expression in $(2.4-5)$ we get:

$$t'' = \left[\gamma(v^2)\gamma(u'^2) - \alpha^2\frac{1}{c^2}(\vec{v}\vec{u}')\right]t + \alpha^2\frac{1}{c^2}\left[\gamma(u'^2)\vec{v} + \gamma(v^2)\vec{u}' + \alpha\frac{1}{c}(\vec{v}\times\vec{u}')\right]\vec{r} = \gamma(u^2)t + \alpha^2\frac{1}{c^2}\vec{u}\vec{r} \tag{2.4-6}$$

The equality $(2.4-6)$ can only happen when the following relations exist for the resultant *absolute velocity* $\vec{u}$.

$$\begin{cases} \gamma(u^2) = \gamma(v^2)\gamma(u'^2) - \alpha^2\frac{1}{c^2}\vec{v}\vec{u}' \\ \vec{u} = \gamma(v^2)\vec{u}' + \gamma(u'^2)\vec{v} + \alpha\frac{1}{c}\vec{v}\times\vec{u}' \end{cases} \tag{2.4-7}$$

Two equations of $(2.4-7)$ are consistent with each other and one can prove the correctness of the first equation using the second equation. From the addition formula $(2.4-7)$ we can obtain a subtraction formula for *absolute velocities*, just by exchanging primes and absolute relative velocity $\vec{v}$ with $-\vec{v}$.

2.4.2 - Differentiation Method to Derive Superposition Formula for Absolute Velocities

For this purpose we need to use the inverse *Armenian transformation equations* $(2.4-3)$ and differentiate *local time t* and radius vector $\vec{r}$ by the *absolute time* τ. Also inserting differentials $\frac{dt'}{d\tau}$ and $\frac{dt}{d\tau}$ from $(2.3-11)$ into it, we obtain the superposition formula for *absolute velocities*.

$$\begin{cases} \frac{dt}{d\tau} = \gamma(v^2)\frac{dt'}{d\tau} - \alpha^2\frac{1}{c^2}\vec{v}\frac{d\vec{r}'}{d\tau} \\ \frac{d\vec{r}}{d\tau} = \gamma(v^2)\frac{d\vec{r}'}{d\tau} + \vec{v}\frac{dt'}{d\tau} + \alpha\frac{1}{c}\vec{v}\times\frac{d\vec{r}'}{d\tau} \end{cases} \quad\Rightarrow\quad \begin{cases} \gamma(u^2) = \gamma(v^2)\gamma(u'^2) - \alpha^2\frac{1}{c^2}\vec{v}\vec{u}' \\ \vec{u} = \gamma(v^2)\vec{u}' + \gamma(u'^2)\vec{v} + \alpha\frac{1}{c}\vec{v}\times\vec{u}' \end{cases} \tag{2.4-8}$$

Remark (7) *Resultant absolute velocity expressions are received in two different ways - by successive transformation* $(2.4-7)$ *and by differentiation* $(2.4-8)$ *coincide to each other. That means the Armenian transformation equations* ***pass the mathematical test to be correct.***

2.4.3 - Addition, Subtraction and Transformation of the Absolute Velocities

Using values for γ from $(2.3-13)$ we can represent addition and subtraction formulas for *absolute velocities* $(2.4-7)$ the following way:

- Addition formula for *absolute velocities* symbolically denoted as $\vec{u}' \oplus \vec{v}$

$$\begin{cases} \sqrt{1-\alpha^2\dfrac{u^2}{c^2}} = \sqrt{1-\alpha^2\dfrac{v^2}{c^2}}\sqrt{1-\alpha^2\dfrac{u'^2}{c^2}} - \alpha^2\dfrac{1}{c^2}(\vec{v}\vec{u}') \\ \vec{u} = \vec{u}' \oplus \vec{v} = \vec{u}'\sqrt{1-\alpha^2\dfrac{v^2}{c^2}} + \vec{v}\sqrt{1-\alpha^2\dfrac{u'^2}{c^2}} + \alpha\dfrac{1}{c}(\vec{v}\times\vec{u}') \end{cases} \qquad 2.4\text{-}9$$

- Subtraction formula for *absolute velocities* symbolically denoted as $\vec{u} \ominus \vec{v}$

$$\begin{cases} \sqrt{1-\alpha^2\dfrac{u'^2}{c^2}} = \sqrt{1-\alpha^2\dfrac{v^2}{c^2}}\sqrt{1-\alpha^2\dfrac{u^2}{c^2}} + \alpha^2\dfrac{1}{c^2}(\vec{v}\vec{u}) \\ \vec{u}' = \vec{u} \ominus \vec{v} = \vec{u}\sqrt{1-\alpha^2\dfrac{v^2}{c^2}} - \vec{v}\sqrt{1-\alpha^2\dfrac{u^2}{c^2}} - \alpha\dfrac{1}{c}(\vec{v}\times\vec{u}) \end{cases} \qquad 2.4\text{-}10$$

One can prove that the addition formula for *absolute velocities* given by the second equations of $(2.4-9)$ satisfy the associative law.

$$(\vec{u}_1 \oplus \vec{u}_2) \oplus \vec{u}_3 = \vec{u}_1 \oplus (\vec{u}_2 \oplus \vec{u}_3) \qquad 2.4\text{-}11$$

Multiplying the first equations of the systems $(2.4-9)$ and $(2.4-10)$ by c and using expressions $(2.2-8)$ and $(2.2-9)$ we can express addition and subtraction formulas by zero and spatial components of the *absolute velocities* only, which is the transformation equations for *absolute velocities*.

$$\begin{cases} u_0 = \dfrac{1}{c}v_0u'_0 - \alpha^2\dfrac{1}{c}(\vec{v}\vec{u}') \\ \vec{u} = \dfrac{1}{c}v_0\vec{u}' + \dfrac{1}{c}u'_0\vec{v} + \alpha\dfrac{1}{c}(\vec{v}\times\vec{u}') \end{cases} \quad \text{and} \quad \begin{cases} u'_0 = \dfrac{1}{c}v_0u_0 + \alpha^2\dfrac{1}{c}(\vec{v}\vec{u}) \\ \vec{u}' = \dfrac{1}{c}v_0\vec{u} - \dfrac{1}{c}u_0\vec{v} - \alpha\dfrac{1}{c}(\vec{v}\times\vec{u}) \end{cases} \qquad 2.4\text{-}12$$

Using *absolute velocity* transformation equations $(2.4-12)$ and making a straight calculation also recalling formulas $(2.2-9)$ and $(2.2-10)$ we obtain the following invariant relation:

$$u_0^2 + \alpha^2u^2 = \frac{1}{c^2}(v_0^2 + \alpha^2v^2)(u_0'^2 + \alpha^2u'^2) = u_0'^2 + \alpha^2u'^2 = c^2 \qquad 2.4\text{-}13$$

Remark (8) *The addition and subtraction formulas* $(2.4-12)$ *also invariant relation* $(2.4-13)$ *clearly shows, that absolute velocity transforms as a four-vector.*

2.4.4 - Addition and Subtraction of the Local Velocities

Dividing the second equations of the $(2.4-9)$ and $(2.4-10)$ into the first equations and using velocity conversion formulas $(2.3-12)$ we obtain addition and subtraction formulas for *local velocities*.

$$\langle\vec{u}\rangle = \frac{\langle\vec{u}'\rangle + \langle\vec{v}\rangle + \alpha\frac{1}{c}\langle\vec{v}\rangle\times\langle\vec{u}'\rangle}{1-\alpha^2\frac{1}{c^2}\langle\vec{v}\rangle\langle\vec{u}'\rangle} \quad \text{or} \quad \langle\vec{u}'\rangle = \frac{\langle\vec{u}\rangle - \langle\vec{v}\rangle - \alpha\frac{1}{c}\langle\vec{v}\rangle\times\langle\vec{u}\rangle}{1+\alpha^2\frac{1}{c^2}\langle\vec{v}\rangle\langle\vec{u}\rangle} \qquad 2.4\text{-}14$$

Remark (9) *Local velocity or Newtonian velocity* $\langle\vec{u}\rangle = \dfrac{d\vec{r}}{dt}$ ***does not transforms as part of a four-vector***.

Using a straight forward calculation from $(2.4-14)$ we can obtain the expressions for *local velocities* similar to the first equations of $(2.4-9)$ and $(2.4-10)$.

$$\sqrt{1+\alpha^2\frac{\langle u\rangle^2}{c^2}} = \frac{\sqrt{1+\alpha^2\frac{\langle v\rangle^2}{c^2}}\sqrt{1+\alpha^2\frac{\langle u'\rangle^2}{c^2}}}{1-\alpha^2\frac{1}{c^2}\langle\vec{v}\rangle\langle\vec{u}'\rangle} \quad \text{and} \quad \sqrt{1+\alpha^2\frac{\langle u'\rangle^2}{c^2}} = \frac{\sqrt{1+\alpha^2\frac{\langle v\rangle^2}{c^2}}\sqrt{1+\alpha^2\frac{\langle u\rangle^2}{c^2}}}{1+\alpha^2\frac{1}{c^2}\langle\vec{v}\rangle\langle\vec{u}\rangle} \qquad 2.4\text{-}15$$

The formulas $(2.4-15)$ we can obtain also from the first equations of $(2.4-9)$ using relations $(2.3-12)$ and $(2.3-13)$.

Multiplying together the first and second equations of $(2.4-15)$ we receive an interesting identity between *local velocities*.

$$\left(1-\alpha^2\frac{1}{c^2}\langle\vec{v}\rangle\langle\vec{u}'\rangle\right)\left(1+\alpha^2\frac{1}{c^2}\langle\vec{v}\rangle\langle\vec{u}\rangle\right) = 1+\alpha^2\frac{\langle v\rangle^2}{c^2} \qquad 2.4\text{-}16$$

2.5 - Summary of the Armenian Transformation Equations

In *Armenian transformation equations* $(2.3-8)$ substituting *absolute relative velocity* value $\vec{v} = \gamma(\langle v\rangle^2)\langle\vec{v}\rangle$, we can express by *local relative velocity* $\langle\vec{v}\rangle$ as well.

- Straight *Armenian transformation equations* expressed by *absolute relative velocity* $\vec{v}$ or by *local relative velocity* $\langle\vec{v}\rangle$:

$$\begin{cases} t' = \gamma(v^2)t + \alpha^2\frac{1}{c^2}\vec{v}\vec{r} \\ \vec{r}' = \gamma(v^2)\vec{r} - \vec{v}t - \alpha\frac{1}{c}\vec{v}\times\vec{r} \end{cases} \quad \text{or} \quad \begin{cases} t' = \gamma(\langle v\rangle^2)\left(t + \alpha^2\frac{1}{c^2}\langle\vec{v}\rangle\vec{r}\right) \\ \vec{r}' = \gamma(\langle v\rangle^2)\left(\vec{r} - \langle\vec{v}\rangle t - \alpha\frac{1}{c}\langle\vec{v}\rangle\times\vec{r}\right) \end{cases} \qquad 2.5\text{-}1$$

- Inverse *Armenian transformation equations* expressed by *absolute relative velocity* $\vec{v}$ or by *local relative velocity* $\langle\vec{v}\rangle$:

$$\begin{cases} t = \gamma(v^2)t' - \alpha^2\frac{1}{c^2}\vec{v}\vec{r}' \\ \vec{r} = \gamma(v^2)\vec{r}' + \vec{v}t' + \alpha\frac{1}{c}\vec{v}\times\vec{r}' \end{cases} \quad \text{or} \quad \begin{cases} t = \gamma(\langle v\rangle^2)\left(t' - \alpha^2\frac{1}{c^2}\langle\vec{v}\rangle\vec{r}'\right) \\ \vec{r} = \gamma(\langle v\rangle^2)\left(\vec{r}' + \langle\vec{v}\rangle t' + \alpha\frac{1}{c}\langle\vec{v}\rangle\times\vec{r}'\right) \end{cases} \qquad 2.5\text{-}2$$

- Invariant *Armenian interval:*

$$s^2 = c^2t'^2 + \alpha^2r'^2 = c^2t^2 + \alpha^2r^2 \qquad 2.5\text{-}3$$

Where

$$\gamma(v^2) = \sqrt{1-\alpha^2\frac{v^2}{c^2}} = \frac{1}{\sqrt{1+\alpha^2\frac{\langle v\rangle^2}{c^2}}} = \gamma(\langle v\rangle^2) \qquad 2.5\text{-}4$$

And coefficient α is a real number and also it's a **constant value** characterizing the moving particle or time-space-medium.

Conclusion (5) *Armenian transformation equations expressed by* $(2.5-1)$ *and* $(2.5-2)$ *are the replacement for the Lorentz transformation equations.*

References

1. Aharoni J., *The Special Theory of Relativity*, (Clarendon Press, Oxford, 1959), pp. 41-47, 53-55

2. Clyde Davenport, *A complex Calculus with Applications to Special Relativity*, 1991

3. David Bohm, *The Special Theory of Relativity*, 1965, pp. 36-125

4. Lorentz H. A., *Electromagnetic phenomena in a system moving with any velocity less than that of light*, (Dover, 1923)

5. Louis Brand, *Vector and Tensor Analysis*, (John Wiley & Sons, Inc., New York)

6. Moller C., *The Theory of Relativity*, (Oxford University Press, 1972,1977)

7. Robertson H.P., Thomas W. Noonan, *Relativity and Cosmology*, (W. B. Saunders, London, 1968), pp. 44-45

8. Robert Resnick, David Halliday, *Basic Concepts in Relativity*, (Macmillan Publishing Company, New York, 1992), pp. 47-50

9. Rosser W.G.V., *An Introduction to The Theory of Relativity*, (Butterworths, London, 1964), pp. 96-98

10. Stephenson and Kilminster, *Special relativity for Physicists*, (London, 1958)

11. Thomas L. H., *The Kinematics of an Electron with an Axis*, (Philosophical Magazine and Journal of Science, London, 1927)

12. Yuan Zhong Zhang, *Special Relativity and Its Experimental Foundations*, (World Scientific), pp. 22-49