Ripà's Conjectures on the *k*-dimensions 3 X 3 X … X 3 Dots Problem

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Abstract. The classic thinking problem, the "Nine Dots Puzzle", is widely used in courses on creativity and appears in a lot of games magazines. Here are two mutually exclusive conjectures about the generic solution of the problem of the 3^k dots spread to 3 X 3 X … X 3 points, in a k-dimensional space.

Keywords: dots, straight line, outside the box, graph theory.

MSC2010: Primary 91A43; Secondary 05E30, 91A46.

§1. Introduction

The 3 X 3 X … X 3 dots problem states: "Since the 3^k points as shown in **Fig. 1** (for $k = 4$), we must join with straight line and continuous stroke, using the smallest number of lines possible [1]".

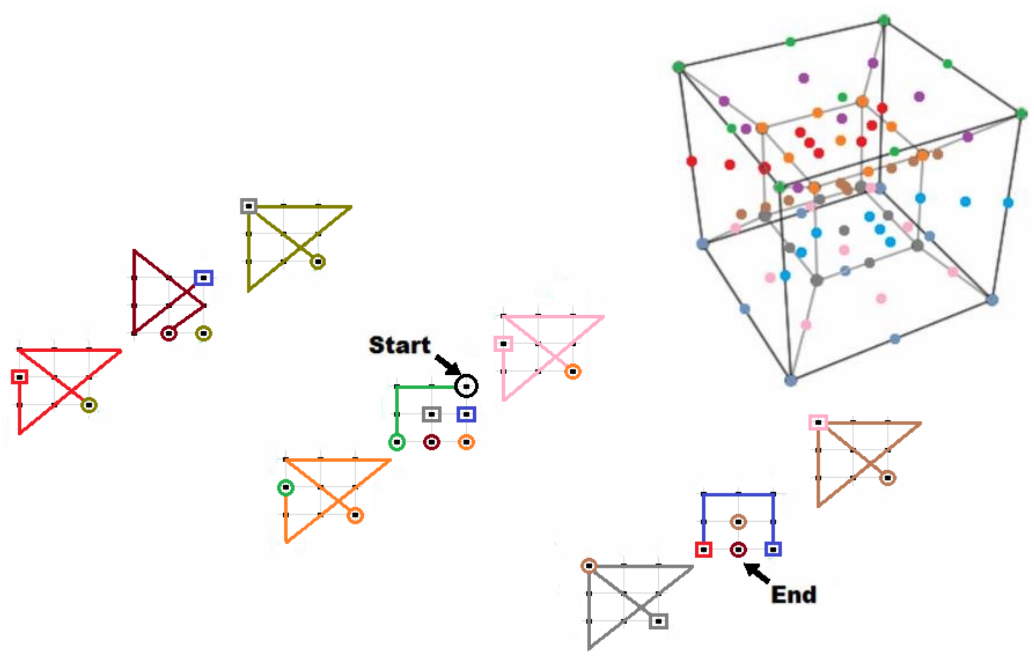

Figure 1. An extension of the 3 X 3 puzzle in a four-dimensional space (k=4), solved using 42 lines.

The interesting thing about this problem is that it requires lateral thinking for its solution [2]. Thinking outside the box (sometimes erroneously called "thinking out of the box" or "thinking outside the square") is to think differently, unconventionally or from a new perspective. This phrase often refers to novel, creative and smart thinking.
In August 2015, Ripà has shown [3] that $h(k)$, the number of straight lines connected at their end-points necessary to join all the 3^k dots, for $k \geq 4$, is

$$\left\lceil \frac{2 \cdot 3^k + 4}{4} \right\rceil - 1 \leq h(k) \leq 43 \cdot 3^{k-4} - 1$$

While, $h(1) = 1$, $h(2) = 4$, and $h(3) = 14$.

Except for $k < 2$, the $h(k)$ represent "outside the box" solutions.

§2. Ripà's first Conjecture

Let us consider the 3 X 3 X … X 3 as stated above, my conjecture says that, for every $k \geq 2$,

$$h(k) = \frac{3^k - 3}{2} + k - 1$$

Hence,

Conjecture 1.

$$\boldsymbol{h(k) = k + \frac{3^k - 5}{2}}, \qquad \boldsymbol{\forall k \geq 2}$$

Table 1. $\mathbf{3 \text{ X } 3 \text{ X} \dots \text{X } 3}$ points puzzle: number of straight lines necessary to join all the $\mathbf{3^k}$ dots according to Conjecture 1.

k	**h(*k*) (Conj. 1)**	**Current Lower Bound (M. Ripà)**	**Current Upper Bound (M. Ripà)**
1	**<u>1</u>**	1	1
2	**<u>4</u>**	4	4
3	**<u>14</u>**	14	14
4	**42**	41	42
5	**124**	122	128
6	**368**	365	386
7	**1098**	1094	1160
8	**3286**	3281	3482
9	**9848**	9842	10448
10	**29532**	29525	31346
11	**88582**	88574	94040
12	**265730**	265721	282122
13	**797172**	797162	846368
14	**2391496**	2391485	2539106
15	**7174466**	7174454	7617320

§3. Ripà's second Conjecture

Proceeding from the premise that Conjecture 1 is wrong, we would take Conjecture 1 as a conservative upper bound if we make a comparison with the next Conjecture 2. In fact, we can consider my second conjecture as a reliable lower bound for this problem, since we are implicitly assuming that there will not be other constraints except the one we have already analyzed.

Therefore, looking carefully at the sequence of the known h(k), we can see that every line joins the maximum theoretical number of dots (one time 3 dots, then 2 dots taking into account every additional line) if $k \leq 2$. For $k \geq 3$, there is no way to achieve an average score at or above 2 and it is reasonable to assume that we will spend one additional line for every k (starting from $k = 3$) in order to join a single "central" dot.

So,

$$3^k \leq 3 + 2 \cdot (h(k) - k + 2 - 1) + 1 \cdot (k - 2)$$

Thus, **Ripà's Conjecture 2** is as follows:

$$\boldsymbol{h(k) = \left\lceil \frac{3^k + k - 3}{2} \right\rceil, \qquad \forall k \geq 1}$$

Table 2. Straight lines necessary to join all the $\mathbf{3^k}$ dots according to Conjecture 2 and according to Conjecture 1.

k	h(k) (Conj. 2)	h(k) (Conj. 1)
1	**1**	**1**
2	**4**	**4**
3	**14**	**14**
4	**41**	42
5	**123**	124
6	**366**	368
7	**1096**	1098
8	**3283**	3286
9	**9845**	9848
10	**29528**	29532
11	**88578**	88582
12	**265725**	265730
13	**797167**	797172
14	**2391490**	2391496
15	**7174460**	7174466

References

[1] Loyd, S., *Cyclopedia* of Puzzles, The Lamb Publishing Company. 1914.

[2] Lung, C. T., Dominowski, R. L., *Effects of strategy instructions and practice on nine-dot problem solving*, Journal of Experimental Psychology: Learning, Memory, and Cognition **11 (4)**: 804–811. 1 January 1985.

[3] Ripà, M., *The n X n X n Points Problem Optimal Solution*, viXra, 30 Aug. 2015, `http://vixra.org/pdf/1508.0201v2.pdf`