On the Size of a Photon

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Abstract: After discussing various examples, the diameter of a single photon is calculated by combining the formulas of quantum mechanics and wave theory. The experimentally known coherence length is the length of the photon.

History

Is light an electromagnetic wave or a stream of photons? About 350 years ago, Gassendi suspected a stream of corpuscles, Newton reaffirmed this idea. Although no one was able to provide experimental evidence, this assumption held for about 150 years. In 1800, the idea of light changed abruptly when Thomas Young showed first interference phenomena. Now light was a transverse wave whose electromagnetic character was discovered by Maxwell. This classic finding experienced a few years of success, because Max Planck recognized in 1900 that these waves could gain or lose energy only in finite amounts related to their frequency. Planck called these "lumps" of light energy "quanta". Within the framework of classical physics that was not only revolutionary, it was incomprehensible. Physics experienced a radical change in the way of thinking.

In 1905, Albert Einstein published an idea that came immediately with fierce resistance, but was nevertheless accepted 15 years later. He proposed that these light quanta had a "real" existence. Later they were called Photons. To date, nobody has been able to measure the size of these particles of light, or at least indicate an order of magnitude. One reason is that the speed of photons can not be reduced.

Today, many physicists believe that photons actually exist, but they always emphasize that we must *not* imagine photons as small particles, such as spheres or bullets. Most properties are set by theory. Textbooks never mention the dimensions of photons and are very silent on this point. Nevertheless, some physicians claim *without proof* that photons are point-like, without any relation to the wavelength. Do experimental facts support this view? Are there any observations which contradict this assertion?

Consider the famous double-slit experiment. How can a dot-shaped photon realize that at a certain distance a second gap exists? Can the distance be any size, regardless of the wavelength? To answer these questions, a rough estimate of the size of a photon would be sufficient.

Interrelation of experiment and theory

Experiments alone provide unsystematic, incoherent garbage. A simple theory can establish relationships that were previously unknown. A good theory leads to at least one surprising and unexpected prediction that can be confirmed experimentally. The invention of theories without experimental basis and without testable predictions is worthless, a collection of unproven claims. Physics depends on observations, measurements and detection of patterns. Based on those results, theorists try to find useful formulas to calculate the results of future measurements. Every inexplicable and reproducible divergence may contribute to the improvement of the theory.

The requirement for manageable formulas requires to confine the theory to clearly defined parts of physics. Outside of their narrow range, they provide either no or wrong solutions. Here, we talk about electromagnetic waves in the region well below X-rays.

On an atomic scale, Quantum mechanics (QM) allows to calculate the results of many experiments

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in advance with amazing accuracy. The results are correct, but often contradict common sense. Quantum Optics is a branch of physics describing the optical phenomena of light. Normally, the experiments are done near the wavelength of visible light, even though the total spectrum of electromagnetic radiation extends far beyond both sides. The limits of those experiments are determined by the size of atoms. On the one hand, the resonators and energy detectors used can not be smaller than several hundred atoms. Therefore, wavelengths shorter than 10 nm are seldom used. On the other hand, electronic rectifiers for frequencies above 10^{11} Hz still do not exist. At higher frequencies, neither amplitude nor phase of electromagnetic waves can be measured, only energies.

Below 10^{11} Hz there are a lot of experimental results, which can not be treated with QM. To construct a sensitive receiver for radar, radio astronomy or satellite communication, QM can not contribute a single solution, nothing. In high-frequency technology below 10^{11} Hz, everything is calculated with Maxwell's equations. This set of formulas provides accurate predictions in the field of electromagnetic waves. Not a single fact gets clearer if the word photon is used.

Some very known experiments with light can only (or better) be explained by electromagnetic waves: In the field of light, no instrument can measure the amplitude (not intensity!) of electromagnetic waves. But without exception, *all results* in nonlinear optics are explained using the terms *amplitude* and *frequency.* Throughout the whole WP article Planck's constant is not even mentioned and the term "photon" is obviously superfluous in this context[[b]].

In 1956, the "Hanbury Brown and Twiss effect" was predicted and quite easily explained with the aid of Maxwell's equations and eventually discovered. From the perspective of QM, the HBT effect was doubted and treated with hostility, although it was experimentally demonstrated. The rules of QM prohibited explicitly that the HBT experiment could work. But QM was faulty and it took many years to expand the formula apparatus of QM so that finally the HBT effect could be explained. The quantum mechanical explanation of the HBT effect is difficult to understand and considerably more complex than the explanation by means of electromagnetic waves. But anyhow, the QM is no longer left empty-handed. Probably future experimental facts will require further supplements of the QM.

The Limits of Theories

The four formulas of Maxwell's equations link only electrical measurement values and therefore can not provide statements about, for example, gravity or energy. They describe error free all phenomena of electromagnetic waves including generation (near field), propagation (far field) and detection by antennas.

Max Planck discovered that the energy of electromagnetic waves at very low energies may not be changed continuously, but only in steps. This enforced the development of quantum mechanics. QM focuses on energy levels, moments and probabilities, but contains no information on electric fields or gravity. (The “waves” of the Schrödinger equation have nothing to do with the electromagnetic waves. It is questionable whether they exist at all, or whether they are just a mathematical trick.)

Because the QM does not contain methods to describe electromagnetic waves, Einstein invented photons. Their description is very scarce: photons are generated at *A* with the energy *hf* and absorbed with exactly the same energy in *B*. Point. (The redshift was not discovered in 1905). Not a word about how big they are and how they look. No description of route and speed between *A* and *B*. No distinction between near and far field. No instructions on how to deal with well known phenomena such as interference or refraction, because they were not required to explain the photoelectric effect. Einstein's speculation is a good approximation, if you transport visible light over very short distances inside a laboratory. But is it accurate enough at much lower frequencies and very large distances? Was that ever checked?

b If light consists of electrically neutral photons, how can the concentration of a huge number of photons generate an electric field?

110 years later, the question arises whether the reduction of electromagnetic waves on energy and angular momentum is not an oversimplification. For some tasks the few quantum numbers of photons may be enough, they are undoubtedly correct. But they may not be sufficient for many other tasks, because the QM does not have appropriate tools to perform calculations of speed, size and electromagnetic fields.

Two Worlds

When analog, finely stepped values of amplitude and phase can be measured or any kind of interference, use wave theory. The Maxwell equations are a set of partial differential equations, expecting readings without discontinuities. The wavelength should be much greater than the spacing of atoms. The wave theory provides detailed answers to questions like: How is the wave generated and propagates between transmitter and receiver and how is the path affected by obstacles. All properties of these transverse waves are well known and checked. But this formula set does not care about energy.

QM provides information about the *probability amplitudes* of position (no exact position!) and momentum and some other physical properties like the spin of a particle. But this theory does not tell us, how a photon looks like. There are experimental efforts to gain some insight. In contrast to the rectifiers of high frequency technology, the detection devices of Quantum Optics are nonlinear (square law detectors) and count individual photons, provided they exceed a certain energy threshold. This minimum energy is limited by the average thermal energy inside the detectors material. At room temperature, only photons with a maximum wavelength of about 10 µm may be measured, because - technically speaking - a photodiode is a digital receiver with poor signal-to-noise ratio. QM makes no statement about the way of the photon between transmitter and receiver. When a photon is registered, the route can not be reconstructed. Some researchers conclude: That's impossible, because QM has no solution. But originally QM had no answer to the HBT effect.

Unfortunately, the two worlds do not overlap, which is why comparative measurements are impossible. Instead, there is a broad gap between these areas, the terahertz gap. Here, viable methods of measurement do not exist.

Estimating the Size of Photons

QM insists that Photons are uncharged elementary particles with zero rest mass. In a vacuum, they move at the speed of light. Size and shape could not be measured in the past 110 years and there are no substantiated estimates. In the following section, the term "electromagnetic wave" is avoided and replaced by "photon" (independent of frequency and wavelength).

Observation 1: A sieve is a simple tool to separate a mixture of different sized parts. A prerequisite is that the particles do not adhere to the sieve. Photons satisfy this requirement, because they are uncharged and can't be influenced by electrical currents or magnetic fields.

In almost every kitchen there is a microwave oven, containing two distinct photon sources: A weak lamp with low power and a magnetron transmitter with very high power. Both generate photons (NO electromagnetic waves!). The weak lamp generates much fewer photons per second than the magnetron because in the range of visible light each photon carries much more energy than one microwave photon.

The door of the cooking chamber usually has a window for easy viewing, with a layer of conductive mesh. Very few of the microwave Photons can pass through the holes of the door, while the photons of visible light (with its much shorter wavelength) can. Why is there a difference when all the photons are point-shaped and only differ in the energy content? Whoever speaks of induced currents in the metal screen must also explain how *electrically neutral* photons can induce currents and may be influenced by them.

A simpler and more convincing explanation would be: The volume of photons increases with increasing wavelength and at a wavelength of 12 cm, the cross-sectional area of the photons is larger than the gaps of the metal sieve. Therefore, the thick photons can no longer slip through, only the much thinner photons of visual light.

Observation 2: Optical fibers permit data transmission over longer distances and at higher rates than wire cables. If light is stream of point-like photons, why should the required diameter of the core be larger than the wavelength? Why are the optimal diameter of core and cladding not calculated using the laws of quantum mechanics, but with the Maxwell equations? These differential equations do not meet the requirements of the QM. Does that mean that the fiber does not guide not photons but ordinary electromagnetic waves?

Observation 3: A television transmitter (f = 0.6 GHz) generates a lot of photons, which are to be guided to the antenna by a long cylindrical waveguide. Why must the pipe diameter be larger than the minimum value 29.3 cm? If the diameter is too small, *all* the TV-photons are reflected to the transmitter, without exception. The tube does not contain any disturbing or absorbing elements or even mirrors. But not a single TV-photon arrives at the antenna 170 m above, although one can look through the voluminous tube. Where, why and how are the uncharged photons forced to turn back?

In none of the three examples, QM provides some sort of reason for the peculiar behavior of the photons. This theory has no methods to describe the path of a photon between between source and destination. QM also has no way to describe size and shape of a photon. The reason for the construction of quantum mechanics was the handling of the quantized energy.

Calculating the Size of one Photon

In quantum optics, light may be considered not only as an electromagnetic wave but also as a "stream" of particles called photons which travel with *c*, the vacuum speed of light. A photon should not be considered to be classical billiard ball, but as quantum mechanical particle described by a wave function spread over a finite region. At this point it should be noted that there are two kinds of wave packets with very different time behavior.

- The wave packet of a *massive* particle (matter wave, proposed by Louis de Broglie) is dispersive. For non-relativistic velocities, the dispersion relation $\omega = h k^2 / (4 \pi m)$ applies. This non-linearity leads to wave-packet spreading.
- Since a photon has *no rest mass*, its wave packet is *non-dispersive*, the shape does *not* change. For plane waves in vacuum and no geometric constraint, the dispersion relation is linear: $\omega = c \cdot k$. The shape of the wave packet is preserved.

In physics, one photon of monochromatic light is described in two very different ways. From the perspective of quantum mechanics, a photon has the energy *hf*, the linear momentum *hf/c* and angular momentum (spin) $h/2\pi$.

From the perspective of the wave theory, the description is more complicated for two reasons.

- The Maxwell equations contain no equation for energy. This requires a detour via the energy density.
- The standard approach $A \cdot \sin(\omega t)$ for a wave is mathematically convenient, but it is no physically correct Ansatz, because it corresponds to an infinite extended wave field with infinite energy. But a photon contains the smallest possible amount of energy and can't extend unlimited.

A wave packet with a limited volume solves both problems, but the shape is unknown. In the score of electromagnetic waves, the spin *h/2π* corresponds to a circularly polarized wave. The formulas for the moment and spin for this type of wave are known (A single photon can never be linearly polarized.). Since QM and wave theory shall describe the same object, the corresponding equations must be equated and yield a surprisingly simple result[1].

The radius of an elementary wave packet, called photon, is

$$R=\frac{3\sqrt{2}c}{4\pi f}=\frac{3\sqrt{2}\lambda}{4\pi}\approx 0.338\cdot\lambda \tag{12}$$

As long as the wave packet is undisturbed, the angular momentum, the energy *hf* and the radius *R* are constant. This can be checked can be checked experimentally.

Unfortunately, the calculation does not provide any information about the length of the wave packet. But there are very reliable experimental data. At high intensity wave packets may overlap, whereby a determination of the length is at least difficult. At low light intensity, it is very unlikely that any wave packets overlap. Then, their length can be measured independently.

There is a simple relationship between the coherence length *L* of a wave packet named photon and the line width Δf of the spectral frequency: $L\cdot\Delta f = c$

Measurements in radio astronomy with emission lines around 5 GHz result in a Full width at half maximum Δf = 0.6 MHz or $L \approx$ 500 m for dilute gas[2]. This corresponds to approximately 8300 wavelengths.

The FWHM of the well known HI 21 cm line is about 20 kHz or less, corresponding to $L \approx$ 15000 m or 71000 wavelengths[3][4].

Precision measurements of the sodium D line[5] yield a FWHM of about 61.4 MHz or $L \approx$ 4.8 m, corresponding to $8.3\cdot10^6$ wavelengths.

Those results allow to describe the shape and size of a photon:

A photon is an approximately cylindrical, circularly polarized wave packet with the diameter of roughly 0.67 λ and a minimum length of about 10,000 λ. It carries the energy *h·f*

The calculation is based on the assumption that the electric field intensity is constant within the wave packet and vanishes outside everywhere. This abrupt change is mathematically convenient, but probably not too realistic. Presumably, the field strength is at its maximum near the center of the wave packet maximum and decreases with increasing distance from the axis of symmetry. For this reason, the diameter of the wave packet can not be precisely defined, but it is most likely smaller than a wavelength.

Summary

A photon is born as an electromagnetic wave packet with the energy *hf* and the spin 1. This elongated, very thin wave packet retains its shape until it is completely absorbed and emits the energy hf. Even if the wave packet is modified while it is flying with the speed of light, it behaves as an electromagnetic wave whose properties (including nonlinear effects, but apart from the precisely defined energy) are fully described with the aid of Maxwell's equations.

[1] H. Weidner, The Size and Energy Loss of a Wave Packet, 2014
https://www.researchgate.net/publication/265013965_The_Size_and_Energy_Loss_of_a_Wave_Packet
[2] C. Quireza et al., Radion Recombination Lines in Galactic HII Regions, AAS 165:338–359, 2006,
http://iopscience.iop.org/article/10.1086/503901/pdf
[3] P. Kalberla, The HI Distribution of the Milky Way, 2009
http://www.ifa.hawaii.edu/users/jpw/ism/reading/kalberla_mw_HI.pdf
[4] The HI 21 cm Line, http://www.cv.nrao.edu/course/astr534/HILine.html
[5] D. Steck, Sodium D Line Data, LANL 2003, http://steck.us/alkalidata/sodiumnumbers.1.6.pdf