Some weighted aggregation operators of trapezoidal neutrosophic numbers and their multiple attribute decision making method

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Abstract: Based on the combination of single valued neutrosophic sets (SVNSs) and fuzzy numbers, this paper proposes the concepts of a neutrosophic number and a trapezoidal neutrosophic number (TNN) as the extension of an intuitionistic fuzzy number and an intuitionistic trapezoidal fuzzy number (ITFN), the basic operational relations of TNNs and the score function of TNN. Then, we develop a trapezoidal neutrosophic weighted arithmetic averaging (TNWAA) operator and a trapezoidal neutrosophic weighted geometric averaging (TNWGA) operator to aggregate TNN information and investigate their properties. Furthermore, a multiple attribute decision making method based on the proposed TNWAA and TNWGA operators and the score function of TNN is established under TNN environment. Finally, an illustrative example of investment alternatives is given to demonstrate the application and effectiveness of the developed approach.

Keywords: Neutrosophic number; Trapezoidal neutrosophic number; Score function; Trapezoidal neutrosophic weighted arithmetic averaging (TNWAA) operator; Trapezoidal neutrosophic weighted geometric averaging (TNWGA) operator; Multiple attribute decision making

1. Introduction

Smarandache (1999) originally gave a concept of a neutrosophic set, which is a part of neutrosophy and generalizes fuzzy sets (Zadeh, 1965), interval valued fuzzy sets (IVFSs) (Turksen, 1986), intuitionistic fuzzy sets (IFSs) (Atanassov, 1986), and interval-valued intuitionistic fuzzy sets (IVIFSs) (Atanassov and Gargov, 1989) from philosophical point of view. To obtain the real applications, Wang et al. (2005, 2010) presented single valued neutrosophic sets (SVNSs) and interval neutrosophic sets (INSs), which are the subclasses of neutrosophic sets. They can independently express the truth-membership degree, indeterminacy-membership degree, and false-membership degree. SVNSs and INSs, as the generalization of IFSs and IVIFSs, can handle incomplete, indeterminate and inconsistent information which exists commonly in real situations, while IFSs and IVIFSs only express truth-membership degree and false-membership degree, but cannot deal with indeterminate and inconsistent information. Hence, SVNSs and INSs are very suitable for applications in decision making. Ye (2013) developed the correlation coefficient of SVNSs as the extension of the correlation coefficient of IFSs and proved that the cosine similarity measure of SVNSs is a special case of the correlation coefficient of SVNSs, and then applied it to single valued neutrosophic decision-making problems. Chi and Liu (2013) proposed an extended TOPSIS method for multiple attribute decision making under interval neutrosophic environment. Moreover, Ye (2014a) presented the Hamming and Euclidean distances between INSs and the distances-based similarity

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measures of INSs, and then a multicriteria decision-making method based on the similarity measures of INSs was established in interval neutrosophic setting. Furthermore, Ye (2014b) proposed a cross-entropy measure of SVNSs and applied it to multicriteria decision making problems with single valued neutrosophic information. Ye (2014c) further introduced a simplified neutrosophic set (SNS) as a subclass of a neutrosophic set, which includes SVNS and INS, and developed the simplified neutrosophic weighted averaging (SNWA) operator and the simplified neutrosophic weighted geometric (SNWG) operator, and their applications in multicriteria decision-making under simplified neutrosophic environment. Liu et al. (2014) further proposed some generalized single valued neutrosophic number Hamacher aggregation operators and applied them to group decision making. Then, Zhang et al. (2014) defined the score, accuracy, and certainty functions for INSs and presented a comparison approach, and then they also developed some aggregation operators for INNs and a multicriteria decision making method by means of the aggregation operators. On the other hand, Ye (2014d) put forward vector similarity measures, including the Dice, Jaccard, and cosine measures of SNSs, and applied them to multicriteria decision making problems in the simplified neutrosophic setting. Biswas et al. (2014 established a single valued neutrosophic multiple attribute decision-making method with unknown weight information, where optimization models were used to determine unknown attribute weights and the grey relational coefficient of each alternative from ideal alternative was utilized to rank alternatives. Zhang and Wu (2014) also developed a method for solving single valued neutrosophic multicriteria decision making problems with incomplete weight information, in which the criterion values are given in the form of single-valued neutrosophic sets (SVNSs), and the information about criterion weights is incompletely known or completely unknown.

Intuitionistic fuzzy numbers (IFNs) and intuitionistic trapezoidal fuzzy numbers (ITFNs) introduced in (Wang and Zhang, 2009) are the extending of IFSs in another way, which extends discrete set to continuous set. Then the domains of SVNSs and INSs are discrete sets, but not continuous sets in existing literature. The advantage of continuous sets is that they include much information and the fuzziness in multiple attribute decision making has the better character because of the proposal of fuzzy number (Wang and Zhang, 2009). At present, there are no studies on neutrosophic numbers and trapezoidal neutrosophic numbers (TNNs) in above mentioned decision-making problems. Motivated by the reference (Wang and Zhang, 2009), we should make the truth-membership, indeterminacy-membership, falsity-membership degrees in a SVNS no longer relative to single values, but relative to fuzzy numbers or trapezoidal fuzzy numbers. Thus we can introduce the concepts of a neutrosophic number and a TNN, which extends discrete set to continuous set, as the extension of IFN and ITFN (Wang and Zhang, 2009). However, TNN is a special case of a neutrosophic number and useful in practical applications, and is of importance for neutrosophic multiple attribute decision making problems. Therefore, the purposes of this article are: (1) to introduce the concepts of a neutrosophic number and a TNN, some basic operational relations of TNNs and a score function for TNN, (2) to propose two aggregation operators: a trapezoidal neutrosophic weighted arithmetic averaging (TNWAA) operator and a trapezoidal neutrosophic weighted geometric averaging (TNWGA) operator, and (3) to establish a decision making approach based on the TNWAA and TNWGA operators and the score function of TNN under TNN environment.

The rest of the article is organized as follows. Section 2 briefly describes some concepts of IFNs, ITFNs and operational relations for ITFNs. Section 3 proposes the concepts of a neutrosophic number and a TNN and defines some basic operations of TNNs and the score function of TNN. In Section 4, we develop TNWAA and TNWGA operators for TNNs and investigate their properties. Section 5 establishes a decision making approach based on the TNWAA and TNWGA operators and the score function under TNN environment. In Section 6, an illustrative example is provided to illustrate the application of the developed method. Section 7 contains a conclusion and future research.

2. Intuitionistic fuzzy numbers and Intuitionistic trapezoidal fuzzy numbers

In this section, we briefly describe some concepts of IFNs, ITFNs and operational relations for ITFNs.

Definition 1 (Wang and Zhang, 2009)**.** Let $\tilde{a}$ be an IFN in the set of real numbers *R*, then its membership function is defined as

$$\mu_{\tilde{a}}(x)=\begin{cases} f_{\tilde{a}}(x), & a_1 \le x < a_2, \\ \mu_{\tilde{a}}, & a_2 \le x \le a_3, \\ g_{\tilde{a}}(x), & a_3 < x \le a_4, \\ 0, & otherwise \end{cases}, \tag{1}$$

and its nonmembership function is defined as

$$\nu_{\tilde{a}}(x)=\begin{cases} h_{\tilde{a}}(x), & b_1 \le x < b_2, \\ \nu_{\tilde{a}}, & b_2 \le x \le b_3, \\ k_{\tilde{a}}(x), & b_3 < x \le b_4, \\ 1, & otherwise \end{cases}, \tag{2}$$

where $\mu_{\tilde{a}}, \nu_{\tilde{a}} \in [0, 1]$, $0 \le \mu_{\tilde{a}} + \nu_{\tilde{a}} \le 1$ and $a_1, a_2, a_3, a_4, b_1, b_2, b_3, b_4 \in R$, and four functions $f_{\tilde{a}}, g_{\tilde{a}}, h_{\tilde{a}}, k_{\tilde{a}}: R \to [0, 1]$ are called the side of a fuzzy number. The functions $f_{\tilde{a}}, k_{\tilde{a}}$ are increasing continuous functions and the functions $g_{\tilde{a}}, h_{\tilde{a}}$ are decreasing continuous functions.

Particularly, if the increasing functions $f_{\tilde{a}}, k_{\tilde{a}}$ and decreasing functions $g_{\tilde{a}}, h_{\tilde{a}}$ are linear, then we have ITFNs, which are preferred in practice.

Definition 2. Let $\tilde{a}$ be an ITFN. Then, the membership function and nonmembership function can be defined, respectively, as follows (Wang and Zhang, 2009):

$$\mu_{\tilde{a}}(x)=\begin{cases} \dfrac{x-a_1}{a_2-a_1}\mu_{\tilde{a}}, & a_1 \le x < a_2, \\ \mu_{\tilde{a}}, & a_2 \le x \le a_3, \\ \dfrac{a_4-x}{a_4-a_3}\mu_{\tilde{a}}, & a_3 < x \le a_4, \\ 0, & otherwise \end{cases}, \tag{3}$$

$$\nu_{\tilde{a}}(x)=\begin{cases} \dfrac{b_2-x+\nu_{\tilde{a}}(x-b_1)}{b_2-b_1}, & b_1 \le x < b_2, \\ \nu_{\tilde{a}}, & b_2 \le x \le b_3, \\ \dfrac{x-b_3+\nu_{\tilde{a}}(b_4-x)}{b_4-b_3}, & b_3 < x \le b_4, \\ 1, & otherwise \end{cases}, \tag{4}$$

where $\mu_{\tilde{a}}, \nu_{\tilde{a}} \in [0, 1]$, $0 \le \mu_{\tilde{a}} + \nu_{\tilde{a}} \le 1$ and $a_1, a_2, a_3, a_4, b_1, b_2, b_3, b_4 \in R$. Generally, if $[a_1, a_2, a_3, a_4] =$

$[b_1, b_2, b_3, b_4]$ in an ITFN $\tilde{a}$, then the ITFN $\tilde{a}$ is denoted as $\tilde{a} = \langle(a_1, a_2, a_3, a_4); \mu_{\tilde{a}}, \nu_{\tilde{a}}\rangle$.

ITFNs have the following operational relations (Wang and Zhang, 2009):

Definition 3 (Wang and Zhang, 2009). Let $\tilde{a} = \langle(a_1, a_2, a_3, a_4); \mu_{\tilde{a}}, \nu_{\tilde{a}}\rangle$ and $\tilde{b} = \langle(b_1, b_2, b_3, b_4); \mu_{\tilde{b}}, \nu_{\tilde{b}}\rangle$ be two ITFNs and $\lambda \geq 0$. Then there are the following operational relations:

(1) $\tilde{a} + \tilde{b} = \langle(a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4); \mu_{\tilde{a}} + \mu_{\tilde{b}} - \mu_{\tilde{a}}\mu_{\tilde{b}}, \nu_{\tilde{a}}\nu_{\tilde{b}}\rangle$;

(2) $\tilde{a} \cdot \tilde{b} = \langle(a_1b_1, a_2b_2, a_3b_3, a_4b_4); \mu_{\tilde{a}}\mu_{\tilde{b}}, \nu_{\tilde{a}} + \nu_{\tilde{b}} - \nu_{\tilde{a}}\nu_{\tilde{b}}\rangle$;

(3) $\lambda\tilde{a} = \langle(\lambda a_1, \lambda a_2, \lambda a_3, \lambda a_4); 1 - (1 - \mu_{\tilde{a}})^\lambda, \nu_{\tilde{a}}^\lambda\rangle$;

(4) $\tilde{a}^\lambda = \langle(a_1^\lambda, a_2^\lambda, a_3^\lambda, a_4^\lambda); \mu_{\tilde{a}}^\lambda, 1 - (1 - \nu_{\tilde{a}})^\lambda\rangle$.

3. Neutrosophic numbers and trapezoidal neutrosophic numbers

In this section, motivated by IFNs and ITFNs, we propose neutrosophic numbers and TNNs based on the combination of SVNSs and fuzzy numbers as the generalization of IFNs and ITFNs, which extend discrete sets to continuous sets.

Smarandache (1999) firstly presented a neutrosophic set from philosophical point of view. To easily apply the neutrosophic set to practical problems. Wang et al (2010) introduced the concept of a SVNS, which is a subclass of the neutrosophic set.

Definition 4 (Wang et al, 2010). Let X be a space of points (objects) with generic elements in X denoted by x. A SVNS N in X is characterized by a truth-membership function $T_N(x)$, an indeterminacy-membership function $I_N(x)$, and a falsity-membership function $F_N(x)$. Then, a SVNS N can be denoted by

$$N = \{\langle x, T_N(x), I_N(x), F_N(x)\rangle \mid x \in X\},$$

where the sum of $T_N(x)$, $I_N(x)$, $F_N(x) \in [0, 1]$ satisfies $0 \leq T_N(x) + I_N(x) + F_N(x) \leq 3$ for each point x in X. For convenience, we can use the simplified symbol $n_x = <T_x, I_x, F_x>$ to represent a basic element in a SVNS N, and call it a single valued neutrosophic number (SVNN).

Different from the definition of SVNS, we make the truth-membership, indeterminacy-membership and falsity-membership degrees no longer relative to single values, but relative to fuzzy numbers. Then, we can give the following definitions of a neutrosophic number and a TNN.

Definition 5. Let $\tilde{n}$ be a neutrosophic number in the set of real numbers R, then its truth-membership function is defined as

$$T_{\tilde{n}}(x) = \begin{cases} f_{\tilde{n}}(x), & a_1 \leq x < a_2, \\ T_{\tilde{n}}, & a_2 \leq x \leq a_3, \\ g_{\tilde{n}}(x), & a_3 < x \leq a_4, \\ 0, & otherwise \end{cases}, \quad (5)$$

its indeterminacy-membership function is defined as

$$I_{\tilde{n}}(x)=\begin{cases} h_{\tilde{n}}(x), & b_1 \le x < b_2, \\ I_{\tilde{n}}, & b_2 \le x \le b_3, \\ k_{\tilde{n}}(x), & b_3 < x \le b_4, \\ 1, & otherwise \end{cases}, \tag{6}$$

and its falsity-membership function is defined as

$$F_{\tilde{n}}(x)=\begin{cases} p_{\tilde{n}}(x), & c_1 \le x < c_2, \\ F_{\tilde{n}}, & c_2 \le x \le c_3, \\ q_{\tilde{n}}(x), & c_3 < x \le c_4, \\ 1, & otherwise \end{cases}, \tag{7}$$

where $T_{\tilde{n}}, I_{\tilde{n}}, F_{\tilde{n}} \in [0, 1]$, $0 \le T_{\tilde{n}} + I_{\tilde{n}} + F_{\tilde{n}} \le 3$ and $a_1, a_2, a_3, a_4, b_1, b_2, b_3, b_4$, $c_1, c_2, c_3, c_4 \in R$, and six functions $f_{\tilde{n}}$, $g_{\tilde{n}}$, $h_{\tilde{n}}$, $k_{\tilde{n}}$, $p_{\tilde{n}}$, $q_{\tilde{n}}$: $R \to [0, 1]$ are called the side of a fuzzy number. The functions $f_{\tilde{n}}, k_{\tilde{n}}, q_{\tilde{n}}$ are increasing continuous functions and the functions $g_{\tilde{n}}, h_{\tilde{n}}, p_{\tilde{n}}$ are decreasing continuous functions.

Especially, if the increasing functions $f_{\tilde{n}}, k_{\tilde{n}}, q_{\tilde{n}}$ and decreasing functions $g_{\tilde{n}}, h_{\tilde{n}}, p_{\tilde{n}}$ are linear, then we have a TNN, which is preferred in practice.

Definition 6. Let $\tilde{n}$ be a TNN. Then, the truth-membership function, indeterminacy-membership function, and falsity-membership function can be defined, respectively, as follows:

$$T_{\tilde{n}}(x)=\begin{cases} \dfrac{x-a_1}{a_2-a_1}T_{\tilde{n}}, & a_1 \le x < a_2, \\ T_{\tilde{n}}, & a_2 \le x \le a_3, \\ \dfrac{a_4-x}{a_4-a_3}T_{\tilde{n}}, & a_3 < x \le a_4, \\ 0, & otherwise \end{cases}, \tag{8}$$

$$I_{\tilde{n}}(x)=\begin{cases} \dfrac{b_2-x+I_{\tilde{n}}(x-b_1)}{b_2-b_1}, & b_1 \le x < b_2, \\ I_{\tilde{n}}, & b_2 \le x \le b_3, \\ \dfrac{x-b_3+I_{\tilde{n}}(b_4-x)}{b_4-b_3}, & b_3 < x \le b_4, \\ 1, & otherwise \end{cases}, \tag{9}$$

$$F_{\tilde{n}}(x) = \begin{cases} \dfrac{c_2 - x + F_{\tilde{n}}(x - c_1)}{c_2 - c_1}, & c_1 \le x < c_2, \\ F_{\tilde{n}}, & c_2 \le x \le c_3, \\ \dfrac{x - c_3 + F_{\tilde{n}}(c_4 - x)}{c_4 - c_3}, & c_3 < x \le c_4, \\ 1, & otherwise \end{cases}, \tag{10}$$

where $T_{\tilde{n}}, I_{\tilde{n}}, F_{\tilde{n}} \in [0, 1]$, $0 \le T_{\tilde{n}} + I_{\tilde{n}} + F_{\tilde{n}} \le 3$ and $a_1, a_2, a_3, a_4, b_1, b_2, b_3, b_4, c_1, c_2, c_3, c_4 \in R$. Then, $\tilde{n} = \langle([a_1, a_2, a_3, a_4]; T_{\tilde{n}}), ([b_1, b_2, b_3, b_4]; I_{\tilde{n}}), ([c_1, c_2, c_3, c_4]; F_{\tilde{n}})\rangle$ is called a TNN. Generally, if $[a_1, a_2, a_3, a_4] = [b_1, b_2, b_3, b_4] = [c_1, c_2, c_3, c_4]$ in a TNN $\tilde{n}$, then the TNN $\tilde{n}$ can be denoted as $\tilde{n} = \langle(a_1, a_2, a_3, a_4); T_{\tilde{n}}, I_{\tilde{n}}, F_{\tilde{n}}\rangle$.

If $a_2 = a_3$ in a TNN $\tilde{n}$, The TNN $\tilde{n}$ reduces to the triangular neutrosophic number, which is considered as a special case of the TNN $\tilde{n}$. If $a_1 = a_2 = a_3 = a_4 = 1$ in a TNN $\tilde{n}$, then the TNN $\tilde{n}$ reduces to the SVNN.

If $0 \le a_1 \le a_2 \le a_3 \le a_4$, then $\tilde{n}$ is called a positive TNN. If $a_1 \le a_2 \le a_3 \le a_4 \le 0$, then $\tilde{n}$ is called a negative TNN. If $0 \le a_1 \le a_2 \le a_3 \le a_4 \le 1$ and $T_{\tilde{n}}, I_{\tilde{n}}, F_{\tilde{n}} \in [0, 1]$, then $\tilde{n}$ is called a normalized TNN, which is used for this paper.

Thus, we can introduce the following operational relations of TNNs:

Definition 7. Let $\tilde{n}_1 = \langle(a_1, a_2, a_3, a_4); T_{\tilde{n}_1}, I_{\tilde{n}_1}, F_{\tilde{n}_1}\rangle$ and $\tilde{n}_2 = \langle(b_1, b_2, b_3, b_4); T_{\tilde{n}_2}, I_{\tilde{n}_2}, F_{\tilde{n}_2}\rangle$ be two TNNs and $\lambda \ge 0$. Then there are the following operational relations:

(1) $\tilde{n}_1 + \tilde{n}_2 = \langle(a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4); T_{\tilde{n}_1} + T_{\tilde{n}_2} - T_{\tilde{n}_1}T_{\tilde{n}_2}, I_{\tilde{n}_1}I_{\tilde{n}_2}, F_{\tilde{n}_1}F_{\tilde{n}_2}\rangle$;

(2) $\tilde{n}_1 \cdot \tilde{n}_2 = \langle(a_1b_1, a_2b_2, a_3b_3, a_4b_4); T_{\tilde{n}_1}T_{\tilde{n}_2}, I_{\tilde{n}_1} + I_{\tilde{n}_2} - I_{\tilde{n}_1}I_{\tilde{n}_2}, F_{\tilde{n}_1} + F_{\tilde{n}_2} - F_{\tilde{n}_1}F_{\tilde{n}_2}\rangle$;

(3) $\lambda\tilde{n}_1 = \langle(\lambda a_1, \lambda a_2, \lambda a_3, \lambda a_4); 1 - (1 - T_{\tilde{n}_1})^{\lambda}, I_{\tilde{n}_1}^{\lambda}, F_{\tilde{n}_1}^{\lambda}\rangle$;

(4) $\tilde{n}_1^{\lambda} = \langle(a_1^{\lambda}, a_2^{\lambda}, a_3^{\lambda}, a_4^{\lambda}); T_{\tilde{n}_1}^{\lambda}, 1 - (1 - I_{\tilde{n}_1})^{\lambda}, 1 - (1 - F_{\tilde{n}_1})^{\lambda}\rangle$.

Based on the expected value of an ITFN (Wang and Zhang, 2009) and the score function of an interval neutrosophic value (Zhang et al, 2014), we can give the following definition of a score function for a TNN.

Definition 8. Let $\tilde{n} = \langle(a_1, a_2, a_3, a_4); T_{\tilde{n}}, I_{\tilde{n}}, F_{\tilde{n}}\rangle$ be a TNN. Then there is the score function of $\tilde{n}$:

$$S(\tilde{a}) = \frac{1}{12}(a_1 + a_2 + a_3 + a_4)(2 + T_{\tilde{a}} - I_{\tilde{a}} - F_{\tilde{a}}),\ S(\tilde{a}) \in [0,1]. \tag{11}$$

For the comparison between two TNNs, a comparative method based on the score function is defined as follows.

Definition 9. Let $\tilde{n}_1 = \langle(a_1, a_2, a_3, a_4); T_{\tilde{n}_1}, I_{\tilde{n}_1}, F_{\tilde{n}_1}\rangle$ and $\tilde{n}_2 = \langle(b_1, b_2, b_3, b_4); T_{\tilde{n}_2}, I_{\tilde{n}_2}, F_{\tilde{n}_2}\rangle$ be two TNNs. Then, if $S(\tilde{n}_1) > S(\tilde{n}_2)$, then $\tilde{n}_1 > \tilde{n}_2$; if $S(\tilde{n}_1) = S(\tilde{n}_2)$, then $\tilde{n}_1 = \tilde{n}_2$.

For example, let two TNNs be $\tilde{n}_1$ = ⟨(0.4, 0.5, 0.6, 0.7); 0.4, 0.2, 0.3⟩ and $\tilde{n}_2$ = ⟨(0.6, 0.7, 0.8, 0.9); 0.6, 0.3, 0.4⟩. In this case, we can compare them according to the score values. Since $S(\tilde{n}_1)$ = (0.4 + 0.5 + 0.6 + 0.7)(2 + 0.4 - 0.2 - 0.3)/12 = 0.3483 and $S(\tilde{n}_2)$ = (0.6 + 0.7 + 0.8 + 0.9)(2 + 0.6 - 0.3 - 0.4)/12 = 0.475, by Definition 9, there is $\tilde{n}_1 < \tilde{n}_2$.

4. Two weighted aggregation operators of TNNs

Since aggregation operators are an important tool for aggregated information in decision-making process, this section proposes two weighted aggregation operators to aggregate TNNs as a generalization of the weighted aggregation operators for ITFNs (Wang and Zhang, 2009), which are usually used in decision making.

4.1. Trapezoidal neutrosophic weighted arithmetic averaging operator

Definition 10. Let $\tilde{n}_j = \left\langle (a_{j1}, a_{j2}, a_{j3}, a_{j4}); T_{\tilde{n}_j}, I_{\tilde{n}_j}, F_{\tilde{n}_j} \right\rangle$ (j = 1, 2, . . . , n) be a collection of TNNs, then a TNWAA operator is defined as follows:

$$TNWAA(\tilde{n}_1, \tilde{n}_2, \cdots, \tilde{n}_n) = \sum_{j=1}^{n} w_j \tilde{n}_j, \tag{12}$$

where w_j is the weight of $\tilde{n}_j$ (j = 1, 2, . . . , n) such that $w_j > 0$ and $\sum_{j=1}^{n} w_j = 1$. Specially, when $w_j = 1/n$ for j = 1, 2, . . . , n, the TNWAA operator reduces to the trapezoidal neutrosophic arithmetic averaging operator.

According to Definitions 7 and 10, we can introduce the following theorem.

Theorem 1. Let $\tilde{n}_j = \left\langle (a_{j1}, a_{j2}, a_{j3}, a_{j4}); T_{\tilde{n}_j}, I_{\tilde{n}_j}, F_{\tilde{n}_j} \right\rangle$ (j = 1, 2, …, n) be a collection of TNNs, then according to Definitions 7 and 10, we can give the following TNWAA operator:

$$\begin{aligned} TNWAA(\tilde{n}_1, \tilde{n}_2, \cdots, \tilde{n}_n) &= \sum_{j=1}^{n} w_j \tilde{n}_j \\ &= \left\langle \left(\sum_{j=1}^{n} w_j a_{j1}, \sum_{j=1}^{n} w_j a_{j2}, \sum_{j=1}^{n} w_j a_{j3}, \sum_{j=1}^{n} w_j a_{j4} \right); 1 - \prod_{j=1}^{n} (1 - T_{\tilde{n}_j})^{w_j}, \prod_{j=1}^{n} I_{\tilde{n}_j}^{w_j}, \prod_{j=1}^{n} F_{\tilde{n}_j}^{w_j} \right\rangle \end{aligned}, \tag{13}$$

where w_j is the weight of $\tilde{n}_j$ (j = 1, 2, . . . , n) such that $w_j > 0$ and $\sum_{j=1}^{n} w_j = 1$.

Theorem 1 can be proved by means of mathematical induction.

Proof:

(1) When n = 2, then,

$$w_1 \tilde{n}_1 = \left\langle (w_1 a_{11}, w_1 a_{12}, w_1 a_{13}, w_1 a_{14}); 1 - (1 - T_{\tilde{n}_1})^{w_1}, I_{\tilde{n}_1}^{w_1}, F_{\tilde{n}_1}^{w_1} \right\rangle,$$

$$w_2 \tilde{n}_2 = \left\langle (w_2 a_{21}, w_2 a_{22}, w_2 a_{23}, w_2 a_{24}); 1 - (1 - T_{\tilde{n}_2})^{w_2}, I_{\tilde{n}_2}^{w_2}, F_{\tilde{n}_2}^{w_2} \right\rangle.$$

Thus

$$TNWAA(\tilde{n}_1,\tilde{n}_2)=w_1\tilde{n}_1+w_2\tilde{n}_2=\big\langle(w_1a_{11}+w_2a_{21}+w_1a_{12}+w_2a_{22}+w_1a_{13}+w_2a_{23}+w_1a_{14}+w_2a_{24});$$
$$1-(1-T_{\tilde{n}_1})^{w_1}+1-(1-T_{\tilde{n}_2})^{w_2}-(1-(1-T_{\tilde{n}_1})^{w_1})(1-(1-T_{\tilde{n}_2})^{w_2}),I_{\tilde{n}_1}^{w_1}I_{\tilde{n}_2}^{w_2},F_{\tilde{n}_1}^{w_1}F_{\tilde{n}_2}^{w_2}\big\rangle \tag{14}$$
$$=\big\langle(w_1a_{11}+w_2a_{21}+w_1a_{12}+w_2a_{22}+w_1a_{13}+w_2a_{23}+w_1a_{14}+w_2a_{24});$$
$$1-(1-T_{\tilde{n}_1})^{w_1}(1-T_{\tilde{n}_2})^{w_2},I_{\tilde{n}_1}^{w_1}I_{\tilde{n}_2}^{w_2},F_{\tilde{n}_1}^{w_1}F_{\tilde{n}_2}^{w_2}\big\rangle.$$

(2) When $n = k$, by applying Eq. (13), we get

$$TNWAA(\tilde{n}_1,\tilde{n}_2,\cdots,\tilde{n}_k)=\sum_{j=1}^{k}w_j\tilde{n}_j$$
$$=\left\langle\left(\sum_{j=1}^{k}w_ja_{j1},\sum_{j=1}^{k}w_ja_{j2},\sum_{j=1}^{k}w_ja_{j3},\sum_{j=1}^{k}w_ja_{j4}\right);1-\prod_{j=1}^{k}(1-T_{\tilde{n}_j})^{w_j},\prod_{j=1}^{k}I_{\tilde{n}_j}^{w_j},\prod_{j=1}^{k}F_{\tilde{n}_j}^{w_j}\right\rangle. \tag{15}$$

(3) When $n = k + 1$, by applying Eqs. (14) and (15), we can yield

$$TNWAA(\tilde{n}_1,\tilde{n}_2,\cdots,\tilde{n}_{k+1})=\sum_{j=1}^{k+1}w_j\tilde{n}_j=\left\langle\left(\sum_{j=1}^{k+1}w_ja_{j1},\sum_{j=1}^{k+1}w_ja_{j2},\sum_{j=1}^{k+1}w_ja_{j3},\sum_{j=1}^{k+1}w_ja_{j4}\right);\right.$$
$$\left.1-\prod_{j=1}^{k}(1-T_{\tilde{n}_j})^{w_j}+1-(1-T_{\tilde{n}_{k+1}})^{w_{k+1}}-\left(1-\prod_{j=1}^{k}(1-T_{\tilde{n}_j})^{w_j}\right)\left(1-(1-T_{\tilde{n}_{k+1}})^{w_{k+1}}\right),\prod_{j=1}^{k+1}I_{\tilde{n}_j}^{w_j},\prod_{j=1}^{k+1}F_{\tilde{n}_j}^{w_j}\right\rangle. \tag{16}$$
$$=\left\langle\left(\sum_{j=1}^{k+1}w_ja_{j1},\sum_{j=1}^{k+1}w_ja_{j2},\sum_{j=1}^{k+1}w_ja_{j3},\sum_{j=1}^{k+1}w_ja_{j4}\right);1-\prod_{j=1}^{k+1}(1-T_{\tilde{n}_j})^{w_j},\prod_{j=1}^{k+1}I_{\tilde{n}_j}^{w_j},\prod_{j=1}^{k+1}F_{\tilde{n}_j}^{w_j}\right\rangle$$

Therefore, considering the above results, we have Eq. (13) for any n. This completes the proof. □

Obviously, the *TNWAA* operator satisfies the following properties:

(1) Idempotency: Let $\tilde{n}_j$ (j = 1, 2, . . . , n) be a collection of TNNs. If $\tilde{n}_j$ (j = 1, 2, . . . , n) is equal, i.e. $\tilde{n}_j = \tilde{n}$ for j = 1, 2, . . . , n, then $TNWAA(\tilde{n}_1,\tilde{n}_2,\cdots,\tilde{n}_n)=\tilde{n}$.

(2) Boundedness: Let $\tilde{n}_j$ (j = 1, 2, . . . , n) be a collection of TNNs and let

$$\tilde{n}^-=\left\langle(\min_j a_{j1},\min_j a_{j2},\min_j a_{j3},\min_j a_{j4});\min_j T_{\tilde{n}_j},\max_j I_{\tilde{n}_j},\max_j F_{\tilde{n}_j}\right\rangle,$$
$$\tilde{n}^+=\left\langle(\max_j a_{j1},\max_j a_{j2},\max_j a_{j3},\max_j a_{j4});\max_j T_{\tilde{n}_j},\min_j I_{\tilde{n}_j},\min_j F_{\tilde{n}_j}\right\rangle.$$

Then $\tilde{n}^-\le TNWAA(\tilde{n}_1,\tilde{n}_2,\cdots,\tilde{n}_n)\le\tilde{n}^+$.

(3) Monotonity: Let $\tilde{n}_j$ (j = 1, 2, . . . , n) be a collection of TNNs. If $\tilde{n}_j \le \tilde{n}_j^*$ for j = 1, 2, . . . , n, then $TNWAA(\tilde{n}_1,\tilde{n}_2,\cdots,\tilde{n}_n)\le TNWAA(\tilde{n}_1^*,\tilde{n}_2^*,\cdots,\tilde{n}_n^*)$.

Proof:

(1) Since $\tilde{n}_j = \tilde{n}$ for j = 1, 2, . . . , n, we have

$$TNWAA(\tilde{n}_1, \tilde{n}_2, \cdots, \tilde{n}_n) = \sum_{j=1}^{n} w_j \tilde{n}_j$$
$$= \left\langle \left(\sum_{j=1}^{n} w_j a_{j1}, \sum_{j=1}^{n} w_j a_{j2}, \sum_{j=1}^{n} w_j a_{j3}, \sum_{j=1}^{n} w_j a_{j4} \right); 1 - \prod_{j=1}^{n} (1 - T_{\tilde{n}_j})^{w_j}, \prod_{j=1}^{n} I_{\tilde{n}_j}^{w_j}, \prod_{j=1}^{n} F_{\tilde{n}_j}^{w_j} \right\rangle$$
$$= \left\langle \left(a_1 \sum_{j=1}^{n} w_j, a_2 \sum_{j=1}^{n} w_j, a_3 \sum_{j=1}^{n} w_j, a_4 \sum_{j=1}^{n} w_j \right); 1 - (1 - T_{\tilde{n}})^{\sum_{j=1}^{n} w_j}, I_{\tilde{n}}^{\sum_{j=1}^{n} w_j}, F_{\tilde{n}}^{\sum_{j=1}^{n} w_j} \right\rangle$$
$$= \langle (a_1, a_1, a_1.a_1); 1 - (1 - T_{\tilde{n}}), I_{\tilde{n}}, F_{\tilde{n}} \rangle = \tilde{n}$$

(2) Since minimum TNN is $\tilde{n}^-$ and maximum TNN is $\tilde{n}^+$, there is $\tilde{n}^- \leq \tilde{n}_j \leq \tilde{n}^+$. Thus, there is $\sum_{j=1}^{n} w_j \tilde{n}^- \leq \sum_{j=1}^{n} w_j \tilde{n}_j \leq \sum_{j=1}^{n} w_j \tilde{n}^+$. According to the above property (1), there is $\tilde{n}^- \leq \sum_{j=1}^{n} w_j \tilde{n}_j \leq \tilde{n}^+$, i.e., $\tilde{n}^- \leq TNWAA(\tilde{n}_1, \tilde{n}_2, \cdots, \tilde{n}_n) \leq \tilde{n}^+$.

(3) Since $\tilde{n}_j \leq \tilde{n}_j^*$ for j = 1, 2, . . . , n, there is $\sum_{j=1}^{n} w_j \tilde{n}_j \leq \sum_{j=1}^{n} w_j \tilde{n}_j^*$, i.e., $TNWAA(\tilde{n}_1, \tilde{n}_2, \cdots, \tilde{n}_n) \leq TNWAA(\tilde{n}_1^*, \tilde{n}_2^*, \cdots, \tilde{n}_n^*)$.

Thus, we complete the proofs of these properties. □

4.2. Trapezoidal neutrosophic weighted geometric averaging operator

Definition 11. Let $\tilde{n}_j = \langle (a_{j1}, a_{j2}, a_{j3}, a_{j4}); T_{\tilde{n}_j}, I_{\tilde{n}_j}, F_{\tilde{n}_j} \rangle$ (j = 1, 2, . . . , n) be a collection of TNNs, then a TNWGA operator is defined as follows:

$$TNWGA(\tilde{n}_1, \tilde{n}_2, \cdots, \tilde{n}_n) = \prod_{j=1}^{n} \tilde{n}_j^{w_j}, \tag{17}$$

where w_j is the weight of $\tilde{n}_j$ (j = 1, 2, . . . , n) such that $w_j > 0$ and $\sum_{j=1}^{n} w_j = 1$. Specially, when $w_j = 1/n$ for j = 1, 2, . . . , n, the TNWGA operator reduces to the trapezoidal neutrosophic geometric averaging operator.

According to Definitions 7 and 11, we introduce the following theorem.

Theorem 2. Let $\tilde{n}_j = \langle (a_{j1}, a_{j2}, a_{j3}, a_{j4}); T_{\tilde{n}_j}, I_{\tilde{n}_j}, F_{\tilde{n}_j} \rangle$ (j = 1, 2, …, n) be a collection of TNNs, then according to Definitions 7 and 11, the following TNWGA operator is given by

$$TNWGA(\tilde{n}_1, \tilde{n}_2, \cdots, \tilde{n}_n) = \prod_{j=1}^{n} \tilde{n}_j^{w_j}$$
$$= \left\langle \left(\prod_{j=1}^{n} \tilde{n}_{j1}^{w_j}, \prod_{j=1}^{n} \tilde{n}_{j2}^{w_j}, \prod_{j=1}^{n} \tilde{n}_{j3}^{w_j}, \prod_{j=1}^{n} \tilde{n}_{j4}^{w_j} \right); \prod_{j=1}^{n} T_{\tilde{n}_j}^{w_j}, 1 - \prod_{j=1}^{n} (1 - I_{\tilde{n}_j})^{w_j}, 1 - \prod_{j=1}^{n} (1 - F_{\tilde{n}_j})^{w_j} \right\rangle, \tag{18}$$

where w_j is the weight of $\tilde{n}_j$ (j = 1, 2, . . . , n) such that $w_j > 0$ and $\sum_{j=1}^{n} w_j = 1$.

By a similar proof manner of Theorem 1, we can prove Theorem 2, which is not repeated here.

Obviously, the *TNWGA* operator satisfies the following properties:

(1) Idempotency: Let $\tilde{n}_j$ (j = 1, 2, . . . , n) be a collection of TNNs. If $\tilde{n}_j$ (j = 1, 2, . . . , n) is equal, i.e. $\tilde{n}_j = \tilde{n}$ for j = 1, 2, . . . , n, then $TNWGA(\tilde{n}_1, \tilde{n}_2, \cdots, \tilde{n}_n) = \tilde{n}$.

(2) Boundedness: Let $\tilde{n}_j$ (j = 1, 2, . . . , n) be a collection of TNNs and let

$$\tilde{n}^- = \left\langle (\min_j a_{j1}, \min_j a_{j2}, \min_j a_{j3}, \min_j a_{j4}); \min_j T_{\tilde{n}_j}, \max_j I_{\tilde{n}_j}, \max_j F_{\tilde{n}_j} \right\rangle,$$

$$\tilde{n}^+ = \left\langle (\max_j a_{j1}, \max_j a_{j2}, \max_j a_{j3}, \max_j a_{j4}); \max_j T_{\tilde{n}_j}, \min_j I_{\tilde{n}_j}, \min_j F_{\tilde{n}_j} \right\rangle.$$

Then $\tilde{n}^- \le TNWGA(\tilde{n}_1, \tilde{n}_2, \cdots, \tilde{n}_n) \le \tilde{n}^+$.

(3) Monotonity: Let $\tilde{n}_j$ (j = 1, 2, . . . , n) be a collection of TNNs. If $\tilde{n}_j \le \tilde{n}_j^*$ for j = 1, 2, . . . , n, then $TNWGA(\tilde{n}_1, \tilde{n}_2, \cdots, \tilde{n}_n) \le TNWGA(\tilde{n}_1^*, \tilde{n}_2^*, \cdots, \tilde{n}_n^*)$.

By a similar proof manner of the above properties, we can prove these properties (omitted).

5. Decision making method with TNNs

In this section, we apply the TNWAA and TNWGA operators and the score function to multiple attribute decision making problems under the TNN environment.

For a multiple attribute decision making problem, assume that there are a set of alternatives $A = \{A_1, A_2, \ldots, A_m\}$ based on a set of attributes $C = \{C_1, C_2, \ldots, C_n\}$. The weigh vector of the attributes is $W = (w_1, w_2, \ldots, w_n)^T$, which is given by the decision maker. Then, the decision maker can evaluate the alternatives on the attributes by the linguistic values of TNNs from the linguistic term set L = {Very poor, poor, Fairly poor, Medium, Fairly good, Good, Very good}, which are shown in Table 1. In the evaluation process, the decision maker can easily assign the linguistic values of TNNs to the attributes according to the linguistic terms, hence the evaluation information of the alternative A_i on the attributes is represented by the form of a TNN $\tilde{n}_{ij} = \langle (a_{ij1}, a_{ij2}, a_{ij3}, a_{ij4}); T_{ij}, I_{ij}, F_{ij} \rangle$ (i = 1, 2, …, m; j = 1, 2, …, n). Thus, we can establish a trapezoidal neutrosophic decision matrix $D = (\tilde{n}_{ij})_{m\times n}$.

Then, we apply the TNWAA or TNWGA operator and the score function to the multiple attribute decision-making problems with trapezoidal neutrosophic information to rank the alternatives and to select the best one. The steps of the decision-making process are described as follows:

Step 1: Utilize the TNWAA operator of Eq. (13) to obtain the collective overall number $\tilde{n}_i$ for A_i (i = 1, 2, …, m) with respect to the weight vector $\boldsymbol{W} = (w_1, w_2, \ldots, w_n)^T$ for C_j (j = 1, 2, …, n) or the TNWGA operator of Eq. (18) to obtain the collective overall value $\tilde{n}_i$ for A_i (i = 1, 2, …, m) with respect to the weight vector $\boldsymbol{W} = (w_1, w_2, \ldots, w_n)^T$ for C_j (j = 1, 2, …, n).

Step 2: Calculate the score function $S(\tilde{n}_i)$ (i = 1, 2,…, m) of the collective overall number $\tilde{n}_i$ (i = 1, 2, …, m).

Step 3: Rank the alternatives according to the score values, and then select the best alternative.

Step 4: End.

6. An illustrative example

In order to demonstrate the application of the proposed method, an example about the investment selection of a company is adapted from (Ye, 2014c). There is a company, which wants to invest a sum of money to an industry. A panel considers four alternatives: (1) A_1 is a car company; (2) A_2 is a food company; (3) A_3 is a computer company; (4) A_4 is an arms company. The evaluation on the alternatives is based on three attributes: (1) C_1 is the risk; (2) C_2 is the growth; (3) C_3 is the environmental impact. The weigh vector of the three attributes is $\boldsymbol{W} = (0.35, 0.25, 0.4)^{\mathrm{T}}$. When the four possible alternatives with respect to the above three attributes are evaluated by the expert or decision maker according to the linguistic values of TNNs for the linguistic term set in Table 1.

Table 1. Linguistic values of TNNs for the linguistic term set

Linguistic term	Linguistic value of TNNs
Very poor	⟨(0.1, 0.1, 0.1, 0.1); 0.5, 0.3, 0.3⟩
Poor	⟨(0.2, 0.3, 0.4, 0.5); 0.6, 0.2, 0.2⟩
Fairly poor	⟨(0.3, 0.4, 0.5, 0.6); 0.7, 0.1, 0.1⟩
Medium	⟨(0.4, 0.5, 0.6, 0.7); 0.8, 0.0, 0.1⟩
Fairly good	⟨(0.5, 0.6, 0.7, 0.8); 0.7, 0.3, 0.3⟩
Good	⟨(0.7, 0.8, 0.9, 1.0); 0.8, 0.2, 0.2⟩
Very good	⟨(1.0, 1.0, 1.0, 1.0); 0.9, 0.1, 0.1⟩

Thus, we can establish the following trapezoidal neutrosophic decision matrix D:

$$D = \begin{bmatrix} \langle(0.2,0.3,0.4,0.5);0.6,0.2,0.2\rangle & \langle(0.3,0.4,0.5,0.6);0.7,0.1,0.1\rangle & \langle(0.2,0.3,0.4,0.5);0.6,0.2,0.2\rangle \\ \langle(0.4,0.5,0.6,0.7);0.8,0.0,0.1\rangle & \langle(0.5,0.6,0.7,0.8);0.7,0.3,0.3\rangle & \langle(0.3,0.4,0.5,0.6);0.7,0.1,0.1\rangle \\ \langle(0.2,0.3,0.4,0.5);0.6,0.2,0.2\rangle & \langle(0.3,0.4,0.5,0.6);0.7,0.1,0.1\rangle & \langle(0.3,0.4,0.5,0.6);0.7,0.1,0.1\rangle \\ \langle(0.7,0.8,0.9,1.0);0.8,0.2,0.2\rangle & \langle(0.5,0.6,0.7,0.8);0.7,0.3,0.3\rangle & \langle(0.3,0.4,0.5,0.6);0.7,0.1,0.1\rangle \end{bmatrix}.$$

Hence, the proposed method can be applied to this decision making problem according to the following computational process:

Step 1: Utilize the TNWAA operator of Eq. (13) to obtain the collective overall value $\tilde{n}_i$ for A_i (i =1, 2, 3, 4) as follows:

$\tilde{n}_1$ = ⟨(0.2250, 0.3250, 0.4250, 0.5250); 0.6278, 0.1682, 0.1682⟩, $\tilde{n}_2$ = ⟨(0.3850, 0.4850, 0.5850, 0.6850); 0.7397, 0, 0.1316⟩, $\tilde{n}_3$ = ⟨(0.2650, 0.3650, 0.4650, 0.5650); 0.6682, 0.1275, 0.1275⟩, and $\tilde{n}_4$ = ⟨(0.4900, 0.5900, 0.6900, 0.7900); 0.7397, 0.1677, 0.1677⟩.

Step 2: Calculate the score values of $S(\tilde{n}_i)$ (i =1, 2, 3, 4) of the collective overall value $\tilde{n}_i$ (i =1, 2, 3, 4) by Eq. (11), we can obtain:

$S(\tilde{n}_1) = 0.2864$, $S(\tilde{n}_2) = 0.4651$, $S(\tilde{n}_3) = 0.3338$, and $S(\tilde{n}_4) = 0.5129$.

Step 3: Ranking order of the four alternatives is $A_4 \succ A_2 \succ A_3 \succ A_1$ according to the score values. Thus, the alternative A_4 is the best choice among the four alternatives.

On the other hand, we can also utilize the TNWGA operator to give the following computational procedure:

Step 1': By utilizing the TNWAA operator of Eq. (18) for A_i (i = 1, 2, 3, 4), then each collective overall value $\tilde{n}_i$ (i =1, 2, 3, 4) is obtained as follows:

$\tilde{n}_1 = \langle(0.2213, 0.3224, 0.4229, 0.5233); 0.6236, 0.1761, 0.1761\rangle$, $\tilde{n}_2 = \langle(0.3770, 0.4786, 0.5797, 0.6805); 0.7335, 0.1231, 0.1548\rangle$, $\tilde{n}_3 = \langle(0.2603, 0.3617, 0.4624, 0.5629); 0.6632, 0.1363, 0.1363\rangle$, and $\tilde{n}_4 = \langle(0.4585, 0.5642, 0.6681, 0.7710); 0.7335, 0.1889, 0.1889\rangle$.

Step 2': By using Eq. (11), we calculate the score values of $S(\tilde{n}_i)$ (i =1, 2, 3, 4) of the collective overall value $\tilde{n}_i$ (i =1, 2, 3, 4) as follows:

$S(\tilde{n}_1) = 0.2820$, $S(\tilde{n}_2) = 0.4330$, $S(\tilde{n}_3) = 0.3282$, and $S(\tilde{n}_4) = 0.4833$.

Step 3': Hence, the ranking order of the four alternatives is $A_4 \succ A_2 \succ A_3 \succ A_1$. Thus, the alternative A_4 is still the best choice among the four alternatives.

Obviously, above two kinds of ranking orders and the best alternative are the same, which are in agreement with Ye's results (Ye, 2014c).

Compared with the relevant paper (Wang and Zhang, 2009) which proposed the intuitionistic trapezoidal fuzzy decision-making approach, the decision information used in (Wang and Zhang, 2009) is ITFNs, whereas the decision information in this paper is TNNs. As mentioned above, the TNN is a further generalization of the ITFN. So the decision-making method proposed in this paper is more typical and more general in applications since the decision-making method proposed in (Wang and Zhang, 2009) is a special case of the decision-making method proposed in this paper. Furthermore, compared with the relevant papers (Wang and Zhang, 2009; Ye, 2013; Chi and Liu, 2013; Ye, 2014a, 2014b, 2014c, 2014d; Liu et al, 2014; Zhang et al, 2014; Biswas et al, 2014; Zhang and Wu, 2014), the decision-making approach proposed in this paper can be used to solve decision-making problems with triangular and trapezoidal neutrosophic information, whereas the decision-making methods in (Wang and Zhang, 2009; Ye, 2013; Chi and Liu, 2013; Ye, 2014a, 2014b, 2014c, 2014d; Liu et al, 2014; Zhang et al, 2014; Biswas et al, 2014; Zhang and Wu, 2014) are not suitable for the decision-making problems in this paper. Therefore, the method proposed in the paper is a generalization of existing methods since existing methods cannot represent and handle TNN information in decision making.

7. Conclusion

This paper proposed neutrosophic numbers and TNNs and the operational relations of TNNs as the extension of IFNs and ITFNs and the score function of TNN. Then we developed the TNWAA and TNWGA operators to aggregate TNNs and investigated their properties. Further, we established a decision making method based on the TNWAA or TNWGA operator and the score function to solve multiple attribute decision-making problems with TNN information. Finally, an illustrative example was given to show the application of the developed decision making method. An efficient method is provided to solve fuzzy multiple attribute decision making problems based on TNNs. In the future research, it is necessary to investigate the applications of these aggregation operators to the other domains such as pattern recognition and medical diagnosis.

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