Neutrosophic Sets and Systems

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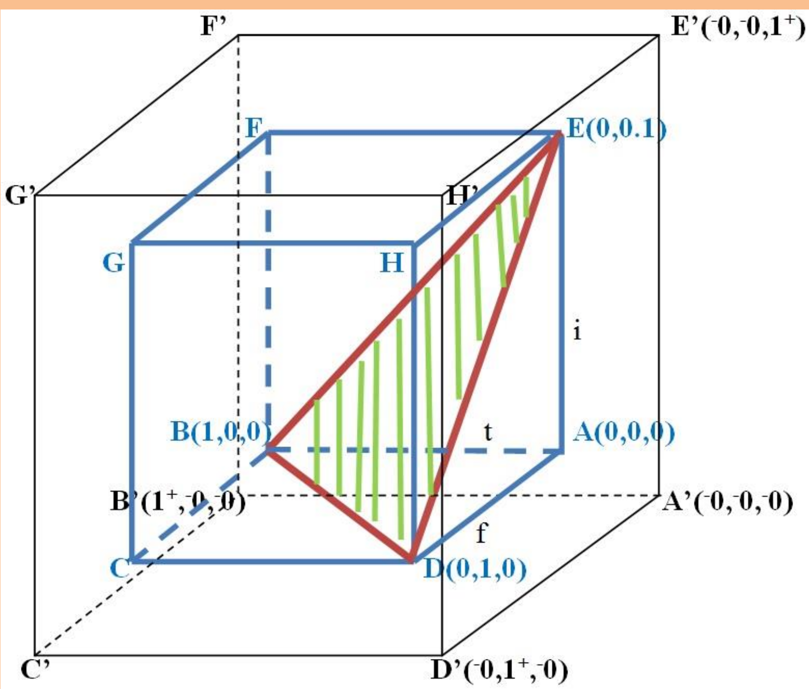

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Neutrosophic Sets and Systems

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Information for Authors and Subscribers

"Neutrosophic Sets and Systems" has been created for publications on advanced studies in neutrosophy, neutrosophic set, neutrosophic logic, neutrosophic probability, neutrosophic statistics that started in 1995 and their applications in any field, such as the neutrosophic structures developed in algebra, geometry, topology, etc.

The submitted papers should be professional, in good English, containing a brief review of a problem and obtained results.

Neutrosophy is a new branch of philosophy that studies the origin, nature, and scope of neutralities, as well as their interactions with different ideational spectra.

This theory considers every notion or idea <A> together with its opposite or negation <antiA> and with their spectrum of neutralities <neutA> in between them (i.e. notions or ideas supporting neither <A> nor <antiA>). The <neutA> and <antiA> ideas together are referred to as <nonA>.

Neutrosophy is a generalization of Hegel's dialectics (the last one is based on <A> and <antiA> only).

According to this theory every idea <A> tends to be neutralized and balanced by <antiA> and <nonA> ideas - as a state of equilibrium.

In a classical way <A>, <neutA>, <antiA> are disjoint two by two. But, since in many cases the borders between notions are vague, imprecise, Sorites, it is possible that <A>, <neutA>, <antiA> (and <nonA> of course) have common parts two by two, or even all three of them as well.

Neutrosophic Set and *Neutrosophic Logic* are generalizations of the fuzzy set and respectively fuzzy logic (especially of intui-tionistic fuzzy set and respectively intuitionistic fuzzy logic). In neutrosophic logic a proposition has a degree of truth (*T*), a de-gree of indeterminacy (*I*), and a degree of falsity (*F*), where *T, I, F* are standard or non-standard subsets of $]^-0, 1^+[$.

Neutrosophic Probability is a generalization of the classical probability and imprecise probability.

Neutrosophic Statistics is a generalization of the classical statistics.

What distinguishes the neutrosophics from other fields is the <neutA>, which means neither <A> nor <antiA>.

<neutA>, which of course depends on <A>, can be indeterminacy, neutrality, tie game, unknown, contradiction, ignorance, imprecision, etc.

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Pauli Exclusion Principle and the Law of Included Multiple-Middle

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Abstract: It has been found that bosons are not subject to the Pauli Exclusion Principle. This paper argues that in some cases the exclusion principle is also invalid for fermions. According to the Law of Included Multiple-Middle and the like, the 4 neutralities between Pauli Exclusion Principle's validity and invalidity are as follows: first, according to Neutrosophy, any proposition has three situations of truth, falsehood and indeterminacy respectively; second, some scholars have pointed out that the exclusion principle may be broken in high-energy state; third, due to the existance of man created law (man-made law), the broken exclusion principle and the man-made (instantaneous) magnetic monopole can be artificially created; fourth, the exclusion principle is not compatible with law of conservation of energy, and in physics the principles that are not compatible with law of conservation of energy will be invalid in some cases.

Keywords: Neutrosophy, Law of Included Multiple-Middle, exclusion principle, error, law of conservation of energy, man created law (man-made law), man-made (instantaneous) magnetic monopole

1 Introduction

As well-known, it has been found that bosons are not subject to the Pauli Exclusion Principle. Then there is the question: whether or not that in some cases the exclusion principle is also invalid for fermions? This paper tries to discuss this issue from four aspects based on Law of Included Multiple-Middle and the like.

According to the Law of Included Multiple-Middle presented in reference [1], for the notion or idea <Neut-A> (its meaning can be found below), it can be split into a multitude of neutralities between <A> and <Anti-A>, such as <neut1A>, <neut2A>, and the like. The <Neut-A> value (i.e. neutrality or indeterminacy related to <A> and <Anti-A>) actually comprises the included middle value. For example, for the Pauli Exclusion Principle, between it is completely valid and it is completely invalid, there are four neutralities or aspects that the Pauli Exclusion Principle is only valid under certain conditions.

Now we will explicit the 4 neutralities between Pauli Exclusion Principle's validity and invalidity in sections 2-5.

2 According to Neutrosophy, any proposition has three situations of truth, falsehood and indeterminacy respectively

Neutrosophy is a new branch of philosophy that studies the origin, nature, and scope of neutralities, as well as their interactions with different ideational spectra.

This theory considers every notion or idea <A> together with its opposite or negation <Anti-A> and the spectrum of "neutralities" <Neut-A> (i.e. notions or ideas located between the two extremes, supporting neither <A> nor <Anti-A>). The <Neut-A> and <Anti-A> ideas together are referred to as <Non-A>.

Neutrosophy is the base of neutrosophic logic, neutrosophic set, neutrosophic probability and statistics used in engineering applications (especially for software and information fusion), medicine, military, cybernetics, and physics.

Neutrosophic Logic is a general framework for unification of many existing logics, such as fuzzy logic (especially intuitionistic fuzzy logic), paraconsistent logic, intuitionistic logic, etc. The main idea of NL is to characterize each logical statement in a 3D Neutrosophic Space, where each dimension of the space represents respectively the truth (T), the falsehood (F), and the indeterminacy (I) of the statement under consideration, where T, I, F are standard or non-standard real subsets of $]^-0, 1^+[$ without necessarily connection between them.

More information about Neutrosophy may be found in references [2,3].

Because the exclusion principle is invalid for bosons, the viewpoint of Neutrosophy that "any proposition is falsehood in some cases" has been vindicated.

Similarly, according to the viewpoint of Neutrosophy, the exclusion principle also should have three situations of truth, falsehood and indeterminacy respectively for fermions.

3 Some scholars have pointed out that the exclusion principle may be broken in high-energy state

It is well known that some scholars have doubted the validity of exclusion principle.

For example, in reference [4], it presents that for high-energy celestial bodies such as neutron stars and the like, the broken Pauli exclusion principle will be observed; and points out that the exclusion principle may be broken in high-energy state.

4 The broken exclusion principle and the man-made (instantaneous) magnetic monopole can be artificially created

The conventional viewpoint considers that man cannot create law. This is a one-sided viewpoint. In some cases, man can create law, including change the rule into law. So the laws can be divided into at least three kinds: the objective law, the man created subjective law, as well as the synthetic law formed by the above mentioned two kinds of laws.

Now we discuss various man created laws (man-made laws).

In the social science: (1)in stock market the banker created the law of stock, (2)for various goods,the whole-sale price calculation formula is decided by the owner, (3) the laws of Chinese new year firecrackers and the Mid-Autumn Festival cake.

In the natural science: (1)the law of gravity and the theory of general relativity were created by Newton and Einstein respectively, (2)some geometries built from a set of axioms, (3)various carry-systems in mathematics, (4)the operation of fountain with man created law, (5)the temperature law of the greenhouse.

In thinking science: one divides into two or one divides into three (such as the three worlds) and one divides into five (such as the five elements in Chinese ancient times), and the different laws to learn the knowledge such as the sequence of easy-difficult or difficult-easy.

In the virtual world (the laws don't need to be tested by practice): (1)in science fiction the Hubble constant can be given arbitrarily as well as the speed of airship can reach ten thousand times of the speed of light, (2)in the ancient Chinese novel “The Pilgrimage to the West”, Tang Monk’s law to punish the Monkey King, (3)in artistic works the law of the hero and the beauty, (4)the law to steal vegetables from the online game.

Finally the optimum synthetic law formed by subjective law and objective law, such as Earth's best seasonal variation, can be created by people.

In physics, the man-made laws have not been paid enough attention. However, some scholars have presented some issues connected with man-made laws. For example, some scholars say that "magnetic monopole" can exist. "magnetic monopole can exist" is a man-made law, because in nature "magnetic monopole" does not exist.

Now, we give an artificial method to create "man-made (instantaneous) magnetic monopole".

Suppose there is a long uniform rectangular-shaped magnet, along its middle section (the demarcation section of N-pole and S-pole) to cut it at very high speed, as the disconnected instant moment, one half of the magnet is the pure N-pole, and the other half is the pure S-pole.

Due to the existence of man-made laws, especially the "man-made (instantaneous) magnetic monopole" can be created as above mentioned, we can say that the broken exclusion principle can be artificially created for fermions.

5 The exclusion principle is not compatible with law of conservation of energy, and in physics the principles that are not compatible with law of conservation of energy will be invalid in some cases

Firstly the exclusion principle can be written as a symmetry form.

In order to connect the exclusion principle with a conserved quantity, supposing "1" (or any other constant) denote “valid”, and "does not equal 1" denote “invalid”, in this way the exclusion principle (denoted as P) can be written as the following form of conserved quantity

$$P=1$$

According to Noether's theorem, each continuous symmetry of a physical system implies that some physical property of that system is conserved. Conversely, each conserved quantity has a corresponding symmetry.

In reference [5] we already point out that for any symmetry, we can find the example of violation of symmetry or broken symmetry. As a kind of symmetry, the exclusion principle (P=1) cannot make an exception. As for the reason, in reference [5] we point out: there is no strict symmetry in nature. For example, the symmetry for law of conservation of energy cannot be the exception.

The prerequisite of law of conservation of energy is the existance of a closed system, but the strictly closed system does not exist, there are only approximately closed systems. Therefore, the symmetry for law of conservation is only approximately correct.

Although the symmetry for law of conservation of energy is only approximately correct, theoretically it could be considered as the unique symmetry in physics that is strictly correct. For other symmetries, they are correct only in the cases that they are not contradicted with this unique symmetry or they can be derived by this unique symmetry.

In reference [6], the examples deriving the improved Newton's second law and improved law of gravity according to law of conservation of energy are discussed. Namely deriving the symmetry for improved Newton's second law and symmetry for improved law of gravity according to the symmetry for law of conservation of energy.

In reference [6] we also point out: besides law of conservation of energy, all other laws of conservation in physics may not be correct (or their probabilities of correctness are all less than 100%). In reference [6] we also discuss the examples that law of conservation of momentum and law of conservation of angular momentum are not correct (their results are contradicted with law of conservation of energy).

The essential reason for the exclusion principle may be invalid is that it does not take into account the law of conservation of energy, and in physics the principles that are not compatible with law of conservation of energy will be invalid in some cases

6 Conclusions

According to Neutrosophy, any proposition has three situations of truth, falsehood and indeterminacy respectively. We already explicit the 4 neutralities between Pauli Exclusion Principle's validity and invalidity. For the reason that the exclusion principle may be invalid for fermions, we can reach the following conclusions: In physics, the law of conservation of energy is the unique truth; for other principles, laws and the like, as they are established, the law of conservation of energy should be considered, otherwise they may be invalid in some cases; for many existing principles, laws and the like that do not consider the law of conservation of energy, we should renewly consider their relationship with the law of conservation of energy, in order to determine their fate or discuss the problems to modify them.

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NSS

Weighted Neutrosophic Soft Sets

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Abstract. In this paper we study the concept of neutrosophic soft sets. Imposing some weights on the parameters considered we introduce here weighted neutrosophic soft sets. Some operations like union, intersection, complement, AND, OR etc. have been defined on this new concept. Some properties of these newly defined operations have also been verified .

Keywords: Soft sets, neutrosophic sets, neutrosophic soft sets, weighted neutrosophic soft sets.

1 Introduction

The soft set theory initiated by Molodtsov [1] has been proved as a generic mathematical tool to deal with problems involving uncertainties or imprecise data. So called traditional tools such as fuzzy sets [2], rough sets [3], vague sets [4], probability theory etc. can not be used successfully because of inadequecy present in the parametrization of the tools. Consequently, Molodtsov has shown that soft set theory has a potential to use in variety of many fields [1]. After its initiation a detailed theoretical construction has been introduced by Maji et al in [5] . Works on soft set theory is growing very rapidly with all its potentiality and is being used in different fields [6 – 11, 17,19]. In case of soft set the parametrization is done with the help of words, sentences, functions etc.. For different characteristics of the parameters present in soft set theory different hybridization viz. fuzzy soft sets [12], soft rough sets [13], intuitionistic fuzzy soft sets [14], vague soft sets [15], neutrosophic soft sets [16] etc. have been introduced. In [16] the parameters considered are neutrosophic in nature. Imposing the weights on the parameters (may be in a particular parameter also) we have introduced weighted neutrosophic soft sets in this paper. In section 2 of this paper we have a relevant recapitulation of some preliminaries for better understanding of the paper. In section 3 after defining weighted neutrosophic soft set we have defined some operations like union, intersection, AND, OR etc.. Some properties of these operations have also been verified in this section. Conclusions are there in the concluding section 4.

2 Preliminaries

In this section we recall some relevant definitions.

Definition 2.1 [18] A neutrosophic set A on the universe of discourse X is defined as $A = \{< x, T_A(x), I_A(x), F_A(x) >, x \in X\}$, where $T, I, F : X \to]^-0, 1^+[$ and $^-0 \le T_A(x) + I_A(x) + F_A(x) \le 3^+$.

From philosophical point of view, the neutrosophic set takes the value from real standard or non-standard subsets of $]^-0, 1^+[$. But in real life application in scientific and engineering problems it is difficult to use neutrosophic set with value from real standard or non-standard subset of $]^-0, 1^+[$. Hence we consider the neutrosophic set which takes the value from the subset of [0, 1].

Definition 2.2 [18] A neutrosophic set A is contained in another neutrosophic set B i.e. $A \subseteq B$ if $\forall x \in X$, $T_A(x) \le T_B(x)$, $I_A(x) \le I_B(x)$, $F_A(x) \ge F_B(x)$.

Definition 2.3 [16] Let U be an initial universe set and E be a set of parameters.

Consider $A \subset E$. Let P(U) denotes the set of all neutrosophic sets of U.

The collection (F, A) is termed to be the neutrosophic soft set over U, where F is a mapping given by $F : A \to P(U)$.

Definition 2.4 [16] Let (F, A) and (G, B) be two neutrosophic soft sets over the common universe U. (F, A) is said to be neutrosophic soft subset of (G, B) if $A \subset B$, and $T_{F(e)}(x) \le T_{G(e)}(x)$, $I_{F(e)}(x) \le I_{G(e)}(x)$, $F_{F(e)}(x) \ge T_{G(e)}(x)$,

$\forall e \in A$, $x \in U$.
We denote it by $(F, A) \subseteq (G, B)$.
(F, A) is said to be neutrosophic soft super set of (G, B) if (G, B) is a neutrosophic soft subset of (F, A). We denote it by $(F, A) \supseteq (G, B)$.

Definition 2.5 [16] Equality of two neutrosophic soft sets.

Two NSSs (F, A) and (G, B) over the common universe U are said to be equal if (F, A) is neutrosophic soft subset of (G, B) and (G, B) is neutrosophic soft subset of (F, A). We denote it by (F, A) = (G, B).

Definition 2.6 [16] NOT set of a set of parameters.

Let $E = \{e_1, e_2 , \cdots , e_n \}$ be a set of parameters. The NOT set of E is denoted by ⌉E is defined by ⌉E = {⌉e_1, ⌉e_2, · · ·, ⌉e_n }, where ⌉e_i = not e_i ,∀i (it may be noted that ⌉and ⌉ are different operators).

Definition 2.7 [16] Complement of a neutrosophic soft set.

The complement of a neutrosophic soft set (F, A) denoted by $(F, A)^c$ and is defined as $(F, A)^c = (F^c$, ⌉A), where F^c: ⌉A → P(U) is a mapping given by $F^c(\alpha)$ = neutrosophic soft complement with $T_{F^c(e)}(x) = F_{F(e)}(x)$, $I_{F^c(e)}(x) = I_{F(e)}(x)$ and $F_{F^c(e)}(x) = T_{F(e)}(x)$, ∀x ∈ U and ∀e ∈ ⌉A.

Definition 2.8 [16] Union of two neutrosophic soft sets.

Let (H, A) and (G, B) be two NSSs over the common universe U. Then the union of (H, A) and (G, B) is denoted by '(H, A) ∪ (G, B)' and is defined by (H, A) ∪ (G, B) = (K, C), where C = A ∪ B and the truth-membership, indeterminacy-membership and falsity-membership of (K, C) are as follows:

$T_{K(e)}(x) = T_{H(e)}(x)$, if e ∈ A − B,

$= T_{G(e)}(x)$, if e ∈ B − A,
$= \max (T_{H(e)}(x), T_{G(e)}(x))$, if e ∈ A ∩ B.

$I_{K(e)}(x) = I_{H(e)}(x)$, if e ∈ A − B,

$= I_{G(e)}(x)$, if e ∈ B − A,
$= (I_{H(e)}(x) + I_{G(e)}(x))/2$, if e ∈ A ∩ B.

$F_{K(e)}(x) = F_{H(e)}(x)$, if e ∈ A − B,

$= F_{G(e)}(x)$, if e ∈ B − A,
$= \min (F_{H(e)}(x), F_{G(e)}(x))$, if e ∈ A ∩ B.

Definition 2.9 [16] Intersection of two neutrosophic soft sets.

Let (H, A) and (G, B) be two NSSs over the same universe U. Then the intersection of (H, A) and (G, B) is denoted by '(H, A) ∩ (G, B)' and is defined by (H, A) ∩ (G, B) = (K, C), where C = A ∩ B and the truth-membership, indeterminacy- membership and falsity-membership of (K, C) are as follows:
$T_{K(e)}(x) = \min (T_{H(e)}(x), T_{G(e)}(x))$, if e ∈ A ∩ B.
$I_{K(e)}(x) = (I_{H(e)}(x) + I_{G(e)}(x))/2$, if e ∈ A ∩ B.
$F_{K(e)}(x) = \max (F_{H(e)}(x), F_{G(e)}(x))$, if e ∈ A ∩ B.
Now we are in the position to define weighted neutrosophic soft sets.

3 Weighted Neutrosophic Soft Sets

Definition 3.1 A neutrosophic soft set is termed to be a weighted neutrosophic soft sets if a weight (w_i, a real positive number ≤ 1) be imposed on the parameter of it. The ijth entries of the weighted neutrosophic soft set,

$d_{ij} = w_{ij} \times c_{ij}$ where c_{ij} is the ij-th entry in the table of neutrosophic soft set.

The weighted neutrosophic soft sets (WNSS) for the neutrosophic soft sets (NSS) (F, A) with weights w associated with the parameter A is denoted by (F, A^w).

Example 3.1 For illustration we consider the **example** in [16]. Let U be the set of houses under consideration and E is the set of parameters which consist of neutrosophic words or phases with neutrosophic words. Consider E = { beautiful, wooden, costly, very costly, moderate, green surroundings, in good repair, in bad repair, cheap, expensive }. Suppose that, there are five houses in the universe U given by, U = { h_1, h_2, h_3, h_4, h_5 } and the set of parameters A = {e_1, e_2, e_3, e_4 }, where e_1 stands for the parameter 'beautiful', e_2 stands for the parameter 'wooden', e_3 stands for the parameter 'costly' and the parameter e_4 stands for 'moderate'. Suppose that,
F(beautiful) = {< h_1, 0.5, 0.6, 0.3 >, < h_2 , 0.4, 0.7, 0.6 >, < h_3 , 0.6, 0.2, 0.3 >, < h_4 , 0.7, 0.3, 0.2 >, < h_5 , 0.8, 0.2, 0.3 >},
F(wooden) = {< h_1 , 0.6, 0.3, 0.5 >, < h_2, 0.7, 0.4, 0.3 >, < h_3, 0.8, 0.1, 0.2 >, < h_4 , 0.7, 0.1, 0.3 >, < h_5 , 0.8, 0.3, 0.6 >},
F(costly) = {< h_1, 0.7, 0.4, 0.3 >, < h_2, 0.6, 0.7, 0.2 >, < h_3, 0.7, 0.2, 0.5 >, < h_4, 0.5, 0.2, 0.6 >, < h_5, 0.7, 0.3, 0.4 >},
F(moderate) = {< h_1, 0.8, 0.6, 0.4 >, < h_2 , 0.7, 0.9, 0.6 >, < h_3 , 0.7, 0.6, 0.4 >, < h_4 , 0.7, 0.8, 0.6 >, < h_5 , 0.9, 0.5, 0.7 >}.
Then the neutrosophic soft set (F, A) describing the attractiveness of the houses given in the following tabular form.

U	beautiful	wooden	costly	moderate
h_1	(0.5, 0.6, 0.3)	(0.6, 0.3, 0.5)	(0.7,0.4, 0.3)	(0.8,0.6, 0.4)
h_2	(0.4, 0.7, 0.6)	(0.7, 0.4, 0.3)	(0.6,0.7, 0.2)	(0.7,0.9, 0.6)
h_3	(0.6, 0.2, 0.3)	(0.8, 0.1, 0.2)	(0.7,0.2, 0.5)	(0.7,0.6, 0.4)
h_4	(0.7, 0.3, 0.2)	(0.7, 0.1, 0.3)	(0.5,0.2, 0.6)	(0.7,0.8, 0.6)
h_5	(0.8, 0.2, 0.3)	(0.8, 0.3, 0.6)	(0.7,0.3, 0.4)	(0.9,0.5, 0.7)

Table 1: The Neutrosophic Soft Sets (F, A).

Imposing the weights w_1 = 0.3, w_2 = 0.6, w_3= 0.4, w_4= 0.7 respectively for the parameters 'beautiful', 'wooden', 'costly' and 'moderate' the weighted neutrosophic soft sets (WNSS) corresponding to the neutrosophic soft sets (F, A) denoted by (F, A^w) and is given in the following tabular form:

U	beautiful, w_1 = 0.3	wooden, w_2 = 0.6,	costly, w_3= 0.4,	moderate, w_4= 0.7
h_1	(0.15, 0.18,0.09)	(0.36, 0.18,0.30)	(0.28,0.16, 0.12)	(0.56,0.42, 0.28)

h_2	(0.12, 0.21,0.18)	(0.42, 0.24, 0.18)	(0.24,0.28, 0.08)	(0.49,0.63, 0.42)
h_3	(0.18, 0.06,0.18)	(0.48, 0.06,0.12)	(0.28,0.08, 0.20)	(0.49,0.42, 0.28)
h_4	(0.21, 0.09,0.06)	(0.42, 0.06,0.18)	(0.20,0.08, 0.24)	(0.49,0.56, 0.42)
h_5	(0.24, 0.06,0.09)	(0.48, 0.18,0.36)	(0.28,0.12, 0.16)	(0.63,0.35, 0.49)

Table 2: The Weighted Neutrosophic Soft Sets (F, A^w).

Definition 3.2 Subset of weighted NSS

Let (F, A^w) and (G, B^w) be two weighted neutrosophic soft sets over the common universe U. (F, A^w) is said to be weighted neutrosophic soft subset of (G, B^w) if A ⊂ B, and $T_{F(e)}(x) \leq T_{G(e)}(x)$, $I_{F(e)}(x) \leq I_{G(e)}(x)$, $F_{F(e)}(x) \geq T_{G(e)}(x)$, ∀e ∈ A, x ∈ U.

We denote it by (F, A^w) ⊆ (G, B^w).

(F, A^w) is said to be neutrosophic soft super set of (G, B^w) if (G, B^w) is a neutrosophic soft subset of (F, A^w). We denote it by (F, A) ⊇ (G, B). It is to be noted that the weights w for A and B may not be same.

Definition 3.3 Equality of two weighted neutrosophic soft sets.

Two WNSSs (F, A^w) and (G, B^w) over the common universe U are said to be equal if (F, A^w) is neutrosophic soft subset of (G, B^w) and (G, B^w) is neutrosophic soft subset of (F, A^w). We denote it by (F, A^w) = (G, B^w).

Definition 3.4 NOT set of a set of parameters.

Let E = {e_1, e_2 , · · · , e_n } be a set of parameters. The NOT set of E is denoted by $\rceil$E is defined by E = {$\rceil e_1$, $\rceil e_2$, · · · , $\rceil e_n$ }, where $\rceil e_i$ = not e_i ,∀i (it may be noted that $\rceil$ and $\rceil$ are different operators).

Definition 3.5 Complement of a weighted neutrosophic soft set.

The complement of a weighted neutrosophic soft set (F, A^w) denoted by $(F, A^w)^c$ and is defined as $(F, A^w)^c = (F^c, A^w)$, where $F^c : \rceil A^{\omega} \rightarrow P(U)$ is a mapping given by $F^c(e)$ = neutrosophic soft complement with $T_{F^c(e^w)}(x) = F_{F(e^w)}(x)$, $I_{F^c(e^w)}(x) = I_{F(e^w)}(x)$ and $F_{F^c(e^w)}(x)=T_{F(e^w)}(x)$.

Example 3.2 Consider the WNSS (F, A^w) as in **example 3.1** above.

The tabular representation of the complement of $(F, A^w)^c$ is as below:

U	not beautiful, $w_1 = 0.3$	not wooden, $w_2 = 0.6$,	not costly, $w_3 = 0.4$,	not moderate, $w_4 = 0.7$
h_1	(0.09, 0.18,0.15)	(0.30,0.18, 0.36)	(0.12,0.16, 0.28)	(0.28,0.42, 0.56)
h_2	(0.18, 0.21,0.12)	(0.18, 0.24, 0.42)	(0.08,0.28, 0.24)	(0.42,0.63, 0.49)
h_3	(0.18, 0.06,0.18)	(0.12 0.06, 0.48)	(0.20,0.08, 0.28)	(0.28,0.42, 0.49)
h_4	(0.06,0.09,0.21)	(0.18,0.06, 0.42)	(0.24,0.08, 0.20)	(0.42,0.56, 0.49)
h_5	(0.09, 0.06, 0.24)	(0.36,0.18, 0.48)	(0.16,0.12, 0.28)	(0.49,0.35, 0.63)

Table 3: The Weighted Neutrosophic Soft Sets $(F, A^w)^c$.

Definition 3.6 Empty or Null neutrosophic soft set with respect to a parameter.

A weighted neutrosophic soft set (H, A^w) over the universe U is termed to be empty or weighted null neutrosophic soft set with respect to the parameter A if $T_{H(e^w)}(x) = 0$, $I_{H(e^w)}(x) = 0$ and $F_{H(e^w)}(x) = 0$, ∀x ∈ U, ∀e ∈ A.

In this case the weighted null neutrosophic soft set (WNNSS) is denoted by $\Phi_A{}^w$.

Example 3.3 Let U = { h_1, h_2, h_3, h_4, h_5 } the set of five houses be considered as the universal set and A = { beautiful, wooden, in the green surroundings } be the set of parameters that characterizes the houses. Consider the neutrosophic soft set (H, A^w) which describes the attractiveness of the houses and

H(beautiful, w_1= 0.4)={< h_1, 0,0,0 >, < h_2, 0,0,0 >, < h_3, 0,0,0 >, < h_4, 0,0,0 >, <h_5, 0,0,0> },

H(wooden, w_2= 0.8)={< h_1, 0,0,0 >, < h_2, 0,0,0 >, < h_3, 0,0,0 >, < h_4, 0,0,0 >, <h_5, 0,0,0> },

H(in the green surroundings, w_3= 0.6)={< h_1, 0,0,0 >, <h_2, 0,0,0 >,< h_3, 0,0,0 >, <h_4, 0,0,0 >, <h_5, 0,0,0> }.

Here the (H, A^w) is the weighted null neutrosophic soft set.

Definition 3.7 Union of two weighted neutrosophic soft sets.

Let (F, A^w) and (G, B^w) be two WNSSs over the common universe U. Then the union of (F, A^w) and (G, B^w) is denoted by '(F, A^w) ⊔ (G, B^w)' and is defined by (F, A^w) ⊔ (G, B^w) = (K, C^w), where C = A ∪ B and the truth-membership, indeterminacy-membership and falsity-membership of (K, C^w) are as follows:

$T_{K(e^w)}(x) = T_{F(e^w)}(x)$, if e ∈ A − B,
$= T_{G(e^w)}(x)$, if e ∈ B − A,
$= \max.(w_1,w_2).\max.(T_{F(e^w)}(x),T_{G(e^w)}(x))$, if e ∈ A∩B,

$I_{K(e^w)}(x) = I_{F(e^w)}(x)$, if e ∈ A− B,
$= I_{G(e^w)}(x)$, if e ∈ B − A,
$= (I_{F(e^w)}(x) + I_{G(e^w)}(x))/2$, if e ∈ A∩B,

$F_{K(e^w)}(x) = F_{F(e^w)}(x)$, if e ∈ A − B,
$= F_{G(e^w)}(x)$, if e ∈ B − A,
$= \min.(w_1,w_2).\min.(F_{F(e^w)}(x),F_{G(e^w)}(x))$, if e ∈ A∩B,

Example 3.4 Let (F, A^w) and (G, B^w) be two WNSSs over the common universe U = { h_1, h_2, h_3, h_4, h_5 } and their tabular representations are given below:

	U	beautiful	wooden	moderate
(F, A)	h_1	(0.6.,0.3,0.7)	(0.7,0.3,0.5)	(0.6,0.4,0.5)
	h_2	(0.5,0.4,0.5)	(0.6,0.7,0.3)	(0.6,0.5,0.4)
	h_3	(o.7,0.4,0.3)	(0.7,0.3,0.5)	(0.7,0.4,0.5)
	h_4	(0.8,0.4,0.7)	(0.6,0.3,0.6)	(0.7,0.5,0.6)
	h_5	(0.6,0.7,0.2)	(0.7,0.3,0.4)	(0.8,0.6,0.5)
weight		**w_1 = 0.4**	**w_2 = 0.3**	**w_3 = 0.6**
(F, A^w)	h_1	(0.24,0.12,0.28)	(0.21,0.09,0.15)	(0.36,0.24,0.30)
	h_2	(0.20,0.16,0.20)	(0.18,0.21,0.09)	(0.36,0.30,0.24)
	h_3	(0.28,0.16,0.12)	(0.21,0.09,0.15)	(0.42,0.24,0.30)
	h_4	(0.32,0.16,0.28)	(0.18,0.09,0.18)	(0.42,0.30,0.36)
	h_5	(0.24,0.28,0.08)	(0.21,0.09,0.12)	(0.48,0.36,0.30)

Table 4: The Weighted Neutrosophic Soft Sets (F, A^w).

	U	costly	moderate
(G, B)	h_1	(0.7,0. 6.,0.6)	(0.7,0.8,0.6)
	h_2	(0.8,0.4,0.5)	(0.8,0.8,0.3)
	h_3	(0.7,0.4,0.6)	(0.5,0.6,0.7)
	h_4	(0.6,0.3,0.5)	(0.8,0.5,0.6)
	h_5	(0.8,0.5,0.4)	(0.6,0.3,0.5)
weight		**w_1 = 0.3**	**w_3 = 0.4**
(G, B^w)	h_1	(0.21,0.18,0.18)	(0.28,0.32,0.24)
	h_2	(0.24,0.12,0.15)	(0.32,0.32,0.12)
	h_3	(0.21,0.12,0.18)	(0.20,0.24,0.28)
	h_4	(0.18,0.09,0.15)	(0.32,0.20,0.24)
	h_5	(0.24,0.15,0.12)	(0.24,0.12,0.20)

Table 5: The Weighted Neutrosophic Soft Sets (G, B^w).

Then the tabular representation of their union (K, C^w) = (F, A^w) $\sqcup$ (G, B^w) is as below:

U	beautiful	wooden	costly	moderate
h_1	(0.24,0.12,0.28)	(0.21,0.09,0.15)	(0.21,0.18,0.18)	(0.42,0.28,0.20)
h_2	(0.20,0.16,0.20)	(0.18,0.21,0.09)	(0.24,0.12,0.15)	(0.48,0.31,0.12)
h_3	(0.28,0.16,0.12)	(0.21,0.09,0.15)	(0.21,0.12,0.18)	(0.42,0.24,0.20)
h_4	(0.32,0.16,0.28)	(0.18,0.09,0.18)	(0.18,0.09,0.15)	(0.36,0.25,0.24)
h_5	(0.24,0.28,0.08)	(0.21,0.09,0.12)	(0.24,0.15,0.12)	(0.48,0.24,0.20)

Table 6: The Weighted Neutrosophic Soft Sets (K, C^w).

Definition 3.8 Intersection of two weighted neutrosophic soft sets.
Let (F, A^w) and (G, B^w) be two WNSSs over the common universe U. Then the intersection of (F, A^w) and (G, B^w) is denoted by '(F, A^w) $\sqcap$ (G, B^w)' and is defined by (F, A^w) $\sqcap$ (G, B^w) = (K, C^w), where C = A ∪ B and the truth-membership, indeterminacy-membership and falsity-membership of (K, C^w) are as follows:

$T_{K(e^w)}(x) = T_{F(e^w)}(x)$, if $e \in A - B$,
$= T_{G(e^w)}(x)$, if $e \in B - A$,
$= \min. (w_1,w_2). \min. (T_{F(e^w)}(x),T_{G(e^w)}(x))$, if $e \in A \cap B$,
$I_{K(e^w)}(x) = I_{F(e^w)}(x)$, if $e \in A - B$,
$= I_{G(e^w)}(x)$, if $e \in B - A$,
$= (I_{F(e^w)}(x) + I_{G(e^w)}(x))/2$, if $e \in A \cap B$,
$F_{K(e^w)}(x) = F_{F(e^w)}(x)$, if $e \in A - B$,
$= F_{G(e^w)}(x)$, if $e \in B - A$,
$= \max. (w_1,w_2). \max. (F_{F(e^w)}(x),F_{G(e^w)}(x))$, if $e \in A \cap B$,

Example 3.5 Consider the WNSSs (F, A^w) and (G, B^w) as in **example 3.4,** then their intersection is given in the following tabular form:

U	beautiful	wooden	costly	moderate
h_1	(0.24,0.12,0.28)	(0.21,0.09,0.15)	(0.21,0.18,0.18)	(0.24,0.28,0.36)
h_2	(0.20,0.16,0.20)	(0.18,0.21,0.09)	(0.24,0.12,0.15)	(0.24,0.31,0.24)
h_3	(0.28,0.16,0.12)	(0.21,0.09,0.15)	(0.21,0.12,0.18)	(0.20,0.24,0.42)
h_4	(0.32,0.16,0.28)	(0.18,0.09,0.18)	(0.18,0.09,0.15)	(0.28,0.25,0.36)
h_5	(0.24,0.28,0.08)	(0.21,0.09,0.12)	(0.24,0.15,0.12)	(0.24,0.24,0.30)

Table 7: The Weighted Neutrosophic Soft Sets (F, A^w) $\sqcap$ (G, B^w)

Consider (F, A^w), (G, B^w) and (K, C^w) be three WNSSs over the common universe U. Based on the definitions of union and intersections of them we have the following Propositions:

Proposition: 3.1

i. $(F, A^w) \sqcup (F, A^w) = (F, A^w)$.
ii. $(F, A^w) \sqcup (G, B^w) = (G, B^w) \sqcup (F, A^w)$.
iii. $(F, A^w) \sqcap (F, A^w) = (F, A^w)$.
iv. $(F, A^w) \sqcap (G, B^w) = (G, B^w) \sqcap (F, A^w)$.

Proof: Proofs being straightforward are not given.

Proposition: 3.2

i. $(F, A^w) \sqcup [(G, B^w) \sqcup (K, C^w)] = [(F, A^w) \sqcup (G, B^w)] \sqcup (K, C^w)$.
ii. $(F, A^w) \sqcap [(G, B^w) \sqcap (K, C^w)] = [(F, A^w) \sqcap (G, B^w)] \sqcap (K, C^w)$.
iii. $(F, A^w) \sqcup [(G, B^w) \sqcap (K, C^w)] = [(F, A^w) \sqcup (G, B^w)] \sqcap [(F, A^w) \sqcap (K, C^w)]$.
iv. $(F, A^w) \sqcap [(G, B^w) \sqcup (K, C^w)] = [(F, A^w) \sqcap (G, B^w)] \sqcup [(F, A^w) \sqcap (K, C^w)]$.

Proofs: Proofs being straightforward are not given.

We can verify the De Morgan's laws in case of union and intersection of two WNSSs.

Proposition: 3.3

i. $[(F, A^w) \sqcap (G, B^w)]^c = (F, A^w)^c \sqcup (G, B^w)^c$.
ii. $[(F, A^w) \sqcup (G, B^w)]^c = (F, A^w)^c \sqcap (G, B^w)^c$.

Proof: (i). Let $(K, D^w) = (F, A^w) \sqcap (G, B^w)$. Therfore

$T_{K(e^w)}(x) = T_{F(e^w)}(x)$, if $e \in A - B$,
$= T_{G(e^w)}(x)$, if $e \in B - A$,
$= \min.(w_1,w_2).\ \min.(T_{F(e^w)}(x), T_{G(e^w)}(x))$, if $e \in A \cap B$,
$I_{K(e^w)}(x) = I_{F(e^w)}(x)$, if $e \in A - B$,
$= I_{G(e^w)}(x)$, if $e \in B - A$,
$= (I_{F(e^w)}(x) + I_{G(e^w)}(x))/2$, if $e \in A \cap B$,
$F_{K(e^w)}(x) = F_{F(e^w)}(x)$, if $e \in A - B$,
$= F_{G(e^w)}(x)$, if $e \in B - A$,
$= \max.(w_1,w_2).\ \max.(F_{F(e^w)}(x), F_{G(e^w)}(x))$, if $e \in A \cap B$,

So,

$T_{K^c(e^w)}(x) = F_{F(e^w)}(x)$, if $e \in A - B$,
$= F_{G(e^w)}(x)$, if $e \in B - A$,
$= \max.(w_1,w_2).\ \max.(F_{F(e^w)}(x), F_{G(e^w)}(x))$, if $e \in A \cap B$,
$I_{K^c(e^w)}(x) = I_{F(e^w)}(x)$, if $e \in A - B$,
$= I_{G(e^w)}(x)$, if $e \in B - A$,
$= (I_{F(e^w)}(x) + I_{G(e^w)}(x))/2$, if $e \in A \cap B$,
$F_{K^c(e^w)}(x) = T_{F(e^w)}(x)$, if $e \in A - B$,
$= T_{G(e^w)}(x)$, if $e \in B - A$,
$= \min.(w_1,w_2).\ \min.(T_{F(e^w)}(x), T_{G(e^w)}(x))$, if $e \in A \cap B$.

Again for $(F, A^w)^c \sqcup (G, B^w)^c$, let $(P, D^w) = (H, A^w)^c$, $(Q, E^w) = (G, B^w)^c$ and $(R, S^w) = (P, D^w)^c \sqcup (Q, E^w)$, where $S = D \cup E$.

Therefore,

$T_{R(e^w)}(x) = T_{p^c(e^w)}(x) = F_{H(e^w)}(x)$, if $e \in A - B$,
$= T_{Q^c(e^w)}(x) = F_{G(e^w)}(x)$, if $e \in B - A$,
$= \max.(w_1,w_2).\ \max.(T_{p^c(e^w)}(x), T_{q^c(e^w)}(x)) = \max.(w_1,w_2).\ \max(F_{H(e^w)}(x), F_{G(e^w)}(x))$, if $e \in A \cap B$.

$I_{R(e^w)}(x) = (I_{P^c(e^w)}(x) + I_{q^c(e^w)}(x))/2 = (I_{H(e^w)}(x) + I_{G(e^w)}(x))/2$, if $e \in A \cap B$,
$= I_{p^c(e^w)}(x) = I_{H(e^w)}(x)$, if $e \in A - B$,
$= I_{Q^c(e^w)}(x) = I_{G(e^w)}(x)$, if $e \in B - A$,

$F_{R(e^w)}(x) = F_{p^c(e^w)}(x) = T_{H(e^w)}(x)$, if $e \in A - B$,
$= F_{Q^c(e^w)}(x) = T_{G(e^w)}(x)$, if $e \in B - A$,
$= \min.(w_1,w_2).\ \min.(F_{p^c(e^w)}(x), F_{q^c(e^w)}(x)) = \min.(w_1,w_2).\ \min(T_{H(e^w)}(x), T_{G(e^w)}(x))$, if $e \in A \cap B$.

Thus the result is proved.

Proof (ii). The proof is similar to the proof of (i).

Definition 3.9 AND operations of two WNSSs.

Let (F, A^w) and (G, B^w) be two WNSSs over the common universe U. Then the 'AND' operation of (F, A^w) and (G, B^w) is denoted by '$(F,A^w) \wedge (G,B^w)$' and is defined by $(F,A^w) \wedge (G,B^w) = (K, C^w)$, where $C = A \times B$ and the truth-membership, indeterminacy-membership and falsity-membership of (K, C^w) are as follows:

$T_{K(\alpha^w,\beta^w)}(x) = \min.(w_1,w_2).\ \min.(T_{F(\alpha)}(x), T_{G(\beta)}(x))$, $\forall \alpha \in A$, $\forall \beta \in B$,
$I_{K(\alpha^w,\beta^w)}(x) = (T_{F(\alpha^w)}(x) + T_{G(\beta^w)}(x))/2$, $\forall \alpha \in A$, $\forall \beta \in B$,
$F_{K(\alpha^w,\beta^w)}(x) = \max.(w_1,w_2).\ \max.(F_{F(\alpha)}(x), F_{G(\beta)}(x))$, $\forall \alpha \in A$, $\forall \beta \in B$.

Example 3.6 Consider the **example 3.5** above. The tabular representation of the WNSS $(F,A^w) \wedge (G,B^w)$ is given below:

U	(beautiful, costly)	(beautiful, moderate)	(wooden, costly)	(wooden, moderate)	(moderate, costly)	(moderate, moderate)
h_1	(0.18,0.15, 0.28)	(0.24,0.22,0.28)	(0.21,0.135, 0.18)	(0.21,0.205, 0.24)	(0.18,0.21,0.36)	(0.24,0.28,0.36)
h_2	(0.15,0.14, 0.20)	(0.20,0.24,0.20)	(0.18,0.165, 0.15)	(0.18,0.165, 0.12)	(0.18,0.21,0.30)	(0.24,0.31,0.24)
h_3	(0.21,0.14, 0.24)	(0.20,0.20,0.28)	(0.21,0.105, 0.18)	(0.15,0.165, 0.28)	(0.21,0.18,0.36)	(0.26,0.24,0.42)
h_4	(0.18,0.125,0.28)	(0.32,0.185, 0.28)	(0.18,0.09,0.18)	(0.18,0.145, 0.24)	(0.18,0.195, 0.36)	(0.28,0.25,0.36)
h_5	(0.18,0.215,0.16)	(0.24,0.20,0.20)	(0.21,0.12,0.12)	(0.18,0.105, 0.20)	(0.24,0.255, 0.30)	(0.24,0.24,0.30)

Table 8: The Weighted Neutrosophic Soft Sets $(F,A^w) \wedge (G,B^w)$

Definition 3.10. OR operations of two WNSSs.

If (F, A^w) and (G, B^w) be two WNSSs over the common universe U then '(F, A^w) **OR** (G, B^w)' denoted by

$(F,A^w) \vee (G,B^w)$ is defined by $(F,A^w) \vee (G,B^w)$ = (O, C^w), where C = A × B and the truth-membership, indeterminacy-membership and falsity-membership of (O, C^w) are given as follows:

$T_{O(\alpha^w,\beta^w)}(x) = \max.\,(w_1,w_2).\,\max.\,(T_{F(\alpha)}(x), T_{G(\beta)}(x)),\ \forall\alpha \in A,\ \forall\beta \in B,$

$I_{O(\alpha^w,\beta^w)}(x) = (I_{F(\alpha^w)}(x) + I_{G(\beta^w)}(x))/2,\ \forall\alpha \in A,\ \forall\beta \in B,$

$F_{O(\alpha^w,\beta^w)}(x) = \min.\,(w_1,w_2).\,\min.\,(F_{F(\alpha)}(x), F_{G(\beta)}(x)),\ \forall\alpha \in A,\ \forall\beta \in B.$

Example 3.7 Consider the **example 3.5** above. The tabular representation of the WNSS $(F,A^w) \vee (G,B^w)$ is given below:

U	(beautiful, costly)	(beautiful, moderate)	(wooden, costly)	(wooden, moderate)	(moderate, costly)	(moderate, moderate)
h_1	(0.28,0.15, 0.18)	(0.28,0.22,0.24)	(0.21,0.135, 0.15)	(0.28,0.205, 0.15)	(0.42,0.21,0.15)	(0.42,0.28,0.20)
h_2	(0.32,0.14, 0.15)	(0.32,0.24,0.12)	(0.24,0.165, 0.09)	(0.32,0.165, 0.09)	(0.48,0.21,0.12)	(0.48,0.31,0.12)
h_3	(0.32,0.14, 0.09)	(0.28,0.20,0.12)	(0.21,0.105, 0.15)	(0.28,0.165, 0.15)	(0.42,0.18,0.15)	(0.42,0.24,0.20)
h_4	(0.32,0.125,0.15)	(0.32,0.185, 0.24)	(0.18,0.09,0.15)	(0.32,0.145, 0.18)	(0.42,0.195, 0.15)	(0.48,0.25,0.24)
h_5	(0.32,0.215,0.06)	(0.24,0.20,0.08)	(0.24,0.12,0.12)	(0.28,0.105, 0.12)	(0.48,0.255, 0.12)	(0.48,0.24,0.20)

Table 9 : The Weighted Neutrosophic Soft Sets $(F,A^w) \vee (G,B^w)$

It is to be noted that for either AND or OR operations on two WNSSs the set of parameter is a subset of E × E whereas for three WNSSs the associated parameters are subset of E × E × E.

Conclusion

In this paper we introduce the concept of weighted neutrosophic soft sets which is a hybridization of soft sets and weighted parameter of neutrosophic soft sets. We have also introduced some operations like union, intersection, AND, OR etc. on this newly defined concept. Some properties of these operations have also been investigated.

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*- Neutrosophic Crisp Set & *- Neutrosophic Crisp relations

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Abstract. Since the world is full of indeterminacy, the neutrosophics found their place into contemporary research. The purpose of this paper is to introduce a new type of neutrosophic crisp set as the *- neutrosophic crisp sets as a generalization to star intuitionistic set due to Indira et al.[4], and study some of its properties. Finally we introduce and study the notion of *- neutrosophic relation and some of its properties.

Keywords: Neutrosophic Crisp Set; Star Intuitionistic Sets; Neutrosophic Relations; Neutrosophic Data.

1 Introduction

The fundamental concepts of neutrosophic set, introduced by Smarandache in [31, 32, 33], and Salama et al. in [5, 6, 7, 8, 9, 10, 11, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30], provides a natural foundation for treating mathematically the neutrosophic phenomena which exist pervasively in our real world and for building new branches of neutrosophic mathematics. Neutrosophy has laid the foundation for a whole family of new mathematical theories generalizing both their classical and fuzzy counterparts [1, 2, 12, 22, 34] such as a neutrosophic set theory. In this paper we introduce a new type of neutrosophic crisp set as the *- neutrosophic crisp set, and study some of its properties. Finally we introduce and study the notion of *- neutrosophic relation and some of its properties. Possible applications to mathematical computer are touched upon.

2 Terminologies

We recollect some relevant basic preliminaries, and in particular, the work of Smarandache in [31, 32, 33], and Salama et al. in [5, 6, 7, 8, 9, 10, 11, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30]. Smarandache introduced the neutrosophic components T, I, F which represent the membership, indeterminacy, and non-membership values respectively, where $\left]^{-}0, 1^{+}\right[$ is nonstandard unit interval.

3 *- Neutrosophic Crisp Sets

We shall now consider some possible definitions for a new type of neutrosophic crisp set

Definition 3.1

Let X be a non-empty fixed set. A neutrosophic crisp set (NCS for short) A is an object having the form $A = \langle A_1, A_2, A_3 \rangle$.

Then we define the *- neutrosophic set A^* as $A^* = \left\langle A_1 \cap (A_2 \cup A_3)^c, A_2 \cap (A_1 \cup A_3)^c, A_3 \cap (A_1 \cup A_2)^c \right\rangle$ where A_1, A_2 and A_3 are subsets of X such that

$M = A_1 \cap (A_2 \cup A_3)^c$, $S = A_2 \cap (A_1 \cup A_3)^c$ and

$R = A_3 \cap (A_1 \cup A_2)^c$.

A *- neutrosophic crisp set is an object having the form $A^* = \langle M, S, R \rangle$

Lemma 3.1

Let X be a non-empty fixed sample space. A neutrosophic crisp set (NCS for short) A is an object having the form $A = \langle A_1, A_2, A_3 \rangle$. Then

$A^* = \left\langle A_1 \cap (A_2 \cup A_3)^c, A_2 \cap (A_1 \cup A_3)^c, A_3 \cap (A_1 \cup A_2)^c \right\rangle$ is also a neutrosophic crisp set.

Proof

It's clear.

Corollary 3.1

Let X be a non-empty fixed set. Then $\phi_N{}^*$ and $X_N{}^*$ are also neutrosophic crisp set.

Theorem 3.1

Let X be a non-empty fixed sample space, two neutrosophic crisp sets A, B are having the form

$A = \langle A_1, A_2, A_3 \rangle$, $B = \langle B_1, B_2, B_3 \rangle$, and two *- neutrosophic sets $A^* = \langle M_1, S_1, R_1 \rangle, B^* = \langle M_2, S_2, R_3 \rangle$ where

$M_1 = A_1 \cap (A_2 \cup A_3)$, $S_1 = A_2 \cap (A_1 \cup A_3)^c$,

$R_1 = A_3 \cap (A_1 \cup A_2)^c$, $M_2 = B_1 \cap (B_2 \cup B_3)^c$,
$S_2 = B_2 \cap (B_1 \cup B_3)^c$, and

$R_2 = B_3 \cap (B_1 \cup B_2)$, Then $A \subseteq B$ implies $A^* \subseteq B^*$.

Proof

Given $A \subseteq B$. Then it is easy to prove that $M_1 \subseteq M_2$, $S_1 \subseteq S_2, R_1 \supseteq R_2$ or $M_1 \subseteq M_2$, $S_1 \subseteq S_2, R_1 \supseteq R_2$ So $A^* \subseteq B^*$.

Remark 3.1

1) All types of $\phi_N{}^*$ and ϕ_N are concedes.
2) All types of $X_N{}^*$ and X_N are concedes.
3) $A^* = B^*$ iff $A^* \subseteq B^*$ and $B^* \subseteq A^*$.

Definition 3.8

Let X be a non-empty set, and $A^* = \langle M, S, R\rangle$ be a *-neutrosophic crisp set on a NCS $A = \langle A_1, A_2, A_3\rangle$ where $M = A_1 \cap (A_2 \cup A_3)^c$, $S = A_2 \cap (A_1 \cup A_3)^c$, $R = A_3 \cap (A_1 \cup A_2)^c$, Then the complement of the set A^* (A^{*c}, for short) may be defined as three kinds of complements

(C_1) Type1: $A^{*c} = \langle M^c, S^c, R^c\rangle$,

(C_2) Type2: $A^{*c} = \langle R, S, M\rangle$,

(C_3) Type3: $A^{*c} = \langle R, S^c, M\rangle$.

Definition 2.3

Let X be a non-empty fixed set, two neutrosophic crisp sets A, B are having the form $A = \langle A_1, A_2, A_3\rangle$, $B = \langle B_1, B_2, B_3\rangle$, and two *- neutrosophic crisp sets $A^* = \langle M_1, S_1, R_1\rangle, B^* = \langle M_2, S_2, R_2\rangle$ where
$M_1 = A_1 \cap (A_2 \cup A_3)^c$, $S_1 = A_2 \cap (A_1 \cup A_3)^c$,
$R_1 = A_3 \cap (A_1 \cup A_2)^c$, $M_2 = B_1 \cap (B_2 \cup B_3)^c$,
$S_2 = B_2 \cap (B_1 \cup B_3)^c$, and
$R_2 = B_3 \cap (B_1 \cup B_2)^c$, Then

1) $A^* \cap B^*$ may be defined as two types:
 i) Type1: $A^* \cap B^* = \langle M_1 \cap M_1, S_2 \cap S_2, R_3 \cup R_3\rangle$ or
 i. Type2: $A^* \cap B^* = \langle M_1 \cap M_1, S_2 \cup S_2, R_3 \cup R_3\rangle$

4) $A^* \cup B^*$ may be defined as two types:
 i) Type1: $A^* \cup B^* = \langle M_1 \cup M_1, S_2 \cap S_2, R_3 \cap R_3\rangle$ or
 ii) Type2: $A^* \cup B^* = \langle M_1 \cup M_1, S_2 \cup S_2, R_3 \cap R_3\rangle$.

Lemma 3.1

Let A^*, B^* are *- neutrosophic crisp sets. Then $A^* - B^* = A^* \cap B^{*C}$

It easy to show that L. H. S is also a *- neutrosophic crisp sets.

Example 3.2

Let $X = \{a,b,c,d,e,f\}$, $A = \langle\{a,b,c,d\},\{e\},\{f\}\rangle$, $B = \langle\{a,b,c\},\{d\},\{e\}\rangle$, $C = \langle\{a,b\},\{c,d\},\{e,f,a\}\rangle$
$D = \langle\{a,b\},\{e,c\},\{f,d\}\rangle$ are NCS. Then
$A^* = \langle\{a,b,c,d\},\{e\},\{f\}\rangle$, $B^* = \langle\{a,b,c\},\{d\},\{e\}\rangle$,
$C^* = \langle\{b\},\{c,d\},\{e,f\}\rangle$,

The complement may be equal as:

1)
$A^{*C} = \langle\{e,f\},\{a,b,c,d,f\},\{a,b,c,d\}\rangle$,
$A^{*C} = \langle\{\{f\},\{e\},\{a,b,c,d\}\rangle$, $A^{*C} = \langle\{\{f\},\{a,b,c,d\},\{a,b,c,d\}\rangle$,

2) $C^{*C} = \langle\{a,c,d,f\},\{a,b,e,f\},\{a,b,c,d\}\rangle$,
$C^{*C} = \langle\{e,f\},\{c,d\},\{b\}\rangle$, $C^{*C} = \langle\{e,f\},\{a,b,e,f\},\{b\}\rangle$.

3) $A^* \cup B^*$ may be equals the following forms
$A^* \cup B^* = \langle\{a,b,c,d\},\{e\},\phi\rangle$,
$A^* \cup B^* = \langle\{a,b,c,d\},\phi,\{f\}\rangle$,

4) $A^* \cap B^*$ may be equals the following forms
$A^* \cap B^* = \langle\{a,b,c\},\{e,d\},\{f,e\}\rangle$,
$A^* \cup B^* = \langle\{a,b,c\},\phi,\{f,e\}\rangle$,

Proposition 3.1

Let $\{A_j{}^* : j \in J\}$ be arbitrary family of *- neutrosophic crisp subsets on X, then

1) $\cap A_j{}^*$ may be defined two types as :
 i) Type1: $\cap A^*{}_j = \langle \cap M_j, \cap S_j, \cup R_j\rangle$, or
 ii) Type2: $\cap A^*{}_j = \langle \cap M_j, \cup S_j, \cup R_j\rangle$.
2) $\cup A^*{}_j$ may be defined two types as :
 i) Type1: $\cup A^*{}_j = \langle \cup M_j, \cap S_j, \cap R_j\rangle$ or
 ii) Type2: $\cup A^*{}_j = \langle \cup M_j, \cup S_j, \cap R_j\rangle$.

Corollary 3.2

Let $\{A_i\}$ be a NCSs in X where $i \in J$, where J is an index set and $\{A_i{}^*\}$ are corresponding *- neutrosophic crisp subsets on X then

a) $A_i{}^* \subseteq B^*$ for each $i \in J \Rightarrow \cup A_i{}^* \subseteq B^*$.
b) $B^* \subseteq A_i{}^*$ for each $i \in J \Rightarrow B^* \subseteq \cup A_i{}^*$.
c) $(\cup A_i{}^*)^c = \cap A_i{}^{*c}$; $(\cap A_i{}^*)^c = \cup A_i{}^{*c}$.

d) $A_i^* \subseteq B^* \Leftrightarrow B^{*c} \subseteq A^{*c}$.

e) $A^{*c^c} = A$,

f) $\phi_N^{*c} = X_N ; X_N^{*c} = \phi_N^*$.

Now we shall define the image and preimage of *- neutrosophic crisp set.

Let X, Y be two non-empty fixed sets and $f : X \to Y$, be a function and $A = \langle A_1, A_2, A_3 \rangle$, $B = \langle B_1, B_2, B_3 \rangle$ are neutrosophic crisp sets on X and Y respectively, $A^* = \langle M_1, S_1, R_1 \rangle$, $B^* = \langle M_2, S_2, R_2 \rangle$ be the *- neutrosophic crisp sets on X and Y respectively.

Definition 3.9

(a) If B^* is a *- NCS in Y, then the preimage of B^* under f, denoted by $f^{-1}(B^*)$, is a *- NCS in X defined by $f^{-1}(B^*) = \langle f^{-1}(M_2), f^{-1}(S_2), f^{-1}(R_2) \rangle$

(b) If A^* is a *- NCS in X, then the image of A^* under f, denoted by $f(A^*)$, is the *- NCS in Y defined by $f(A^*) = \langle f(M_1), f(S_1), f(R_1)^c \rangle$.

Here we introduce the properties of images and preimages some of which we shall frequently use in the following.

Corollary 3.2

Let A^*, $\{A_i^* : i \in J\}$, be a family of *- NCS in X, and B^*, $\{B^*_j : j \in K\}$ *- NCS in Y, and $f : X \to Y$ a function. Then

(a) $A^*_1 \subseteq A^*_2 \Leftrightarrow f(A^*_1) \subseteq f(A^*_2)$, $B^*_1 \subseteq B^*_2 \Leftrightarrow f^{-1}(B^*_1) \subseteq f^{-1}(B^*_2)$,

(b) $A^* \subseteq f^{-1}(f(A^*))$ and if f is injective, then $A^* = f^{-1}(f(A^*))$,

(c) $f^{-1}(f(B^*)) \subseteq B^*$ and if f is surjective, then $f^{-1}(f(B^*)) = B^*$,

(d) $f^{-1}(\cup B^*_i)) = f^{-1}(B^*_i)$, $f^{-1}(\cap B^*_i)) = \cap f^{-1}(B^*_i)$,

(e) $f(\cup A^*_{ii}) = \cup f(A^*_{ii})$; $f(\cap A^*_{ii}) \subseteq \cap f(A^*_{ii})$; and if f is injective, then $f(\cap A^*_{ii}) = \cap f(A^*_{ii})$;

(f) $f^{-1}(Y^*_N) = X^*_N$, $f^{-1}(\phi^*_N) = \phi^*_N$.

(g) $f(\phi^*_N) = \phi^*_N$, $f(X^*_N) = Y^*_N$, if f is surjective.

(h) If f is surjective, then $(f(A^*))^c \subseteq f(A^*)^c$. if furthermore f is injective, then have $(f(A^*))^c = f(A^*)^c$.

(i) $(f^{-1}(B^*)^c = (f^{-1}(B^*))^c$.

Proof

Clear by definitions.

4 *- Neutrosophic Crisp Set Relations

Here we give the definition relation on *- neutrosophic crisp sets and study of its properties.

Let X, Y and Z be three ordinary nonempty sets

Definition 4.1

Let X be a non-empty fixed set, two neutrosophic crisp sets A, B are having the form $A = \langle A_1, A_2, A_3 \rangle$, $B = \langle B_1, B_2, B_3 \rangle$, and two *- neutrosophic crisp sets $A^* = \langle M_1, S_1, R_1 \rangle$, $B^* = \langle M_2, S_2, R_2 \rangle$ where

$M_1 = A_1 \cap (A_2 \cup A_3)$, $S_1 = A_2 \cap (A_1 \cup A_3)$,

$R_1 = A_3 \cap (A_1 \cup A_2)$,

$M_2 = B_1 \cap (B_2 \cup B_3)$, $S_2 = B_2 \cap (B_1 \cup B_3)$, and

$R_2 = B_3 \cap (B_1 \cup B_2)$, Then

i) The product of two *- neutrosophic crisp sets A^* and B^* is a *- neutrosophic crisp set $A^* \times B^*$ given by

$A^* \times B^* = \langle M_1 \times M_2, S_1 \times S_2, R_1 \times R_2 \rangle$ on $X \times Y$.

ii) We will call a *- neutrosophic crisp relation $R^* \subseteq A^* \times B^*$ on the direct product $X \times Y$.

The collection of all *- neutrosophic crisp relations on $X \times Y$ is denoted as $SNCR(X \times Y)$

Definition 4.2

Let R^* be a *- neutrosophic crisp relation on $X \times Y$, then the inverse of R^* is donated by R^{*-1} where $R^* \subseteq A^* \times B^*$ on $X \times Y$ then $R^{*-1} \subseteq B^* \times A^*$ on $Y \times X$.

Example 4.1

Let $X = \{a, b, c, d, e, f\}$, $A = \langle \{a, b, c, d\}, \{e\}, \{f\} \rangle$, $B = \langle \{a, b, c\}, \{d\}, \{e\} \rangle$, are NCS.

Then $A^* = \langle \{a, b, c, d\}, \{e\}, \{f\} \rangle$, $B^* = \langle \{a, b, c\}, \{d\}, \{e\} \rangle$, then the product of two *- neutrosophic crisp sets given by

$A^* \times B^* = \langle \{(a,a),(a,b),(a,c),(b,a),(b,b),(b,c),(c,a),(c,b),(c,c)\}, \{(e,d)\}, \{(f,e)\} \rangle$

and

$B^* \times A^* = \langle \{(a,a),(a,b),(a,c),(a,d),(b,a),(b,b),(b,c),(b,d),(c,a),(c,b),(c,c),(c,d)\}, \{(d,e)\}, \{(e,f)\} \rangle$

, and $R_1^* = \langle \{(a,a)\}, \{(c,c)\}, \{(d,d)\} \rangle$, $R_1^* \subseteq A^* \times B^*$ on $X \times X$,

$R_2^* = \langle \{(a,b)\}, \{(c,c)\}, \{(d,d),(b,d)\} \rangle$ $R_2^* \subseteq B^* \times A^*$ on $X \times X$, $R_1^{*-1} = \langle \{(a,a)\}, \{(c,c)\}, \{(d,d)\} \rangle \subseteq B^* \times A^*$ and $R_2^{*-1} = \langle \{(b,a)\}, \{(c,c)\}, \{(d,d),(d,b)\} \rangle \subseteq B^* \times A^*$.

We can define the operations of *- neutrosophic crisp relations.

Definition 4.3

Let R^* and S^* be two *- neutrosophic crisp relations between X and Y for every $(x, y) \in X \times Y$ and NCSS A

and B in the form $A=\langle A_1,A_2,A_3\rangle$, A^* on X, $B=\langle B_1,B_2,B_3\rangle$, B^* on Y Then we can defined the following operations

i) $R\subseteq S$ may be defined as two types

a) Type1: $R^*\subseteq S^* \Leftrightarrow M_{1_R}\subseteq M_{1_S}$, $S_{1_R}\subseteq S_{1_s}$, $R_{1_R}\supseteq R_{1_s}$

b) Type2: $R^*\subseteq S^* \Leftrightarrow M_{1_R}\subseteq M_{1_S}$, $S_{1_R}\supseteq S_{1_s}$, $R_{1_R}\supseteq R_{1_s}$

ii) $R^*\cup S^*$ may be defined as two types

a) Type1: $R^*\cup S^*=\langle M_{1R}\cup M_{1S}, S_{1R}\cup S_{1S}, R_{1R}\cap R_{1S}\rangle$,

b) Type2: $R^*\cup S^*=\langle M_{1R}\cup M_{1S}, S_{1R}\cap S_{1S}, R_{1R}\cap R_{1S}\rangle$.

iii) $R^*\cap S^*$ may be defined as two types

a) Type1: $R^*\cap S^*=\langle M_{1R}\cap M_{1S}, S_{1R}\cup S_{1S}, R_{1R}\cup R_{1S}\rangle$,

b) Type2: $R^*\cap S^*=\langle M_{1R}\cap M_{1S}, S_{1R}\cap S_{1S}, R_{1R}\cup R_{1S}\rangle$.

Theorem 4.1

Let R^*, S^* and Q^* be three *- neutrosophic crisp relations between X and Y for every $(x,y)\in X\times Y$, then

i) $R^*\subseteq S^*\Rightarrow R^{*-1}\subseteq S^{*-1}$.

ii) $\left(R^*\cup S^*\right)^{-1}\Rightarrow R^{*-1}\cup S^{*-1}$.

iii) $\left(R^*\cap S^*\right)^{-1}\Rightarrow R^{*-1}\cap S^{*-1}$.

iv) $\left(R^{*-1}\right)^{-1}=R^*$.

v) $R^*\cap\left(S^*\cup Q^*\right)=\left(R^*\cap S^*\right)\cup\left(R^*\cap Q^*\right)$.

vi) $R^*\cup\left(S^*\cap Q^*\right)=\left(R^*\cup S^*\right)\cap\left(R^*\cup Q^*\right)$.

vii) If $S^*\subseteq R^*$, $Q^*\subseteq R^*$, then $S^*\cup Q^*\subseteq R^*$.

Proof

Clear

Definition 5.4

The *- neutrosophic crisp relation $I^*\in SNCR^*(X\times X)$, the *- neutrosophic crisp relation of identity may be defined as two types

i) Type1: $I^*=\left\{<\{A^*\times A^*\},\{A^*\times A^*\},\phi^*>\right\}$

ii) Type2: $I^*=\left\{<\{A^*\times A^*\},\phi^*,\phi^*>\right\}$

Now we define two composite relations of *- neutrosophic crisp sets.

Definition 5.5

Let R^* be a *- neutrosophic crisp relation in $X\times Y$, and S^* be a neutrosophic crisp relation in $Y\times Z$. Then the composition of R^* and S^*, $R^*\circ S^*$ be a *- neutrosophic crisp relation in $X\times Z$ as a definition may be defined as two types

i) Type1:

$R^*\circ S^*\leftrightarrow (R^*\circ S^*)(x,z)$
$=\cup\{<\{(M_1\times M_2)_R\cap(M_1\times M_2)_S\}$,
$\{(S_1\times S_2)_R\cap(S_1\times S_2)_S\}$, $\{(R_1\times R_2)_R\cap(R_1\times R_2)_S\}>$.

ii) Type2:

$R^*\circ S^*\leftrightarrow (R^*\circ S^*)(x,z)$
$=\cap\{<\{(M_1\times M_2)_R\cup(M_1\times M_2)_S\}$,
$\{(S_1\times S_2)_R\cup(S_1\times S_2)_S\}$, $\{(R_1\times R_2)_R\cup(R_1\times R_2)_S\}>$.

Theorem 4.2

Let R^* be a *- neutrosophic crisp relation in $X\times Y$, and S be a *- neutrosophic crisp relation in $Y\times Z$ then $(R^*\circ S^*)^{-1}=S^{*-1}\circ R^{*-1}$.

Proof

Let $R^*\subseteq A^*\times B^*$ on $X\times Y$ then $R^{*-1}\subseteq B\times A$, $S^*\subseteq B^*\times D^*$ on $Y\times Z$ then $S^{*-1}\subseteq D^*\times B^*$, from Definition 4.3 and similarly we can $I^*{}_{(R^*\circ S^*)^{-1}}(x,z)=I^*{}_{S^{*-1}}(x,z)$ and $I^*{}_{R^{*-1}}(x,z)$ then $(R^*\circ S^*)^{-1}=S^{*-1}\circ R^{*-1}$.

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Interval Valued Neutrosophic Soft Topological Spaces

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Abstract. In this paper we introduce the concept of interval valued neutrosophic soft topological space together with interval valued neutrosophic soft finer and interval valued neutrosophic soft coarser topology. We also define interval valued neutrosophic interior and closer of an interval valued neutrosophic soft set. Some theorems and examples are cites. Interval valued neutrosophic soft subspace topology are studied. Some examples and theorems regarding this concept are presented..

Keywords: Soft set, interval valued neutrosophic set, interval valued neutrosophic soft set, interval valued neutrosophic soft topological space.

1 Introduction

In 1999, Molodtsov [9] introduced the concept of soft set theory which is completely new approach for modeling uncertainty. In this paper [9] Molodtsov established the fundamental results of this new theory and successfully applied the soft set theory into several directions. Maji et al. [7] defined and studied several basic notions of soft set theory in 2003. Pie and Miao [11], Aktas and Cagman [1] and Ali et. al. [2] improved the work of Maji et al [7]. The intuitionistic fuzzy set is introduced by Atanaasov [4] as a generalization of fuzzy set [15] where he added degree of non-membership with degree of membership. Neutrosophic set introduced by F. Smarandache in 1995 [12]. Smarandache [13] introduced the concept of neutrosophic set which is a mathematical tool for handling problems involving imprecise, indeterminacy and inconstant data. Maji [8] combined neutrosophic set and soft set and established some operations on these sets. Wang et al. [14] introduced interval neutrosophic sets. Deli [6] introduced the concept of interval-valued neutrosophic soft sets.

In this paper we form a topological structure on interval valued neutrosophic soft sets and establish some properties of interval valued neutrosophic soft topological space with supporting proofs and examples.

2 Preliminaries

In this section we recall some basic notions relevant to soft sets, interval-valued neutrosophic sets and interval-valued neutrosophic soft sets.

Definition 2.1: [9] Let U be an initial universe and E be a set of parameters. Let $P(U)$ denotes the power set of U and $A \subseteq E$. Then the pair (f,A) is called a *soft set* over U, where f is a mapping given by $f : A \to P(U)$.

Definition 2.2: [13] A neutrosophic set A on the universe of discourse U is defined as $A = \{(x, \mu_A(x), \gamma_A(x), \delta_A(x)) : x \in U\}$, where $\mu_A, \gamma_A, \delta_A : U \to]^-0,1^+[$ are functions such that the condition: $\forall x \in U$, $^-0 \le \mu_A(x) + \gamma_A(x) + \delta_A(x) \le 3^+$ is satisfied.

Here $\mu_A(x), \gamma_A(x), \delta_A(x)$ represent the truth-membership, indeterminacy-membership and falsity-membership respectively of the element $x \in U$. From philosophical point of view, the neutrosophic set takes the value from real standard or non-standard subsets of $]^-0,1^+[$. But in real life application in scientific and engineering problems it is difficult to use neutrosophic set with value from real standard or non-standard subset of $]^-0,1^+[$. Hence we consider the neutrosophic set which takes the value from the subset of $[0,1]$.

Definition 2.3: [14] An *interval valued neutrosophic set* A on the universe of discourse U is defined as $A = \{(x, \mu_A(x), \gamma_A(x), \delta_A(x)) : x \in U\}$, where $\mu_A, \gamma_A, \delta_A : U \to Int]^-0,1^+[$ are functions such that the

condition:
$\forall x \in U$, $^{-}0 \le sup\mu_A(x) + sup\gamma_A(x) + sup\delta_A(x) \le 3^{+}$ is satisfied.

In real life applications it is difficult to use interval valued neutrosophic set with interval-value from real standard or non-standard subset of $Int(]^{-}0,1^{+}[)$. Hence we consider the interval valued neutrosophic set which takes the interval-value from the subset of $Int([0,1])$ (where $Int([0,1])$ denotes the set of all closed sub intervals of $[0,1]$). The set of all interval valued neutrosophic sets on U is denoted by *IVNS(U)*.

Definition 2.4: [6] Let U be an universe set, E be a set of parameters and $A \subseteq E$. Let *IVNs(U)* denotes the set of all interval valued neutrosophic sets of U. Then the pair (f, A) is called an *interval valued neutrosophic soft set* (*IVNSs* in short) over U, where f is a mapping given by $f : A \to IVNs(U)$. The collection of all interval valued neutrosophic soft sets over U is denoted by *IVNSs(U)*.

Definition 2.5: [6] Let U be a universe set and E be a set of parameters. Let $(f, A), (g, B) \in IVNSs(U)$, where $f : A \to IVNs(U)$ is defined by

$$f(a) = \left\{\left(x, \mu_{f(a)}(x), \gamma_{f(a)}(x), \delta_{f(a)}(x)\right) : x \in U\right\}$$

and $g : B \to IVNs(U)$ is defined by

$$g(b) = \left\{\left(x, \mu_{g(b)}(x), \gamma_{g(b)}(x), \delta_{g(b)}(x)\right) : x \in U\right\}$$

where

$\mu_{f(a)}(x), \gamma_{f(a)}(x), \delta_{f(a)}(x), \mu_{g(b)}(x), \gamma_{g(b)}(x), \delta_{g(b)}(x) \in Int([0,1])$ for $x \in U$. Then

(i) (f, A) is called *interval valued neutrosophic subset* of (g, B) (denoted by $(f, A) \subseteq (g, B)$) if $A \subseteq B$ and $\mu_{f(e)}(x) \le \mu_{g(e)}(x)$, $\gamma_{f(e)}(x) \ge \gamma_{g(e)}(x)$, $\delta_{f(e)}(x) \ge \delta_{g(e)}(x)$ $\forall e \in A, \forall x \in U$. Where

$\mu_{f(e)}(x) \le \mu_{g(e)}(x)$ iff $inf\,\mu_{f(e)} \le inf\,\mu_{g(e)}$ and $sup\mu_{f(e)} \le sup\mu_{g(e)}$

$\gamma_{f(e)}(x) \ge \gamma_{g(e)}(x)$ iff $inf\,\gamma_{f(e)} \ge inf\,\gamma_{g(e)}$ and $sup\gamma_{f(e)} \ge sup\gamma_{g(e)}$

$\delta_{f(e)}(x) \ge \delta_{g(e)}(x)$ iff $inf\,\delta_{f(e)} \ge inf\,\delta_{g(e)}$ and $sup\delta_{f(e)} \ge sup\delta_{g(e)}$.

(ii) Their *union*, denoted by $(f, A) \cup (g, B) = (h, C)$ (say), is an interval valued neutrosophic soft set over U, where $C = A \cup B$ and for $e \in C$, $h : C \to IVNS(U)$ is defined by

$h(e) = \left\{\left(x, \mu_{h(e)}(x), \gamma_{h(e)}(x), \delta_{h(e)}(x)\right) : x \in U\right\}$, where for $x \in U$,

$$\mu_{h(e)}(x) = \begin{cases} \mu_{f(e)}(x) & if\ e \in A - B \\ \mu_{g(e)}(x) & if\ e \in B - A \\ \mu_{f(e)}(x) \vee \mu_{g(e)}(x) & if\ e \in A \cap B \end{cases}$$

$$\gamma_{h(e)}(x) = \begin{cases} \gamma_{f(e)}(x) & if\ e \in A - B \\ \gamma_{g(e)}(x) & if\ e \in B - A \\ \gamma_{f(e)}(x) \wedge \gamma_{g(e)}(x) & if\ e \in A \cap B \end{cases}$$

$$\delta_{h(e)}(x) = \begin{cases} \delta_{f(e)}(x) & if\ e \in A - B \\ \delta_{g(e)}(x) & if\ e \in B - A \\ \delta_{f(e)}(x) \wedge \delta_{g(e)}(x) & if\ e \in A \cap B \end{cases}$$

(iii) Their *intersection*, denoted by $(f, A) \cap (g, B) = (h, C)$ (say), is an interval valued neutrosophic soft set of over U, where $C = A \cap B$ and for $e \in C$, $h : C \to IVNS(U)$ is defined by

$h(e) = \left\{\left(x, \mu_{h(e)}(x), \gamma_{h(e)}(x), \delta_{h(e)}(x)\right) : x \in U\right\}$, where for $x \in U$ and $e \in C$,

$\mu_{h(e)}(x) = \mu_{f(e)}(x) \wedge \mu_{g(e)}(x), \gamma_{h(e)}(x) = \gamma_{f(e)}(x) \vee \gamma_{g(e)}(x)$ and $\delta_{h(e)}(x) = \delta_{f(e)}(x) \vee \delta_{g(e)}(x)$.

(iv) The *complement* of (f, A), denoted by $(f, A)^c$ is an interval valued neutrosophic soft set over U and is defined as $(f, A)^c = (f^c, \rceil A)$, where $f^c : \rceil A \to IVNS(U)$ is defined by

$f^c(a) = \left\{\left(x, \delta_{f(a)}(x), \left[1 - sup\gamma_{f(a)}(x), 1 - inf\,\gamma_{f(a)}(x)\right], \mu_{f(a)}(x)\right) : x \in U\right\}$

for $a \in A$.

Definition 2.6:[5,6] An *IVNSs* (f, A) over the universe U is said to be universe *IVNSs* with respect to A if $\mu_{f(a)}(x) = [1,1]$, $\gamma_{f(a)}(x) = [0,0]$, $\delta_{f(a)}(x) = [0,0]$ $\forall x \in U, \forall a \in A$. It is denoted by I.

Definition 2.7: An *IVNSs* (f, A) over the universe U is said to be null *IVNSs* with respect to A if $\mu_{f(a)}(x)=[0,0]$, $\gamma_{f(a)}(x)=[1,1]$, $\delta_{f(a)}(x)=[1,1]$ $\forall x \in U, \forall a \in A$. It is denoted by ϕ.

3 Interval Valued Neutrosophic Soft Topological Spaces

In this section, we give the definition of interval valued neutrosophic soft topological spaces with some examples and results. We also define discrete and indiscrete interval valued neutrosophic soft topological space along with interval valued neutrosophic soft finer and coarser topology.

Let U be an universe set, E be the set of parameters, $\wp(U)$ be the set of all subsets of *U*, *IVNs(U)* be the set of all interval valued neutrosophic sets in *U* and *IVSNs(U;E)* be the family of all interval valued neutrosophic soft sets over *U* via parameters in *E*.

Definition 3.1: Let (ζ_A, E) be an element of *IVNSs(U;E)*, $\wp(\zeta_A, E)$ be the collection of all interval valued neutrosophic soft subsets of (ζ_A, E). A sub family τ of $\wp(\zeta_A, E)$ is called an interval valued neutrosophic soft topology (in short *IVNS*-topology) on (ζ_A, E) if the following axioms are satisfied:

(i) $(\phi_{\zeta_A}, E), (\zeta_A, E) \in \tau$

(ii) $\{(f_A^k, E): k \in K\} \subseteq \tau \Rightarrow \bigcup_{k\in K}(f_A^k, E) \in \tau$

(iii) If $(g_A, E), (h_A, E) \in \tau$ then $(g_A, E) \cap (h_A, E) \in \tau$

The triplet (ζ_A, E, τ) is called interval valued neutrosophic soft topological space (in short *IVNS*-topological space) over (ζ_A, E). The members of τ are called τ –open *IVNS* sets (or simply open sets). Here $\phi_{\zeta_A}: A \to IVNS(U)$ is defined as $\phi_{\zeta_A}(e)=\{(x,[0,0],[1,1],[1,1]): x \in U\}$ $\forall e \in A$.

Example 3.2: Let $U=\{u_1, u_2, u_3\}$, $E=\{e_1, e_2, e_3, e_4\}$, $A=\{e_1, e_2, e_3\}$. The tabular representation of (ζ_A, E) given by

U	e_1	e_2	e_3
u_1	([.5,.8],[.3,.5],[.2,.7])	([.4,.7],[.2,.3],[.1,.3])	([.3,.9],[0,.1],[0,.2])
u_2	([.4,.7],[.3,.4],[.1,.2])	([.6,.9],[.1,.2],[.1,.2])	([.4,.8],[.1,.2],[0,.5])
u_3	([.5,1],[0,.1],[.3,.6])	([.6,.8],[.2,.4],[.1,.3])	([.4,.9],[.1,.3],[.2,.4])

Table1:Tabular representation of (ζ_A, E)

The tabular representation of (ϕ_{ζ_A}, E) is given by

U	e_1	e_2	e_3
u_1	([0,0],[1,1],[1,1])	([0,0],[1,1],[1,1])	*([0,0],[1,1],[1,1])*
u_2	([0,0],[1,1],[1,1])	([0,0],[1,1],[1,1])	*([0,0],[1,1],[1,1])*
u_3	([0,0],[1,1],[1,1])	([0,0],[1,1],[1,1])	*([0,0],[1,1],[1,1])*

Table2:Tabular representation of (ϕ_{ζ_A}, E)

The tabular representation of (f_A^1, E) is given by

U	e_1	e_2	e_3
u_1	*([.1,.7],[.4,.8],[.3,1])*	*([.1,.3],[.4,.6],[.2,.6])*	*([.2,.5],[.8,.9],[.4,.9])*
u_2	*([.1,.3],[.6,.7],[.2,.8])*	*([0,.5],[.5,.8],[.4,1])*	*([0,.3],[.6,.9],[.1,.7])*
u_3	*([.4,.8],[.6,.7],[.6,.9])*	*([0,.3],[.4,.7],[.2,.8])*	*([.1,.3],[.6,.8],[.3,.7])*

Table3:Tabular representation of (f_A^1, E)

The tabular representation of (f_A^2, E) is given by

U	e_1	e_2	e_3
u_1	([.4,.7],[.5,.7],[.4,.9])	([.2,.3],[.4,.5],[.7,.9])	([.3,.7],[.5,.8],[.1,.2])
u_2	([.3,.5],[.4,.8],[.1,.4])	([.4,.6],[.3,.5],[.2,.5])	([.1,.3],[.3,.5],[.6,.8])
u_3	([.3,.9],[.1,.2],[.6,.7])	([.5,.7],[.6,.7],[.3,.4])	([.2,.6],[.3,.5],[.5,.8])

Table4: Tabular representation of (f_A^2, E)

Let $(f_A^3, E)=(f_A^1, E)\cap(f_A^2, E)$ then the tabular representation of (f_A^3, E) is given by

U	e_1	e_2
u_1	([.1,.7],[.5,.8],[.4,1])	([.1,.3],[.4,.6],[.7,.9])
u_2	([.1,.3],[.6,.8],[.2,.8])	([0,.5],[.5,.8],[.4,1])
u_3	([.3,.8],[.6,.7],[.6,.9])	([0,.3],[.6,.7],[.3,.8])

e_3
([.2,.5],[.8,.9],[.4,.9])
([0,.3],[.6,.9],[.6,.8])
([.1,.3],[.6,.8],[.5,.8])

Table5:Tabular representation of $\left(f_A^3, E\right)$

Let $\left(f_A^4, E\right)=\left(f_A^1, E\right) \cup\left(f_A^2, E\right)$ then the tabular representation of $\left(f_A^4, E\right)$ is given by

U	e_1	e_2
u_1	([.4,.7],[.4,.7],[.3,.9])	([.2,.3],[.4,.5],[.2,.6])
u_2	([.3,.5],[.4,.7],[.1,.4])	([.4,.6],[.3,.5],[.2,.5])
u_3	([.4,.9],[.1,.2],[.6,.7])	([.5,.7],[.4,.7],[.2,.4])

e_3
([.3,.7],[.5,.8],[.1,.2])
([.1,.3],[.3,.5],[.1,.7])
([.2,.6],[.3,.5],[.3,.7])

Table6:Tabular representation of $\left(f_A^4, E\right)$

Here we observe that the sub-family $\tau_1=\left\{\left(\phi_{\zeta_A}, E\right),\left(\zeta_A, E\right),\left(f_A^1, E\right),\left(f_A^2, E\right),\left(f_A^3, E\right),\left(f_A^4, E\right)\right\}$ of $\wp\left(\zeta_A, E\right)$ is a *IVNS*-topology on $\left(\zeta_A, E\right)$, as it satisfies the necessary three axioms of topology and $\left(\zeta_A, E, \tau\right)$ is a *IVNS*-topological space. But the sub-family $\tau_2=\left\{\left(\phi_{\zeta_A}, E\right),\left(\zeta_A, E\right),\left(f_A^1, E\right),\left(f_A^2, E\right)\right\}$ of $\wp\left(\zeta_A, E\right)$ is not an *IVNS*-topology on $\left(\zeta_A, E\right)$, as the union $\left(f_A^4, E\right)=\left(f_A^1, E\right) \cup\left(f_A^2, E\right)$ does not belong to τ_2.

Definition 3.3: As every *IVNS*-topology on $\left(\zeta_A, E\right)$ must contains the sets $\left(\phi_{\zeta_A}, E\right)$ and $\left(\zeta_A, E\right)$, so the family $\vartheta=\left\{\left(\phi_{\zeta_A}, E\right),\left(\zeta_A, E\right)\right\}$ forms a *IVNS*-topology on $\left(\zeta_A, E\right)$. The topology is called indiscrete *IVNS*-topology and the triplet $\left(\zeta_A, E, \vartheta\right)$ is called an indiscrete interval valued neutrosophic soft topological space (or simply indiscrete *IVNS*-topological space).

Definition 3.4: Let ξ denotes the family of all *IVNS*-subsets of $\left(\zeta_A, E\right)$. Then we observe that ξ satisfies all the axioms of topology on $\left(\zeta_A, E\right)$. This topology is called discrete interval valued neutrosophic soft topology and the triplet $\left(\zeta_A, E, \xi\right)$ is called discrete interval valued neutrosophic soft topological space (or simply discrete *IVNS*-topological space).

Theorem 3.5: Let $\left\{\tau_i : i \in I\right\}$ be any collection of *IVNS*-topology on $\left(\zeta_A, E\right)$. Then their intersection $\bigcap_{i \in I} \tau_i$ is also a *IVNS*-topology on $\left(\zeta_A, E\right)$.

Proof: (i) Since $\left(\phi_{\zeta_A}, E\right),\left(\zeta_A, E\right) \in \tau_i$ for each $i \in I$. Hence $\left(\phi_{\zeta_A}, E\right),\left(\zeta_A, E\right) \in \bigcap_{i \in I} \tau_i$.

(ii) Let $\left\{\left(f_A^k, E\right): k \in K\right\}$ be an arbitrary family of interval valued neutrosophic soft sets where $\left(f_A^k, E\right) \in \bigcap_{i \in I} \tau_i$ for each $k \in K$. Then for each $i \in I$, $\left(f_A^k, E\right) \in \tau_i$ for $k \in K$ and since for each $i \in I$, τ_i ia a *IVNS*-topology, therefore $\bigcup_{k \in K}\left(f_A^k, E\right) \in \tau_i$ for each $i \in I$. Hence $\bigcup_{k \in K}\left(f_A^k, E\right) \in \bigcap_{i \in I} \tau_i$.

(iii) Let $\left(f_A^1, E\right),\left(f_A^2, E\right) \in \bigcap_{i \in I} \tau_i$, then $\left(f_A^1, E\right),\left(f_A^2, E\right) \in \tau_i$ for each $i \in I$. Since for each $i \in I$, τ_i is an *IVNS*-topology, therefore $\left(f_A^1, E\right) \cap\left(f_A^2, E\right) \in \tau_i$ for each $i \in I$. Hence $\left(f_A^1, E\right) \cap\left(f_A^2, E\right) \in \bigcap_{i \in I} \tau_i$.

Thus $\bigcap_{i \in I} \tau_i$ satisfies all the axioms of topology. Hence $\bigcap_{i \in I} \tau_i$ forms a *IVNS*-topology. But union of *IVNS*-topologies need not be a *IVNS*-topology. Let us show this with the following example.

Example 3.6: In example 3.2, the sub families $\tau_3=\left\{\left(\phi_{\zeta_A}, E\right),\left(\zeta_A, E\right),\left(f_A^1, E\right)\right\}$ and $\tau_4=\left\{\left(\phi_{\zeta_A}, E\right),\left(\zeta_A, E\right),\left(f_A^2, E\right)\right\}$ are IVNS-topologies in $\left(\zeta_A, E\right)$. But their union $\tau_3 \cup \tau_4=\left\{\left(\phi_{\zeta_A}, E\right),\left(\zeta_A, E\right),\left(f_A^1, E\right),\left(f_A^2, E\right)\right\}$ is not a IVNS-topology in $\left(\zeta_A, E\right)$.

Definition 3.7: Let $\left(\zeta_A, E, \tau\right)$ be an *IVNS*-topological space over $\left(\zeta_A, E\right)$. An interval valued neutrosophic soft subset $\left(f_A, E\right)$ of $\left(\zeta_A, E\right)$ is called interval valued neutrosophic soft closed set (in short *IVNS*-closed set) if its complement $\left(f_A, E\right)^c$ is a member of τ.

Example 3.8: Let us consider example 3.2. then the IVNS-closed sets in $\left(\zeta_A, E, \tau_1\right)$ are

U	e_1	e_2	e_3
u_1	([.2,.7],[.5,.7],[.5,.8])	([.1,.3],[.7,.8],[.4,.7])	([0,.2],[.9,1],[.3,.9])
u_2	([.1,.2],[.6,.7],[.4,.7])	([.1,.2],[.8,.9],[.6,.9])	([0,.5],[.8,.9],[.4,.8])
u_3	([.3,.6],[.9,1],[.5,1])	([.1,.3],[.6,.8],[.6,.8])	([.2,.4],[.7,.9],[.4,.9])

Table7:Tabular representation of $\left(\zeta_A, E\right)^c$

U	e_1	e_2	e_3
u_1	([1,1], [0,0],[0,0])	([1,1], [0,0],[0,0])	([1,1], [0,0],[0,0])
u_2	([1,1], [0,0],[0,0])	([1,1], [0,0],[0,0])	([1,1], [0,0],[0,0])
u_3	([1,1], [0,0],[0,0])	([1,1], [0,0],[0,0])	([1,1], [0,0],[0,0])

Table8:Tabular representation of $\left(\phi_{\zeta_A}, E\right)^c$

U	e_1	e_2	e_3
u_1	([.3,1],[.2,.6],[.1,.7])	([.2,.6],[.4,.6],[.1,.3])	([.4,.9],[.1,.2],[.2,.5])
u_2	([.2,.8],[.3,.4],[.1,.3])	([.4,1],[.2,.5],[0,.5])	([.1,.6],[.1,.4],[0,.3])
u_3	([.6,.9,[.3,.4],[.4,.8])	([.2,.8],[.3,.6],[0,.3])	([.3,.7],[.2,.4],[.1,.3])

Table9:Tabular representation of $\left(f_A^1, E\right)^c$

U	e_1	e_2	e_3
u_1	([.4,.9],[.3,.5],[.4,.7])	([.7,.9],[.5,.6],[.2,.3])	([.1,.2],[.2,.5],[.3,.7])
u_2	([.1,.4],[.2,.6],[.3,.5])	([.2,.5],[.5,.7],[.4,.6])	([.6,.8],[.5,.7],[.1,.3])
u_3	([.6,.7],[.8,.9],[.3,.9])	([.3,.4],[.3,.4],[.5,.7])	([.5,.8],[.5,.7],[.2,.6])

Table10:Tabular representation of $\left(f_A^2, E\right)^c$

U	e_1	e_2	e_3
u_1	([.4,1],[.2,.5],[.1,.7])	([.7,.9],[.4,.6],[.1,.3])	([.4,.9],[.1,.2],[.2,.5])
u_2	([.2,.8],[.2,.4],[.1,.3])	([.4,1],[.2,.5],[0,.5])	([.6,.8],[.1,.4],[0,.3])
u_3	([.6,.9],[.3,.4],[.3,.8])	([.3,.8],[.3,.4],[0,.3])	([.5,.8],[.2,.4],[.1,.3])

Table11:Tabular representation of $\left(f_A^3, E\right)^c$

U	e_1	e_2	e_3
u_1	([.3,.9],[.3,.6],[.4,.7])	([.2,.6],[.5,.6],[.2,.3])	([.1,.2],[.2,.5],[.3,.7])
u_2	([.1,.4],[.3,.6],[.3,.5])	([.2,.5],[.5,.7],[.4,.6])	([.1,.7],[.5,.7],[.1,.3])
u_3	([.6,.7],[.8,.9],[.4,.9])	([.2,.4],[.3,.6],[.5,.7])	([.3,.7],[.5,.7],[.2,.6])

Table12:Tabular representation of $\left(f_A^4, E\right)^c$

are the IVNS-closed sets in $\left(\zeta_A, E, \tau_1\right)$.

Theorem 3.9: Let $\left(\zeta_A, E, \tau\right)$ be an *IVNS*-topological space over $\left(\zeta_A, E\right)$. Then

1. $\left(\phi_{\zeta_A}, E\right)^c$, $\left(\zeta_A, E\right)^c$ are *IVNS*-closed sets.
2. Arbitrary intersection of *IVNS*-closed sets is *IVNS*-closed set.
3. Finite union of IVNS-closed sets is *IVNS*-closed set.

Proof: **1.** Since $\left(\phi_{\zeta_A}, E\right), \left(\zeta_A, E\right) \in \tau$, therefore $\left(\phi_{\zeta_A}, E\right)^c, \left(\zeta_A, E\right)^c$ are *IVNS*-closed sets.

2. Let $\left\{\left(f_A^k, E\right) : k \in K\right\}$ be an arbitrary family of *IVNS*-closed sets in $\left(\zeta_A, E, \tau\right)$ and let $\left(f_A, E\right) = \bigcap_{k \in K} \left(f_A^k, E\right)$.

Now $\left(f_A,E\right)^c=\left(\bigcap_{k\in K}\left(f_A^k,E\right)\right)^c=\bigcup_{k\in K}\left(f_A^k,E\right)^c$ and $\left(f_A^k,E\right)^c\in\tau$ for each $k\in K$, so $\bigcup_{k\in K}\left(f_A^k,E\right)^c\in\tau$. Hence $\left(f_A,E\right)^c\in\tau$. Thus $\left(f_A,E\right)$ is *IVNS*-closed set.

3. Let $\left\{\left(f_A^i,E\right):i=1,2,3,...,n\right\}$ be a family of *IVNS*-closed sets in $\left(\zeta_A,E,\tau\right)$ and let $\left(g_A,E\right)=\bigcup_{i=1}^{n}\left(f_A^i,E\right)$.

Now $\left(g_A,E\right)^c=\left(\bigcup_{i=1}^{n}\left(f_A^i,E\right)\right)^c=\bigcap_{i=1}^{n}\left(f_A^i,E\right)^c$ and $\left(f_A^i,E\right)^c\in\tau$ for $i=1,2,3,...,n$, so $\bigcap_{i=1}^{n}\left(f_A^i,E\right)^c\in\tau$. Hence $\left(g_A,E\right)^c\in\tau$. Thus $\left(g_A,E\right)$ is *IVNS*-closed set.

Definition 3.10: Let $\left(\zeta_A,E,\tau_1\right)$ and $\left(\zeta_A,E,\tau_2\right)$ be two *IVNS*-topological spaces over $\left(\zeta_A,E\right)$. If each $\left(f_A,E\right)\in\tau_2$ implies $\left(f_A,E\right)\in\tau_1$, then τ_1 is called interval valued neutrosophic soft finer topology than τ_2 and τ_2 is called interval valued neutrosophic soft coarser topology than τ_1.

Example 3.11: In example 3.2 and 3.6, τ_1 is interval valued neutrosophic soft finer topology than τ_3 and τ_3 is called interval valued neutrosophic soft coarser topology than τ_1.

Definition 3.12: Let $\left(\zeta_A,E,\tau\right)$ be a *IVNS*-topological space over $\left(\zeta_A,E\right)$ and ß be a subfamily of τ. If every element of τ can be express as the arbitrary interval valued neutrosophic soft union of some elements of ß, then ß is called an interval valued neutrosophic soft basis for the *IVNS*-topology τ.

Example 3.13: In example 3.2, for the *IVNS*-topology $\tau_1=\left\{\left(\phi_{\zeta_A},E\right),\left(\zeta_A,E\right),\left(f_A^1,E\right),\left(f_A^2,E\right),\left(f_A^3,E\right),\left(f_A^4,E\right)\right\}$, the subfamily ß=$\left\{\left(\phi_{\zeta_A},E\right),\left(\zeta_A,E\right),\left(f_A^1,E\right),\left(f_A^2,E\right),\left(f_A^3,E\right)\right\}$ of $\wp\left(\zeta_A,E\right)$ is a interval valued neutrosophic soft basis for the *IVNS*-topology τ_1.

4 Some Properties of Interval Valued Neutrosophic Soft Topological Spaces

In this section some properties of interval valued neutrosophic soft topological spaces are introduced. Some results on IVNSInt and IVNSCl are also intoduced.

Definition 4.1: Let $\left(\zeta_A,E,\tau\right)$ be a *IVNS*-topological space and let $\left(f_A,E\right)\in IVNSS(U;E)$. The interval valued neutrosophic soft interior and closer of $\left(f_A,E\right)$ is denoted by *IVNSInt(fA,E)* and *IVNSCl(fA,E)* are defined as $IVNSInt\left(f_A,E\right)=\bigcup\left\{\left(g_A,E\right)\in\tau:\left(g_A,E\right)\subseteq\left(f_A,E\right)\right\}$ and $INVNSCl\left(f_A,E\right)=$
$\bigcap\left\{\left(g_A,E\right)\in\tau^c:\left(f_A,E\right)\subseteq\left(g_A,E\right)\right\}$ respectively.

Example 4.2: Let us consider example 3.2 and take an IVNSS $\left(f_A^5,E\right)$ as

U	e_1	e_2	e_3
u_1	([.2,.8],[.3,.6],[.2,.8])	([.2,.4],[.4,.6],[.2,.4])	([.2,.6],[.7,.8],[.3,.4])
u_2	([.1,.6],[.4,.5],[.2,.7])	([.2,.6],[.5,.7],[.1,.7])	([.1,.4],[.2,.5],[.1,.5])
u_3	([.5,.8],[.5,.6],[.5,.8])	([.1,.4],[.4,.6],[.1,.5])	([.2,.5],[.5,.8],[.2,.4])

Table13:Tabular representation of $\left(f_A^5,E\right)$

Now $IVNSInt\left(f_A^5,E\right)=\left(f_A^1,E\right)$ and $IVNSCl\left(f_A^5,E\right)=\left(f_A^1,E\right)^c$.

Theorem 4.3: Let $\left(\zeta_A,E,\tau\right)$ be a *IVNS*-topological space and $\left(f_A,E\right)$, $\left(g_A,E\right)\in IVNSS\left(U;E\right)$ then the following properties hold

1. $IVNSInt\left(f_A,E\right)\subseteq\left(f_A,E\right)$
2. $\left(f_A,E\right)\subseteq\left(g_A,E\right)\Rightarrow IVNSInt\left(f_A,E\right)\subseteq IVNSInt\left(g_A,E\right)$
3. $IVNSInt\left(f_A,E\right)\in\tau$
4. $\left(f_A,E\right)\in\tau\Leftrightarrow IVNSInt\left(f_A,E\right)=\left(f_A,E\right)$
5. $IVNSInt\left(IVNSInt\left(f_A,E\right)\right)=IVNSInt\left(f_A,E\right)$
6. $IVNSInt\left(\phi_A,E\right)=\phi_A$, $IVNSInt\left(U_A,E\right)=U_A$

Proof:

1. Straight forward.

2. $\left(f_A,E\right)\subseteq\left(g_A,E\right)$ implies all the *IVNS*-open sets contained in $\left(f_A,E\right)$ also contained in $\left(g_A,E\right)$.

i.e.

$$\left\{\left(f_A^*,E\right)\in\tau:\left(f_A^*,E\right)\subseteq\left(f_A,E\right)\right\}\subseteq\left\{\left(g_A^*,E\right)\in\tau:\left(g_A^*,E\right)\subseteq\left(g_A,E\right)\right\}$$

i.e.
$\cup\left\{\left(f_A^*,E\right)\in\tau:\left(f_A^*,E\right)\subseteq\left(f_A,E\right)\right\}\subseteq\cup\left\{\left(g_A^*,E\right)\in\tau:\left(g_A^*,E\right)\subseteq\left(g_A,E\right)\right\}$

i.e. $IVNSInt\left(f_A,E\right)\subseteq IVNSInt\left(g_A,E\right)$

3. $IVNSInt\left(f_A,E\right)=\cup\left\{\left(f_A^*,E\right)\in\tau:\left(f_A^*,E\right)\subseteq\left(f_A,E\right)\right\}$

It is clear that $\cup\left\{\left(f_A^*,E\right)\in\tau:\left(f_A^*,E\right)\subseteq\left(f_A,E\right)\right\}\in\tau$

So, $IVNSInt\left(f_A,E\right)\in\tau$.

4. Let $\left(f_A,E\right)\in\tau$, then by (1) $IVNSInt\left(f_A,E\right)\subseteq\left(f_A,E\right)$.

Now since $\left(f_A,E\right)\in\tau$ and $\left(f_A,E\right)\subseteq\left(f_A,E\right)$, Therefore
$\left(f_A,E\right)\subseteq\cup\left\{\left(g_A^*,E\right)\in\tau:\left(g_A^*,E\right)\subseteq\left(g_A,E\right)\right\}=IVNSInt\left(f_A,E\right)$

i.e, $\left(f_A,E\right)\subseteq IVNSInt\left(f_A,E\right)$

Thus $IVNSInt\left(f_A,E\right)=\left(f_A,E\right)$

Conversly, let $IVNSInt\left(f_A,E\right)=\left(f_A,E\right)$

Since by (3) $IVNSInt\left(f_A,E\right)\in\tau$

Therefore $\left(f_A,E\right)\in\tau$

5. By (3) $IVNSInt\left(f_A,E\right)\in\tau$

$\therefore$ By (4) $IVNSInt\left(IVNSInt\left(f_A,E\right)\right)=IVNSInt\left(f_A,E\right)$.

6. We know that $\left(\phi_A,E\right),\left(U_A,E\right)\in\tau$

$\therefore$ By (4) $IVNSInt\left(\phi_A,E\right)=\phi_A$, $IVNSInt\left(U_A,E\right)=U_A$

Theorem 4.4: Let $\left(\zeta_A,E,\tau\right)$ be a *IVNS*-topological space and $\left(f_A,E\right)$, $\left(g_A,E\right)\in IVNSs\left(U;E\right)$ then the following properties hold

1. $\left(f_A,E\right)\subseteq IVNSCl\left(f_A,E\right)$
2. $\left(f_A,E\right)\subseteq\left(g_A,E\right)\Rightarrow IVNSCl\left(f_A,E\right)\subseteq IVNSCl\left(g_A,E\right)$
3. $\left(IVNSCl\left(f_A,E\right)\right)^c\in\tau$
4. $\left(f_A,E\right)^c\in\tau\Leftrightarrow IVNSCl\left(f_A,E\right)=\left(f_A,E\right)$
5. $IVNSCl\left(IVNSCl\left(f_A,E\right)\right)=IVNSCl\left(f_A,E\right)$
6. $IVNSCl\left(\phi_A,E\right)=\phi_A$, $IVNSCl\left(U_A,E\right)=U_A$

Proof: straight forward.

Theorem 4.5: Let $\left(\zeta_A,E,\tau\right)$ be an *IVNS*-topological space on $\left(\zeta_A,E\right)$ and let $\left(f_A,E\right),\left(g_A,E\right)\in IVNSs\left(U;E\right)$. Then the following properties hold

1. $IVNSInt\left(\left(f_A,E\right)\cap\left(g_A,E\right)\right)=IVNSInt\left(f_A,E\right)\cap IVNSInt\left(g_A,E\right)$
2. $IVNSInt\left(\left(f_A,E\right)\cup\left(g_A,E\right)\right)\supseteq IVNSInt\left(f_A,E\right)\cup IVNSInt\left(g_A,E\right)$
3. $IVNSCl\left(\left(f_A,E\right)\cup\left(g_A,E\right)\right)=IVNSCl\left(f_A,E\right)\cup IVNSCl\left(g_A,E\right)$
4. $IVNSCl\left(\left(f_A,E\right)\cap\left(g_A,E\right)\right)\subseteq IVNSCl\left(f_A,E\right)\cap IVNSCl\left(g_A,E\right)$
5. $\left(IVNSInt\left(f_A,E\right)\right)^c=IVNSCl\left(f_A,E\right)^c$
6. $\left(IVNSCl\left(f_A,E\right)\right)^c=IVNSInt\left(f_A,E\right)^c$

Proof:

1. By theorem 4.2 (1), $IVNSInt\left(f_A,E\right)\subseteq\left(f_A,E\right)$ and $IVNSInt\left(g_A,E\right)\subseteq\left(g_A,E\right)$. Thus $IVNSInt\left(f_A,E\right)\cap IVNSInt\left(g_A,E\right)\subseteq\left(f_A,E\right)\cap\left(g_A,E\right)$.

Hence
$IVNSInt\left(f_A,E\right)\cap IVNSInt\left(g_A,E\right)\subseteq IVNSInt\left(\left(f_A,E\right)\cap\left(g_A,E\right)\right)$ …………… (i)

Again since $\left(f_A,E\right)\cap\left(g_A,E\right)\subseteq\left(f_A,E\right)$. By theorem 4.2 (2), $IVNSInt\left(\left(f_A,E\right)\cap\left(g_A,E\right)\right)\subseteq IVNSInt\left(f_A,E\right)$.

Similarly
$IVNSInt\left(\left(f_A,E\right)\cap\left(g_A,E\right)\right)\subseteq IVNSInt\left(g_A,E\right)$

Hence
$IVNSInt\left(\left(f_A,E\right)\cap\left(g_A,E\right)\right)\subseteq IVNSInt\left(f_A,E\right)\cap IVNSInt\left(g_A,E\right)$ … ………….. (ii)

Using (i) and (ii) we get, $IVNSInt\left(\left(f_A,E\right)\cap\left(g_A,E\right)\right)=IVNSInt\left(f_A,E\right)\cap IVNSInt\left(g_A,E\right)$.

2. Since $\left(f_A,E\right)\subseteq\left(f_A,E\right)\cup\left(g_A,E\right)$.

By theorem 4.2 (2), $IVNSInt\left(f_A,E\right)\subseteq IVNSInt\left(\left(f_A,E\right)\cup\left(g_A,E\right)\right)$.

Similarly, $IVNSInt\left(g_A,E\right)\subseteq IVNSInt\left(\left(f_A,E\right)\cup\left(g_A,E\right)\right)$.

Hence
$IVNSInt\left(\left(f_A,E\right)\cup\left(g_A,E\right)\right)\supseteq IVNSInt\left(f_A,E\right)\cup IVNSInt\left(g_A,E\right)$.

3. Similar to 1.

4. Similar to 2.

5. $\left(IVNSInt\left(f_A,E\right)\right)^c=\left(\cup\left\{\left(g_A,E\right)\in\tau:\left(g_A,E\right)\subseteq\left(f_A,E\right)\right\}\right)^c$

$=\cap\left\{\left(g_A,E\right)\in\tau^c:\left(f_A,E\right)^c\subseteq\left(g_A,E\right)\right\}$

$=IVNSCl\left(f_A,E\right)^c$

6. Similar to 5.

Equality does not hold in theorem 4.4 (2), (4). Let us show this by an example.

Example 4.6: Let $U=\{u_1,u_2\}$, $E=\{e_1,e_2,e_3\}$, $A=\{e_1,e_2\}$. The tabular representation of (ζ_A,E) is given by

U	e_1	e_2
u_1	([.5,.8],[.3,.5],[.2,.7])	([.3,.9],[.1,.2],[0,.1])
u_2	([.4,.6],[.3,.4],[.1,.2])	([.4,.8],[.1,.3],[.1,.2])

Table14:Tabular representation of (ζ_A,E)

The tabular representation of (ϕ_{ζ_A},E) is given by

U	e_1	e_2
u_1	([0,0], [1,1], [1,1])	([0,0], [1,1], [1,1])
u_2	([0,0], [1,1], [1,1])	([0,0], [1,1], [1,1])

Table15:Tabular representation of (ϕ_{ζ_A},E)

The tabular representation of (f_A,E) is given by

U	e_1	e_2
u_1	([.1,.7],[.4,.8],[.3,1])	([.2,.5],[.7,.9],[.3,.7])
u_2	([.1,.2],[.6,.7],[.2,.7])	([0,.3],[.5,.8],[.4,1])

Table16:Tabular representation of (f_A,E)

Clearly $\tau=\{(\phi_{\zeta_A},E),(\zeta_A,E),(f_A,E)\}$ is a IVNS-topology on (ζ_A,E) . Let us now take two interval valued neutrosophic soft sets (g_A,E) and (h_A,E) as

U	e_1	e_2
u_1	([.1,.6],[.4,.9],[.4,1])	([.1,.5],[.7,.9],[.3,.8])
u_2	([.1,.2],[.6,.7],[.2,.8])	([0,.2],[.5,.9],[.4,1])

Table17:Tabular representation of (g_A,E)

U	e_1	e_2
u_1	([0,.7],[.5,.8],[.3,1])	([.2,.5],[.8,1],[.6,.7])
u_2	([.1,.2],[.6,.8],[.3,.7])	([0,.3],[.6,.8],[.5,1])

Table18:Tabular representation of (h_A,E)

Now $(g_A,E)\cup(h_A,E)=(f_A,E)$

$\therefore$

$IVNSInt((g_A,E)\cup(h_A,E))=IVNSInt(f_A,E)=(f_A,E)$

Also $IVNSInt(g_A,E)=(\phi_{\zeta_A},E)$, $IVNSInt(h_A,E)=(\phi_{\zeta_A},E)$

$\therefore$

$IVNSInt(g_A,E)\cup IVNSInt(h_A,E)=(\phi_{\zeta_A},E)\cup(\phi_{\zeta_A},E)=(\phi_{\zeta_A},E)$

Thus

$IVNSInt((f_A,E)\cup(g_A,E))\neq IVNSInt(f_A,E)\cup IVNSInt(g_A,E)$.

Therefore equality does not hold for (2).

By theorem 4.4 (5),

$IVNSCl(g_A,E)^c=(IVNScl(g_A,E))^c=(\phi_{\zeta_A},E)^c=(\zeta_A,E)$.

Similarly $IVNSCl(h_A,E)^c=(\zeta_A,E)$.

Therefore

$IVNSCl(g_A,E)^c\cap IVNSCl(h_A,E)^c=(\zeta_A,E)\cap(\zeta_A,E)=(\zeta_A,E)$

. Also

$$\begin{aligned}IVNSCl((g_A,E)^c\cap(h_A,E)^c)&=IVNSCl((g_A,E)\cup(h_A,E))^c\\&=(IVNSInt((g_A,E)\cup(h_A,E)))^c\\&=(IVNSInt(f_A,E))^c\\&=(f_A,E)^c\end{aligned}$$

Thus

$IVNSCl((f_A,E)\cap(g_A,E))\neq IVNSCl(f_A,E)\cap IVNSCl(g_A,E)$

. Therefore equality doesnot hold in (4).

5 Interval Valued Neutrosophic Soft Subspace Topology

In this section we introduce the concept of interval valued neutrosophic soft subspace topology along with some examples and results.

Theorem 5.1: Let (ζ_A,E,τ) be an *IVNS*-topological space on (ζ_A,E) and $(f_A,E)\in\wp(\zeta_A,E)$. Then the collection $\tau_{(f_A,E)}=\{(f_A,E)\cap(g_A,E):(g_A,E)\in\tau\}$ is an *IVNS*-topology on (ζ_A,E) .

Proof:

(i) Since $(\phi_{\zeta_A},E),(\zeta_A,E)\in\tau$, therefore $(f_A,E)\cap(\phi_{\zeta_A},E)=(\phi_{f_A},E)\in\tau_{(f_A,E)}$ and $(f_A,E)\cap(\zeta_A,E)=(f_A,E)\in\tau_{(f_A,E)}$.

(ii) Let $(f_A^k,E)\in\tau_{(f_A,E)},\forall k\in K$ .Then $(f_A^k,E)=(f_A,E)\cap(g_A^k,E)$ where $(g_A^k,E)\in\tau$ for each $k\in K$.

Now

$\bigcup_{k\in K}(f_A^k,E)=\bigcup_{k\in K}((f_A,E)\cap(g_A^k,E))=(f_A,E)\cap\left(\bigcup_{k\in K}(g_A^k,E)\right)\in\tau_{(f_A,E)}$

(since $\bigcup_{k\in K}(g_A^k,E)\in\tau$ as each $(g_A^k,E)\in\tau$.

(iii) Let $(f_A^1,E),(f_A^2,E)\in\tau_{(f_A,E)}$ then $(f_A^1,E)=(f_A,E)\cap(g_A^1,E)$ and $(f_A^2,E)=(f_A,E)\cap(g_A^2,E)$ where $(g_A^1,E),(g_A^2,E)\in\tau$.

Now

$$\left(f_A^1,E\right)\cap\left(f_A^2,E\right)=\left(\left(f_A,E\right)\cap\left(g_A^1,E\right)\right)\cap\left(\left(f_A,E\right)\cap\left(g_A^2,E\right)\right)$$
$$=\left(f_A,E\right)\cap\left(\left(g_A^1,E\right)\cap\left(g_A^2,E\right)\right)\in\tau_{(f_A,E)}$$

(since $\left(g_A^1,E\right)\cap\left(g_A^2,E\right)\in\tau$ as $\left(g_A^1,E\right),\left(g_A^2,E\right)\in\tau$).

Definition 5.2: Let (ζ_A,E,τ) be an *IVNS*-topological space on (ζ_A,E) and $(f_A,E)\in\wp(\zeta_A,E)$. Then the IVNS-topology $\tau_{(f_A,E)}=\{(f_A,E)\cap(g_A,E):(g_A,E)\in\tau\}$ is called interval valued neutrosophic soft subspace topology and $\left(f_A,E,\tau_{(f_A,E)}\right)$ is called interval valued neutrosophic soft subspace of (ζ_A,E,τ) .

Example 5.3: Let us consider the IVNS-topology $\tau_1=\left\{\left(\phi_{\zeta_A},E\right),(\zeta_A,E),\left(f_A^1,E\right),\left(f_A^2,E\right),\left(f_A^3,E\right),\left(f_A^4,E\right)\right\}$ as in example 3.2 and an *IVNSS* (f_A,E):

U	e_1	e_2	e_3
u_1	([.4,.6],[.6,.7],[.3,.5])	([.5,.7],[.4,.6],[0,.3])	([.3,.5],[.5,.8],[.2,.3])
u_2	([.2,.3],[.3,.6],[.5,.7])	([.6,.8],[.4,.5],[.2,.3])	([.5,.8],[.5,.7],[.2,.3])
u_3	([.5,.7],[.4,.6],[.3,.4])	([.4,.5],[.7,.9],[.6,.7])	([.1,.3],[.7,.9],[.5,.7])

Table19:Tabular representation of $\left(f_A^1,E\right)$

Then $\left(\phi_{f_A},E\right)=(f_A,E)\cap\left(\phi_{\zeta_A},E\right)$:

U	e_1	e_2	e_3
u_1	([0,0],[1,1],[1,1])	([0,0],[1,1],[1,1])	([0,0],[1,1],[1,1])
u_2	([0,0],[1,1],[1,1])	([0,0],[1,1],[1,1])	([0,0],[1,1],[1,1])
u_3	([0,0],[1,1],[1,1])	([0,0],[1,1],[1,1])	([0,0],[1,1],[1,1])

Table20:Tabular representation of $\left(\phi_{f_A},E\right)$

$\left(g_A^1,E\right)=(f_A,E)\cap\left(f_A^1,E\right)$:

U	e_1	e_2	e_3
u_1	([.1,.6],[.6,.7],[.3,1])	([.1,.3],[.4,.6],[.2,.6])	([.2,.5],[.5,.8],[.4,.9])
u_2	([.1,.3],[.6,.7],[.5,.8])	([0,.5],[.4,.5],[.4,1])	([0,.3],[.6,.9],[.2,.7])
u_3	([.4,.7],[.4,.6],[.6,.9])	([0,.3],[.7,.9],[.6,.8])	([.1,.3],[.7,.9],[.5,.7])

Table21:Tabular representation of $\left(g_A^1,E\right)$

$\left(g_A^2,E\right)=(f_A,E)\cap\left(f_A^2,E\right)$:

U	e_1	e_2	e_3
u_1	([.4,.6],[.6,.7],[.4,.9])	([.2,.3],[.4,.6],[.7,.9])	([.3,.5],[.5,.8],[.2,.3])
u_2	([.2,.3],[.4,.8],[.5,.7])	([.4,.6],[.4,.5],[.2,.5])	([.1,.3],[.5,.7],[.6,.8])
u_3	([.3,.7],[.4,.6],[.6,.7])	([.4,.5],[.7,.9],[.6,.7])	([.1,.3],[.7,.9],[.3,.8])

Table22:Tabular representation of $\left(g_A^2,E\right)$

$\left(g_A^3,E\right)=(f_A,E)\cap\left(f_A^3,E\right)$:

U	e_1	e_2	e_3
u_1	([.1,.6],[.6,.8],[.4,1])	([.1,.3],[.4,.6],[.7,.9])	([.2,.5],[.5,.8],[.4,.9])
u_2	([.1,.3],[.6,.8],[.5,.8])	([0,.5],[.4,.5],[.4,1])	([0,.3],[.6,.9],[.6,.8])
u_3	([.3,.7],[.4,.6],[.6,.9])	([0,.3],[.7,.9],[.6,.8])	([.1,.3],[.7,.9],[.5,.8])

Table23:Tabular representation of $\left(g_A^3,E\right)$

$\left(g_A^4,E\right)=(f_A,E)\cap\left(f_A^4,E\right)$:

U	e_1	e_2	e_3
u_1	([.2,.5],[.5,.8],[.4,.9])	([.2,.5],[.5,.8],[.4,.9])	([.3,.5],[.5,.8],[.2,.3])
u_2	([0,.3],[.6,.9],[.6,.8])	([0,.3],[.6,.9],[.6,.8])	([.1,.3],[.5,.7],[.2,.7])
u_3	([.1,.3],[.7,.9],[.5,.8])	([.1,.3],[.7,.9],[.5,.8])	([.1,.3],[.7,.9],[.5,.7])

Table24:Tabular representation of $\left(g_A^4,E\right)$

Then $\tau_{(f_A,E)}=\left\{\left(\phi_{f_A},E\right),(f_A,E),\left(g_A^1,E\right),\left(g_A^2,E\right),\left(g_A^4,E\right)\right\}$ is an interval valued neutrosophic soft subspace topology for τ_1 and $\left(f_A, E, \tau_{(f_A,E)}\right)$ is called interval valued neutrosophic soft subspace of $\left(\zeta_A, E, \tau_1\right)$.

Theorem 5.4: Let (ζ_A, E, τ) be an *IVNS*-topological space on (ζ_A, E), ß be an *IVNS*-basis for τ and $(f_A, E) \in \wp(\zeta_A, E)$. Then the family $ß_{(f_A,E)} = \left\{(f_A, E) \cap (g_A, E) : (g_A, E) \in ß\right\}$ is an *IVNS*-basis for subspace topology $\tau_{(f_A,E)}$.

Proof: Let $(h_A, E) \in \tau_{(f_A,E)}$ be arbitrary, then there exists an *IVNSS* $(g_A, E) \in \tau$ such that $(h_A, E) = (f_A, E) \cap (g_A, E)$. Since ß is a basis for τ, therefore there exists a sub collection $\left\{\left(\chi_A^i, E\right) : i \in I\right\}$ of ß such that $(g_A, E) = \bigcup_{i \in I}\left(\chi_A^i, E\right)$.

Now $(h_A, E) = (f_A, E) \cap (g_A, E) = \bigcup_{i \in I}\left(\chi_A^i, E\right) = \bigcup_{i \in I}\left((f_A, E) \cap \left(\chi_A^i, E\right)\right)$. Since $(f_A, E) \cap \left(\chi_A^i, E\right) \in ß_{(f_A,E)}$, therefore $ß_{(f_A,E)}$ is an *IVNS*-basis for the subspace topology $\tau_{(f_A,E)}$.

Conclusion

In this paper we introduce the concept of interval valued neutrosophic soft topology. Some basic theorem and properties of the above concept are also studied. IVN interior and IVN closer of an interval valued neutrosophic soft set are also defined. Interval valued neutrosophic soft subspace topology is also studied.

In future there will be more research work in this concept, taking the basic definitions and results from this article**.**

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Multi-criteria Group Decision Making Approach for Teacher Recruitment in Higher Education under Simplified Neutrosophic Environment

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Abstract

Teacher recruitment is a multi-criteria group decision-making process involving subjectivity, imprecision, and fuzziness that can be suitably represented by neutrosophic sets. Neutrosophic set, a generalization of fuzzy sets is characterized by a truth-membership function, falsity-membership function and an indeterminacy-membership function. These functions are real standard or non-standard subsets of] 0^-, 1^+[.There is no restriction on the sum of the functions, so the sum lies between]0^-, 3^+[. A neutrosophic approach is a more general and suitable way to deal with imprecise information, when compared to a fuzzy set. The purpose of this study is to develop a neutrosophic multi-criteria group decision-making model based on hybrid score-accuracy functions for teacher recruitment in higher education. Eight criteria obtained from expert opinions are considered for recruitment process. The criteria are namely academic performance index, teaching aptitude, subject knowledge, research experience, leadership quality, personality, management capacity, and personal values. In this paper we use the score and accuracy functions and the hybrid score-accuracy functions of single valued neutrosophic numbers (SVNNs) and ranking method for SVNNs. Then, multi-criteria group decision-making method with unknown weights for attributes and incompletely known weights for decision makers is used based on the hybrid score-accuracy functions under single valued neutrosophic environments. We use weight model for attributes based on the hybrid score-accuracy functions to derive the weights of decision makers and attributes from the decision matrices represented by the form of SVNNs to decrease the effect of some unreasonable evaluations. Moreover, we use the overall evaluation formulae of the weighted hybrid score-accuracy functions for each alternative to rank the alternatives and recruit the most desirable teachers. Finally, an educational problem for teacher selection is provided to illustrate the effectiveness of the proposed model.

Keywords: Multi-criteria group decision- making, Hybrid score-accuracy function, Neutrosophic numbers (SVNNs), and Single valued Neutrosophic set, Teacher recruitment

Introduction

Teacher recruitment problem can be considered as a multi-criteria group decision-making (MCGDM) problem that generally consists of selecting the most desirable alternative from all the feasible alternatives. Classical MCGDM approaches [1,2,3] deal with crisp numbers i.e. the ratings and the weights of criteria are measured by crisp numbers. However, it is not always possible to present the information by crisp numbers. In order to deal this sutuation fuzzy sets introduced by Zadeh in 1965 [4] can be used. Atanassov [5]extended the concept of fuzzy sets to intuitionistic fuzzy sets(IFSs) in 1986. Fuzzy and intuitionistic MCGDM appraoches [6,7] were studied with fuzzy or intuitionistic fuzzy numbers i.e. the ratings and the weights are expressed by linguistic variables characterized by fuzzy or intuitionistic fuzzy numbers.

Teacher recruitment process for higher education can be considered as a spcial case of personnel selection. The traditional methods for recruiting teahers generally involve subjective judgment of experts, which make the accuracy of the results highly questionable. In order to tackle the problem, new methodology is urgently needed. Liang and Wang [8] studied fuzzy multi-criteria decision making (MCDM) algorithm for personnel selection. Karsak [9] presented fuzzy MCDM approach based on ideal and anti-ideal solutions for the selection of the most suitable candidate. Günör et al.[10] developed analytical hierarchy process (AHP) for personnel selection. Dağdeviren [11] studied a hybrid model based on analytical network process (ANP) and modified technique for order preference by similarity to ideal solution (TOPSIS)[12] for supporting the personnel selection process in the manufacturing systems. Dursun and Karsak [13] discussed fuzzy MCDM approach by using TOPSIS with 2-tuples for personnel selection. Personnel selection studies were well reviewed by Robertson and Smith [14]. In their studies, Robertson and Smith [14] investigated the role of job analysis, contemporary models of work performance, and set of criteria employed in personnel selection process. Ehrgott and Gandibleux [15] presented a comprehensive survey of the state of the art in MCDM. Pramanik and Mukhopadhay [16] presented a intuitionistic fuzzy MCDM approach for teacher selection based grey relational analysis.

Though fuzzy and intuitionistic fuzzy MCDM problems are widely studied, but indeterminacy should be incorportated in the model formulation of the problems. Inderterminacy plays an important role in decision making process. So neutrosophic set [17] generalization of intuitionistic fuzzy sets should be incorporated in the decision making process. Neutrosophic set was introduced to represent mathematical model of uncertainty, imprecision, and inconsistency. Biswas et al. [18] presented entropy based grey relational analysis method for multi-attribute decision-making under single valued neutrosopic assesment. Biswas et al.[19] also studied a new methodology to deal neutrosophic multi-attribute decision-making problem. Ye [20] proposed the correlation coefficient of SVNSs for single valued neutrosophic multi-critera decision-making problems.

The ranking order of alternatives plays an important role in decision-making process. In this study, we present a multi-criteria group decision-making approach for teacher recritment in higher education with unknown weights based on score and accuracy functions, hybrid score-accuracy functions proposed by J. Ye [21] under simplified neutrosophic environment.

Rest of the paper is organized in the following way. Section II presents preliminaries of neutrosophic sets and Section III presents operational definitions. Section IV presents methodology based on hybrid score-accuracy functions Section V is devoted to present an example of teacher selection in higher education based on hybrid score-accuracy functions . Section VI presents conclusion, finally, section VII presents the concluding remarks.

Section II

Mathematical preliminaries on Neutrosophic set

Some basic concepts of SNSs:

The neutrosophic set is a part of neutrosophy and generalizes fuzzy set, IFS, and IVIFS from philosophical point of view [22].

Definition1. Neutrosophic set [22]

Let X be a space of points (objects), with a generic element in X denoted by x. A neutrosophic set A in X is characterized by a truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$, and a falsity-membership function $F_A(x)$. The functions $T_A(x)$, $I_A(x)$ and $F_A(x)$ are real standard or nonstandard subsets of $]0, 1[$, i.e.,$T_A(x)$: $X \to]0^-, 1^+[$,
$I_A(x)$: $X \to]0^-, 1^+[$, and $F_A(x)$: $X \to]0^-, 1^+[$. Hence, there is no restriction on the sum of $T_A(x)$, $I_A(x)$ and $F_A(x)$ and $0^- \leq \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \leq 3^+$.

Definition 2. Single valued neutrosophic sets [23].

Let X be a space of points (objects), with a generic element in X denoted by x. A neutrosophic set A in X is characterized by a truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$ and a falsity-membership function $F_A(x)$. If the functions $T_A(x)$, $I_A(x)$ and $F_A(x)$ are singleton subintervals/subsets in the real standard [0, 1], that is$T_A(x)$: $X \to [0, 1]$, $I_A(x)$: $X \to [0, 1]$, and $F_A(x)$: $X \to [0, 1]$. Then, a simplification of the neutrosophic set A is denoted by $A = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle / x \in X\}$ which is called a SNS. It is a subclass of a neutrosophic set and includes SVNS and INS. In this paper, we shall use the SNS whose values of the functions $T_A(x)$, $I_A(x)$ and $F_A(x)$ can be described by three real numbers (i.e. a SVNS) in the real standard [0, 1].

Definition 3. Single valued neutrosophic number (SNN) [21]

Let X be a universal set. A SVNS A in X is characterized by a truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$, and a falsity-membership function $F_A(x)$. Then, a SVNS A can be denoted by the following symbol:
$A = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle / x \in X\}$, where $T_A(x)$, $I_A(x)$, $F_A(x) \in [0, 1]$ for each point x in X. Therefore, the sum of $T_A(x)$, $I_A(x)$ and $F_A(x)$ satisfies the condition $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$. For a SVNS A in X, the triple $\langle T_A(x), I_A(x), F_A(x)\rangle$ is called single valued neutrosophic number (SVNN), which is the fundamental element of a SVNS.

Definition 4. Complement of SVNS [21]

The complement of a SVNS A is denoted by A^c and defined as $T_A^c(x) = F_A(x)$, $I_A^c(x) = 1 - I_A(x)$,
$F_A^c(x) = T_A(x)$ for any x in X. Then, it can be denoted by the following form:

$$A^c = \{\langle x, F_A(x), 1 - I_A(x), T_A(x)\rangle / x \in X\}$$

For two SVNSs A and B in X, two of their relations are defined as follows: A SVNS A is contained in the other SVNS B, A ⊆ B, if and only if $T_A(x) \le T_B(x)$, $I_A(x) \ge I_B(x)$, $F_A(x) \ge F_B(x)$ for any x in X.

Two SVNSs A and B are equal, written as A= B, if and only if A ⊆ Band B ⊆ A.

Ranking methods for SVNNs

In this subsection, we define the score function, accuracy function, and hybrid score-accuracy function of a SVNN, and the ranking method for SVNNs.

Definition 5 Score function and accuracy function [21]

Let a= $\langle T(a), I(a), F(a) \rangle$ be a SVNN. Then, the score function and accuracy function of the SVNN can be presented, respectively, as follows:

$s(a) = (1 + T(a) - F(a))/2$ for $s(a) \in [0, 1]$ (1)

$h(a) = (2 + T(a) - F(a) - I(a))/3$ for $h(a) \in [0, 1]$ (2)

For the score function of a SVNN a, if the truth-membership T(a) is bigger and the falsity-membership F(a) are smaller, then the score value of the SVNN a is greater. For the accuracy function of a SVNN a, if the sum of T(a), 1−I(a) and 1−F(a) is bigger, then the statement is more affirmative, i.e., the accuracy of the SVNN a is higher. Based on score and accuracy functions for SVNNs, two theorems are stated below.

Theorem 1.

For any two SVNNs a_1 and a_2, if $a_1 > a_2$, then $s(a_1) > s(a_2)$.

Theorem 2.

For any two SVNNs a_1 and a_2, if $s(a_1) = s(a_2)$ and $a_1 \ge a2$, then $h(a_1) \ge h(a_2)$.

For proof, see [21]

Based on theorems 1 and 2, a ranking method between SVNNs can be given by the following definition.

Definition [21]

Let a_1 and a_2 be two SVNNs. Then, the ranking method can be defined as follows:

(1) If $s(a_1) > s(a_2)$, then $a_1 > a_2$;
(2) If $s(a_1) = s(a_2)$ and $h(a_1) \ge h(a_2)$, then $a_1 \ge a_2$;

Section III

Operational definitions of the terms stated in the problem

i) Academic performance: Academic performance implies the percentage of marks (if grades are given, transform it into marks) obtained in post graduate examinations.

ii) Teaching aptitude: Degree of knowledge in strategies of instruction and information communication technology (ICT).

iii) Subject knowledge: Degree of knowledge of a person in his/her respective field of study to be delivered during his/her instruction.

iv) Research experience: Research experience of a person implies his or her contribution of new knowledge in the form of publication in reputed peer reviewed journals with ISSN.

v) Leadership quality: Leadership quality of a person implies the ability a) to challenge status quo b) to implement rational decision

vi) Personality: Defining and explaining personality are of prime importance while recruiting teachers. But how do psychologists measure and study personality? Four distinct methods are most common, namely behavioral observation, interviewing, projective tests, and questionnaires. McCrae & Costa [24] studied five-factor model of personality. Five factors of personality are extraversion versus introversion, agreeableness versus antagonism, conscientiousness versus undirectedness, neuroticism versus emotional stability, and openness versus not openness. In this study personality implies the five factors of personality traits of five factor model.

vii) Management capacity: Management capacity of a person implies his/her ability to manage in the actual teaching learning process.

viii) Values: Values will implicitly refer to personal values that serve as guiding principles about how individuals ought to behave.

Section IV

Multi-criteria group decision-making methods based on hybrid score-accuracy functions

In a multi-criteria group decision-making problem, let A= $\{A_1, A_2, \ldots, A_m\}$ be a set of alternatives and let C= $\{C_1, C_2, \ldots, C_n\}$ be a set of attributes. Then, the weights of decision makers and attributes are not assigned previously, where the information about the weights of the decision makers is completely unknown and the information about the weights of the attributes is incompletely known in the group decision-making problem. In such a case, we develop two methods based on the hybrid score-accuracy functions for multiple attribute group decision-making problems with unknown weights under single valued neutrosophic and interval neutrosophic environments.

Multi-criteria group decision-making method in single valued neutrosophic setting

In the group decision process under single valued neutrosophic environment, if a group of t decision makers or experts is required in the evaluation process, then the kth decision maker can provide the evaluation information of the alternative A_i (i= 1, 2, …, m) on the attribute C_j (j= 1, 2, …, n), which is represented by the form of a SVNS:

$$A_i^k = \left\{ \left\langle C_j, T_{A_i}^k(C_j), I_{A_i}^k(C_j), F_{A_i}^k(C_j) \right\rangle / C_j \in C \right\}$$

Here, $0 \le T_{A_i}^k(C_j) + I_{A_i}^k(C_j) + F_{A_i}^k(C_j) \le 3$,

$T_{A_i}^k(C_j) \in [0,1]$, $I_{A_i}^k(C_j) \in [0,1]$, $F_{A_i}^k(C_j) \in [0,1]$,

for k = 1, 2, …., t, j=1, 2, …., n, i=1, 2, ….,m

For convenience, $a_{ij}^k = \left\langle T_{ij}^k, I_{ij}^k, F_{ij}^k \right\rangle$ is denoted as a SVNN in the SVNS. A_i^k (k= 1, 2, …t; i= 1, 2, …, m; j= 1, 2, …, n). Therefore, we can get the k-th single valued neutrosophic decision matrix $D^k = (A_{ij}^k)_{m\times n}$ (k= 1, 2, …, t). Then, the group decision-making method is described as follows.

Step1:

Calculate hybrid score-accuracy matrix

The hybrid score-accuracy matrix $Y^k = (Y_{ij}^k)_{m\times n}$ (k= 1, 2, …, t; i= 1, 2, …, m; j= 1, 2, …, n) is obtained from the decision matrix $D^k = (A_{ij}^k)_{m\times n}$ by the following formula:

$$Y_{ij}^k = \frac{1}{2}\alpha(1 + T_{ij}^k - F_{ij}^k) + \frac{1}{3}(1-\alpha)(2 + T_{ij}^k - I_{ij}^k - F_{ij}^k) \quad (3)$$

Step2:

Calculatethe average matrix

From the obtained hybrid score-accuracy matrices, the average matrix $Y^* = (Y_{ij}^*)_{m\times n}$ (k= 1, 2, …, t; i= 1, 2, …, m; j= 1, 2, …, n) is calculated by $Y_{ij}^* = \frac{1}{t}\sum_{k=1}^{t}(Y_{ij}^k)$ (4)

The collective correlation coefficient between Y^k (k= 1, 2, …, t) and Y^* represents as follows:

$$e_k = \sum_{i=1}^{m} \frac{\sum_{j=1}^{n} Y_{ij}^k Y_{ij}^*}{\sqrt{\sum_{j=1}^{n} (Y_{ij}^k)^2}\sqrt{\sum_{j=1}^{n} (Y_{ij}^*)^2}}. \quad (5)$$

Step3:

Determination decision maker's weights

In practical decision-making problems, the decision makers may have personal biases and some individuals may give unduly high or unduly low preference values with respect to their preferred or repugnant objects. In this case, we will assign very low weights to these false or biased opinions. Since the ''mean value'' is the ''distributing center'' of all elements in a set, the average matrix Y^* is the maximum compromise among all individual decisions of the group. In mean sense, a hybrid score-accuracy matrix Y^k is closer to the average one Y^*. Then, the preference value (hybrid score-accuracy value) of the k-th decision maker is closer to the average value and his/her evaluation is more reasonable and more important, thus the weight of the k-th decision maker is bigger. Hence, a weight model for decision makers can be defined as:

$$\lambda_k = \frac{e_k}{\sum_{k=1}^{t} e_k} \quad (6)$$

Where $0 \le \lambda_k \le 1$, $\sum_{k=1}^{t}\lambda_k = 1$ for k=1, 2, ….,t.

Step4:

Calculate collective hybrid score-accuracy matrix

For the weight vector $\lambda = (\lambda_1, \lambda_2, \cdots, \lambda_k)^T$ of decision makers obtained from eqation.(6), we accumulate all individual hybrid score-accuracy matrices of $Y^k = (Y_{ij}^k)_{m\times n}$ (k= 1, 2, …, t; i= 1, 2, …, m; j= 1, 2, …, n) into a collective hybrid score-accuracy matrix $Y = (Y_{ij})_{m\times n}$ by the following formula:

$$Y_{ij} = \sum_{k=1}^{t} \lambda_k Y_{ij}^k \quad (7)$$

Step5:

Weight model for attributes

For a specific decision problem, the weights of the attributes can be given in advance by a partially known subset corresponding to the weight information of the attributes, which is denoted by W. Reasonable weight values of the attributes should make the overall averaging value of all alternatives as large as possible because they can enhance the obvious differences and identification of various alternatives under the attributes to easily rank the alternatives. To determine the weight vector of the attributes Ye introduced the following optimization model:

$$\max W = \frac{1}{m}\sum_{i=1}^{m}\sum_{j=1}^{n} W_j Y_{ij}$$

Subject to,

$$\sum_{j=1}^{n} W_j = 1$$

$$W_j > 0 \quad (8)$$

This is a linear programming problem, which can be easily solved to determine the weight vector of the attributes W= $(W_1, W_2, \ldots, W_n)^T$

Step6:

Ranking alternatives

To rank alternatives, we can sum all values in each row of the collective hybrid score-accuracy matrix corresponding to the attribute weights by the overall weighted hybrid score-accuracy value of each alternative A_i (i= 1, 2, …, m):

$$M(A_i) = \sum_{j=1}^{n} W_j Y_{ij} \quad (9)$$

According to the overall hybrid score-accuracy values of $M(A_i)$ (i= 1, 2, …, m), we can rank alternatives A_i (i= 1, 2, …, m) in descending order and choose the best one.

Step7: End

Section V

Example of Teacher Recruitment Process

Suppose that a university is going to recruit in the post of an assistant professor for a particular subject.. After initial screening, five candidates (i.e. alternatives) A_1, A_2, A_3, A_4, A_5 remain for further evaluation. A committee of four decision makers or experts, D_1, D_2, D_3, D_4 has been formed to conduct the interview and select the most appropriate candidate. Eight criteria obtained from expert opinions, namely, academic performances (C_1), subject knowledge (C_2), teaching aptitude (C_3), research- experiences (C_4), leadership quality (C_5), personality (C_6), management capacity (C_7) and values (C_8) are considered for recruitment criteria. If four experts are required in the evaluation process, then the five possible alternatives A_i (i= 1, 2, 3, 4, 5) are evaluated by the form of SVNNs under the above eight attributes on the fuzzy concept "excellence". Thus the four single valued neutrosophic decision matrices can be obtained from the four experts and expressed, respectively, as follows:(see Table 1, 2, 3, 4).

Table1: Single valued neutrosophic decision matrix

D_1=

.	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8
A_1	$\langle .8, .1,.1\rangle$	$\langle .8,.1,.1\rangle$	$\langle .7,.2,.1\rangle$	$\langle .7,.2,.2\rangle$	$\langle .7,.4,.1\rangle$	$\langle .7,.4,.2\rangle$	$\langle .7,.3,.1\rangle$	$\langle .7,.4,.3\rangle$
A_2	$\langle .8,.2,.2\rangle$	$\langle .8,.2,.1\rangle$	$\langle .7,.3,.2\rangle$	$\langle .7,.3,.3\rangle$	$\langle .7,.3,.2\rangle$	$\langle .6,.4,.2\rangle$	$\langle .7,.2,.2\rangle$	$\langle .7,.3,.4\rangle$
A_3	$\langle .8,.1,.2\rangle$	$\langle .8,.3,.2\rangle$	$\langle .7,.4,.3\rangle$	$\langle .7,.3,.1\rangle$	$\langle .7,.2,.3\rangle$	$\langle .6,.3,.3\rangle$	$\langle .7,.1,.3\rangle$	$\langle .7, .3,.3\rangle$
A_4	$\langle .8,.1,.0\rangle$	$\langle .8,.2,.3\rangle$	$\langle .7,.3,.4\rangle$	$\langle .7,.1,.2\rangle$	$\langle .7,.2,.2\rangle$	$\langle .7,.2,.2\rangle$	$\langle .7,.2,.3\rangle$	$\langle .7,.2,.4\rangle$
A_5	$\langle .8,.2,.2\rangle$	$\langle .8,.3,.3\rangle$	$\langle .7,.3,.4\rangle$	$\langle .7,.2,.2\rangle$	$\langle .7,.1,.3\rangle$	$\langle .7,.1,.1\rangle$	$\langle .7,.1,.2\rangle$	$\langle .7,.1,.3\rangle$

Table2: Single valued neutrosophic decision matrix

D_2=

.	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8
A_1	$\langle .8, .2,.1\rangle$	$\langle .8,.1,.1\rangle$	$\langle .7,.2,.2\rangle$	$\langle .7,.1,.2\rangle$	$\langle .7,.4,.2\rangle$	$\langle .7,.4,.2\rangle$	$\langle .7,.3,.2\rangle$	$\langle .7,.3,.3\rangle$
A_2	$\langle .8,.2,.2\rangle$	$\langle .8,.2,.2\rangle$	$\langle .7,.3,.3\rangle$	$\langle .7,.3,.3\rangle$	$\langle .7,.2,.2\rangle$	$\langle .6,.4,.3\rangle$	$\langle .7,.3,.2\rangle$	$\langle .7,.4,.4\rangle$
A_3	$\langle .8,.2,.2\rangle$	$\langle .8,.3,.3\rangle$	$\langle .7,.3,.3\rangle$	$\langle .7,.3,.2\rangle$	$\langle .7,.3,.3\rangle$	$\langle .6,.3,.3\rangle$	$\langle .7,.2,.3\rangle$	$\langle .7,.2,.3\rangle$
A_4	$\langle .8,.1,.0\rangle$	$\langle .8,.2,.2\rangle$	$\langle .7,.3,.4\rangle$	$\langle .7,.1,.2\rangle$	$\langle .7,.3,.2\rangle$	$\langle .7,.2,.2\rangle$	$\langle .7,.3,.3\rangle$	$\langle .7,.3,.4\rangle$
A_5	$\langle .8,.1,.2\rangle$	$\langle .8,.2,.3\rangle$	$\langle .7,.3,.3\rangle$	$\langle .7,.2,.2\rangle$	$\langle .7,.2,.3\rangle$	$\langle .7,.1,.2\rangle$	$\langle .7,.2,.2\rangle$	$\langle .7,.2,.3\rangle$

Table3: Single valued neutrosophic decision matrix

D_3=

.	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8
A_1	$\langle .8, .1,.0\rangle$	$\langle .8,.1,.1\rangle$	$\langle .7,.2,.2\rangle$	$\langle .7,.2,.1\rangle$	$\langle .7,.3,.1\rangle$	$\langle .7,.3,.2\rangle$	$\langle .7,.3,.2\rangle$	$\langle .7,.3,.3\rangle$
A_2	$\langle .8,.2,.1\rangle$	$\langle .8,.2,.1\rangle$	$\langle .7,.3,.2\rangle$	$\langle .7,.2,.3\rangle$	$\langle .7,.3,.2\rangle$	$\langle .6,.4,.4\rangle$	$\langle .7,.3,.2\rangle$	$\langle .7,.3,.3\rangle$
A_3	$\langle .8,.2,.2\rangle$	$\langle .8,.2,.2\rangle$	$\langle .7,.3,.3\rangle$	$\langle .7,.3,.2\rangle$	$\langle .7,.2,.2\rangle$	$\langle .6,.2,.3\rangle$	$\langle .7,.2,.3\rangle$	$\langle .7,.3,.4\rangle$
A_4	$\langle .8,.1,.0\rangle$	$\langle .8,.2,.2\rangle$	$\langle .7,.3,.2\rangle$	$\langle .7,.1,.2\rangle$	$\langle .7,.2,.2\rangle$	$\langle .7,.2,.2\rangle$	$\langle .7,.2,.3\rangle$	$\langle .7,.2,.3\rangle$
A_5	$\langle .8,.1,.2\rangle$	$\langle .8,.2,.3\rangle$	$\langle .7,.2,.4\rangle$	$\langle .7,.1,.2\rangle$	$\langle .7,.1,.3\rangle$	$\langle .7,.1,.2\rangle$	$\langle .7,.2,.2\rangle$	$\langle .7,.2,.3\rangle$

Table4: Single valued neutrosophic decision matrix

D_4=

.	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8
A_1	$\langle .8, .2,.1\rangle$	$\langle .8,.2,.1\rangle$	$\langle .7,.2,.1\rangle$	$\langle .7,.2,.2\rangle$	$\langle .7,.3,.1\rangle$	$\langle .7,.2,.2\rangle$	$\langle .7,.2,.1\rangle$	$\langle .7,.4,.3\rangle$
A_2	$\langle .8,.2,.0\rangle$	$\langle .8,.2,.1\rangle$	$\langle .7,.3,.2\rangle$	$\langle .7,.1,.3\rangle$	$\langle .7,.3,.2\rangle$	$\langle .6,.4,.3\rangle$	$\langle .7,.2,.2\rangle$	$\langle .7,.3,.3\rangle$
A_3	$\langle .8,.1,.2\rangle$	$\langle .8,.2,.2\rangle$	$\langle .7,.3,.3\rangle$	$\langle .7,.3,.2\rangle$	$\langle .7,.2,.2\rangle$	$\langle .6,.3,.2\rangle$	$\langle .7,.3,.3\rangle$	$\langle .7,.3,.3\rangle$
A_4	$\langle .8,.1,.0\rangle$	$\langle .8,.2,.3\rangle$	$\langle .7,.3,.3\rangle$	$\langle .7,.1,.2\rangle$	$\langle .7,.2,.2\rangle$	$\langle .7,.2,.2\rangle$	$\langle .7,.2,.3\rangle$	$\langle .7,.2,.4\rangle$
A_5	$\langle .8,.2,.2\rangle$	$\langle .8,.3,.0\rangle$	$\langle .7,.3,.3\rangle$	$\langle .7,.2,.2\rangle$	$\langle .7,.1,.3\rangle$	$\langle .7,.1,.1\rangle$	$\langle .7,.1,.2\rangle$	$\langle .7,.2,.3\rangle$

Thus, we use the proposed method for single valued neutrosophic group decision-making to get the most suitable teacher. We take α = 0.5 for demonstrating the computing procedure of the proposed method. For the above four decision matrices, the following hybrid score-accuracy matrices are obtained by equation(3):(see Table 5, 6, 7, 8)

Table5: Hybrid score accuracy matrix for D_1

Y_1=

.	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8
A_1	1.7667	1.7167	1.6000	1.5167	1.5333	1.4500	1.5667	1.3667
A_2	1.6000	1.6833	1.4833	1.4000	1.4833	1.3667	1.5167	1.3167
A_3	1.6333	1.6500	1.3667	1.5667	1.4333	1.3167	1.4667	1.4000
A_4	1.8000	1.5167	1.3167	1.5500	1.5167	1.5167	1.4333	1.3500
A_5	1.6000	1.4833	1.3167	1.5167	1.4667	1.6333	1.5500	1.4667

Table6: Hybrid score accuracy matrix for D_2

Y_2=

.	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8
A_1	1.6833	1.7167	1.5167	1.5500	1.4500	1.4500	1.4833	1.4000
A_2	1.6000	1.6000	1.4000	1.4000	1.5167	1.2833	1.4833	1.2833
A_3	1.6000	1.4833	1.4000	1.4833	1.4000	1.3167	1.4333	1.4333
A_4	1.8000	1.6000	1.3167	1.5500	1.4833	1.5167	1.4000	1.3167
A_5	1.6333	1.5167	1.4000	1.5167	1.4333	1.5500	1.5167	1.4333

Table7: Hybrid score accuracy matrix for D_3

Y_3=

.	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8
A_1	1.8000	1.7167	1.5167	1.6000	1.5667	1.4833	1.4833	1.4000
A_2	1.6833	1.6833	1.4833	1.4333	1.4833	1.2000	1.4833	1.4000
A_3	1.6000	1.6000	1.4000	1.4833	1.5167	1.3500	1.4333	1.3167
A_4	1.8000	1.6000	1.4833	1.5500	1.5167	1.5167	1.4333	1.4333
A_5	1.6333	1.5167	1.3500	1.5500	1.4667	1.5500	1.5167	1.4333

Table8: Hybrid score accuracy matrix for D_4

Y_4=

.	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8
A_1	1.6833	1.6833	1.6000	1.5167	1.5667	1.5167	1.6000	1.3667
A_2	1.7333	1.6833	1.4833	1.4667	1.4833	1.2833	1.5167	1.4000
A_3	1.6333	1.6000	1.4000	1.4833	1.5167	1.4000	1.4000	1.4000
A_4	1.8000	1.5167	1.4000	1.5500	1.5167	1.5167	1.4333	1.3500
A_5	1.6000	1.7333	1.4000	1.5167	1.4667	1.6333	1.5500	1.4333

From the above hybrid score-accuracy matrices, by using equation (4) we can yield the average matrix Y^*.(see Table 9)

Table9: The average matrix

$Y^{*}=$

.	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8
A_1	1.7208	1.7084	1.5584	1.5459	1.5292	1.4750	1.5333	1.3834
A_2	1.6417	1.6500	1.4625	1.4375	1.4917	1.2833	1.5000	1.3625
A_3	1.6167	1.5833	1.3917	1.5042	1.4792	1.3459	1.4333	1.3875
A_4	1.8000	1.5584	1.3792	1.5500	1.5084	1.5167	1.4250	1.3625
A_5	1.6167	1.5625	1.3667	1.3450	1.4584	1.5917	1.5334	1.4417

From the equations. (5) and (6), we determine the weights of the three decision makers as follows: $\lambda_1 = 0.2505$ $\lambda_2 = 0.2510$ $\lambda_3 = 0.2491$ $\lambda_3 = 0.2494$

Hence, the hybrid score-accuracy values of the different decision makers' evaluations are aggregated[48] by equation (7) and the following collective hybrid score-accuracy matrix can be obtain as follows(see Table 10):

Table10: Collective hybrid score accuracy- matrix

$Y=$

.	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8
A_1	1.7209	1.7085	1.5584	1.5459	1.5292	1.4751	1.5334	1.3834
A_2	1.6417	1.6500	1.4624	1.4375	1.4918	1.2833	1.5000	1.3624
A_3	1.6168	1.5834	1.3917	1.5043	1.4792	1.3458	1.4332	1.3875
A_4	1.8001	1.5584	1.3793	1.5500	1.5085	1.5167	1.4250	1.3626
A_5	1.6167	1.5626	1.3667	1.3451	1.4584	1.5918	1.5334	1.4417

Assume that the information about attribute weights is incompletely known weight vectors, $0.1 \le w_1 \le 0.2$, $0.1 \le w_2 \le 0.2$, $0.1 \le w_3 \le 0.2$, $0.1 \le w_4 \le 0.2$, $0.1 \le w_5 \le 0.2$, $0.1 \le w_6 \le 0.2$, $0.1 \le w_7 \le 0.2$, $0.1 \le w_8 \le 0.2$ given by the decision makers,

By using the linear programming model (8), we obtain the weight vector of the attributes as:

$W = [0.2, 0.2, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1]^{T}$

By applying eqation (9), we can calculate the overall hybrid score-accuracy values $M(A_i)$ (i=1, 2, 3, 4, 5):

$M(A_1) = 1.58842$, $M(A_2) = 1.51208$, $M(A_3) = 1.49421$, $M(A_4) = 1.54591$, $M(A_5) = 1.50957$

According to the above values of $M(A_i)$ (i= 1, 2, 3, 4, 5), the ranking order of the alternatives is $A_1 > A_4 > A_2 > A_5 > A_3$. Then, the alternative A_1 is the best teacher.

By similar computing procedures, for different values of α the ranking orders of the teachers are shown in the Table 11.

Section VI

Conclusion

In this paper we employ the score and accuracy functions, hybrid score-accuracy functions of SVNNs to recruitt best teacher for higher education under single valued neutrosophic environments, where the weights of decision makers are completely unknown and the weights of attributes are incompletely known. Here, the weight values obtained from these weight models mainly decrease the effect of some unreasonable evaluations, e.g. the decision makers may have personal biases and some individuals may give unduly high or unduly low preference values with respect to their preferred or repugnant objects. Then, we use overall evaluation formulae of the weighted hybrid score-accuracy functions for each alternative to rank the alternatives and select the most desirable teacher. The advantages of the model for group decision-making methods with single valued neutrosophic information is provide simple calculations and good flexibility but also handling with the group decision-making problems with unknown weights by comparisons with other relative decision-making methods under single valued neutrosophic environments. In future, we shall continue working in the extension and application of the methods to other domains, such as best raw material selection for industries.

Table11: The ranking order of the teachers taking different values of α

α	**$M(A_i)$**	**Ranking order**
0.0	$M(A_1)$=1.61872, $M(A_2)$=1.54988, $M(A_3)$=1.54441, $M(A_4)$=1.56961, $M(A_5)$=1.54697	$A_1 > A_4 > A_2 > A_5 > A_3$.
0.3	$M(A_1)$=1.60052, $M(A_2)$=1.52518, $M(A_3)$=1.51429, $M(A_4)$=1.55541, $M(A_5)$=1.52317	$A_1 > A_4 > A_2 > A_5 > A_3$.
0.5	$M(A_1)$=1.58842, $M(A_2)$=1.51208, $M(A_3)$=1.49426, $M(A_4)$=1.54591, $M(A_5)$=1.50957	$A_1 > A_4 > A_2 > A_5 > A_3$.
0.7	$M(A_1)$=1.57632, $M(A_2)$=1.49898, $M(A_3)$=1.47404, $M(A_4)$=1.53651, $M(A_5)$=1.49307	$A_1 > A_4 > A_2 > A_5 > A_3$.
1.0	$M(A_1)$=1.55822, $M(A_2)$=1.48928, $M(A_3)$=1.44392, $M(A_4)$=1.52231, $M(A_5)$=1.48467	$A_1 > A_4 > A_2 > A_5 > A_3$.

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NSS

Generalization of Soft Neutrosophic Rings and Soft Neutrosophic Fields

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Abstract. In this paper we extend soft neutrosophic rings and soft neutrosophic fields to soft neutrosophic birings, soft neutrosophic N-rings and soft neutrosophic bifields and soft neutrosophic N-fields. We also extend soft neutrosophic ideal theory to form soft neutrosophic biideal and soft neutrosophic N-ideals over a neutrosophic biring and soft neutrosophic N-ring . We have given examples to illustrate the theory of soft neutrosophic birings, soft neutrosophic N-rings and soft neutrosophic fields and soft neutrosophic N-fields and display many properties of these.

Keywords: Neutrosophic biring, neutrosophic N-ring, neutrosophic bifield,neutrosophic N-field, soft set, soft neutrosophic biring, soft neutrosophic N-ring, soft neutrosophic bifield, soft neutrosophic N-field.

1 Introduction

Neutrosophy is a new branch of philosophy which studies the origin and features of neutralities in the nature. Florentin Smarandache in 1980 firstly introduced the concept of neutrosophic logic where each proposition in neutrosophic logic is approximated to have the percentage of truth in a subset T, the percentage of indeterminacy in a subset I, and the percentage of falsity in a subset F so that this neutrosophic logic is called an extension of fuzzy logic. In fact neutrosophic set is the generalization of classical sets, conventional fuzzy set, intuitionistic fuzzy set and interval valued fuzzy set. This mathematical tool is used to handle problems like imprecise, indeterminacy and inconsistent data etc. By utilizing neutrosophic theory, Vasantha Kandasamy and Florentin Smarandache dig out neutrosophic algebraic structures. Some of them are neutrosophic fields, neutrosophic vector spaces, neutrosophic groups, neutrosophic bigroups, neutrosophic N-groups, neutrosophic semigroups, neutrosophic bisemigroups, neutrosophic N-semigroup, neutrosophic loops, neutrosophic biloops, neutrosophic N-loop, neutrosophip groupoids, and neutrosophic bigroupoids and so on.

Molodtsov in $[11]$ laid down the stone foundation of a richer structure called soft set theory which is free from the parameterization inadequacy, syndrome of fuzzy se theory, rough set theory, probability theory and so on. In many areas it has been successfully applied such as smoothness of functions, game theory, operations research, Riemann integration, Perron integration, and probability. Recently soft set theory has attained much attention since its appearance and the work based on several operations of soft sets introduced in $[2,9,10]$. Some more exciting properties and algebra may be found in $[1]$. Feng et al. introduced the soft semirings $[5]$. By means of level soft sets an adjustable approach to fuzzy soft sets based decision making can be seen in $[6]$. Some other new concept combined with fuzzy sets and rough sets was presented in $[7,8]$. AygÄunoglu et al. introduced the Fuzzy soft groups $[4]$.

Firstly, fundamental and basic concepts are given for neutrosophic birings, neutrosophic N-rings, neutrosohic bifields and soft neutrosophic N-fields . In the next section we presents the newly defined notions and results in soft neutrosophic birings, soft neutrosophic N-rings and soft neutrosophic bifields and soft neutrosophic N-fields. Various types of soft neutrosophic biideals and N-ideals of birings and N-rings are defined and elaborated with the help of examples.

2 Fundamental Concepts

In this section, we give a brief description of neutrosophic birings, neutrosophic N-rings, neutrosophic bifields and neutrosophic N-fields respectively.

Definition 2.1. Let $(BN(\mathrm{R}),*,\circ)$ be a non-empty set with two binary operations $*$ and $\circ$. $(BN(\mathrm{R}),*,\circ)$ is said to be a neutrosophic biring if $BN(\mathrm{Rs}) = R_1 \cup R_2$ where atleast one of $(\mathrm{R}_1,*,\circ)$ or $(\mathrm{R}_2,*,\circ)$ is a neutrosophic ring and other is just a ring. R_1 and R_2 are proper subsets of $BN(\mathrm{R})$.

Definition 2.2: Let $BN(\mathrm{R}) = (\mathrm{R}_1,*,\circ) \cup (\mathrm{R}_2,*,\circ)$ be a neutrosophic biring. Then $BN(\mathrm{R})$ is called a commutative neutrosophic biring if each $(\mathrm{R}_1,*,\circ)$ and $(\mathrm{R}_2,*,\circ)$ is a commutative neutrosophic ring.

Definition 2.3: Let $BN(\mathrm{R}) = (\mathrm{R}_1,*,\circ) \cup (\mathrm{R}_2,*,\circ)$ be a neutrosophic biring. Then $BN(\mathrm{R})$ is called a pseudo neutrosophic biring if each $(\mathrm{R}_1,*,\circ)$ and $(\mathrm{R}_2,*,\circ)$ is a pseudo neutrosophic ring.

Definition 2.4 Let $(BN(\mathrm{R}) = R_1 \cup R_2;*,\circ)$ be a neutrosophic biring. A proper subset $(T,*,\circ)$ is said to be a neutrosophic subbiring of $BN(\mathrm{R})$ if

1) $T = T_1 \cup T_2$ where $T_1 = R_1 \cap T$ and $T_2 = R_2 \cap T$ and
2) At least one of $(T_1,\circ)$ or $(T_2,*)$ is a neutrosophic ring.

Definition 2.5: If both $(\mathrm{R}_1,*)$ and $(\mathrm{R}_2,\circ)$ in the above definition 2.1 are neutrosophic rings then we call $(BN(\mathrm{R}),*,\circ)$ to be a strong neutrosophic biring.

Definition 2.6 Let $(BN(\mathrm{R}) = R_1 \cup R_2;*,\circ)$ be a neutrosophic biring and let $(T,*,\circ)$ is a neutrosophic subbiring of $BN(\mathrm{R})$. Then $(T,*,\circ)$ is called a neutrosophic biideal of $BN(R)$ if

1) $T = T_1 \cup T_2$ where $T_1 = R_1 \cap T$ and $T_2 = R_2 \cap T$ and
2) At least one of $(T_1,*,\circ)$ or $(T_2,*,\circ)$ is a neutrosophic ideal.

If both $(T_1,*,\circ)$ and $(T_2,*,\circ)$ in the above definition are neutrosophic ideals, then we call $(T,*,\circ)$ to be a strong neutrosophic biideal of $BN(R)$.

Definition 2.7: Let $\{\mathrm{N}(\mathrm{R}),*_1,...,*_2,\circ_1,\circ_2,...,\circ_N\}$ be a non-empty set with two N -binary operations defined on it. We call $N(R)$ a neutrosophic N -ring (N a positive integer) if the following conditions are satisfied.

1) $\mathrm{N}(\mathrm{R}) = R_1 \cup R_2 \cup ... \cup R_N$ where each R_i is a proper subset of $\mathrm{N}(\mathrm{R})$ i.e. $R_i \not\subset R_j$ or $R_j \not\subset R_i$ if $i \neq j$.
2) $(\mathrm{R}_i,*_i,\circ_i)$ is either a neutrosophic ring or a ring for $i = 1,2,3,...,N$.

Definition 2.8: If all the N -rings $(\mathrm{R}_i,*_i)$ in definition 2.7 are neutrosophic rings (i.e. for $i = 1,2,3,...,N$) then we call $\mathrm{N}(\mathrm{R})$ to be a neutrosophic strong N -ring.

Definition 2.9: Let $\mathrm{N}(\mathrm{R}) = \{\mathrm{R}_1 \cup R_2 \cup .... \cup \mathrm{R}_N,*_1,*_2,...,*_N,\circ_1,\circ_2,...,\circ_N\}$ be a neutrosophic N -ring. A proper subset $P = \{\mathrm{P}_1 \cup P_2 \cup .... \mathrm{P}_N,*_1,*_2,...,*_N\}$ of $\mathrm{N}(\mathrm{R})$ is said to be a neutrosophic N -subring if $P_i = P \cap R_i, i = 1,2,...,N$ are subrings of R_i in which atleast some of the subrings are neutrosophic subrings.

Definition 2.10: Let $\mathrm{N}(\mathrm{R}) = \{\mathrm{R}_1 \cup R_2 \cup .... \cup \mathrm{R}_N,*_1,*_2,...,*_N,\circ_1,\circ_2,...,\circ_N\}$ be a neutrosophic N -ring. A proper subset $P = \{\mathrm{P}_1 \cup P_2 \cup .... \cup P_N,*_1,*_2,...,*_N,\circ_1,\circ_2,...,\circ_N\}$ where $P_t = P \cap R_t$ for $t = 1,2,...,N$ is said to be a neutrosophic N -ideal of $N(R)$ if the following conditions are satisfied.

1) Each it is a neutrosophic subring of $R_t, t = 1,2,...,N$.
2) Each it is a two sided ideal of R_t for $t = 1,2,...,N$.

If $(\mathrm{P}_i,*_i,\circ_i)$ in the above definition are neutrosophic ideals, then we call $(\mathrm{P}_i,*_i,\circ_i)$ to be a strong neutrosophic N-ideal of $N(R)$.

Definition 2.11: Let $(BN(\mathrm{F}),*,\circ)$ be a non-empty set with two binary operations $*$ and $\circ$. $(BN(\mathrm{F}),*,\circ)$ is said to be a neutrosophic bifiel if $BN(\mathrm{F}) = F_1 \cup F_2$ where atleast one of $(\mathrm{F}_1, *, \circ)$ or $(\mathrm{F}_2, *, \circ)$ is a neutrosophic field and other is just a field. F_1 and F_2 are proper subsets of $BN(\mathrm{F})$.

If in the above definition both $(\mathrm{F}_1, *, \circ)$ and $(\mathrm{F}_2, *, \circ)$ are neutrosophic fields, then we call $(BN(\mathrm{F}), *, \circ)$ to be a neutrosophic strong bifield.

Definition 2.12: Let $BN(\mathrm{F}) = (\mathrm{F}_1 \cup F_2, *, \circ)$ be a neutrosophic bifield. A proper subset $(T, *, \circ)$ is said to be a neutrosophic subbifield of $BN(\mathrm{F})$ if

1. $T = T_1 \cup T_2$ where $T_1 = F_1 \cap T$ and $T_2 = F_2 \cap T$ and
2. At least one of $(T_1, \circ)$ or $(T_2, *)$ is a neutrosophic field and the other is just a field.

Definition 2.13: Let $\{\mathrm{N(F)}, *_1, \ldots, *_2, \circ_1, \circ_2, \ldots, \circ_N\}$ be a non-empty set with two N -binary operations defined on it. We call $N(R)$ a neutrosophic N -field (N a positive integer) if the following conditions are satisfied.

1. $\mathrm{N(F)} = F_1 \cup F_2 \cup \ldots \cup F_N$ where each F_i is a proper subset of $\mathrm{N(F)}$ i.e. $R_i \not\subset R_j$ or $R_j \not\subset R_i$ if $i \neq j$.
2. $(\mathrm{R}_i, *_i, \circ_i)$ is either a neutrosophic field or just a field for $i = 1, 2, 3, \ldots, N$.

If in the above definition each $(\mathrm{R}_i, *_i, \circ_i)$ is a neutrosophic field, then we call $N(R)$ to be a strong neutrosophic N-field.

Definition 2.14: Let
$\mathrm{N(F)} = \{\mathrm{F}_1 \cup F_2 \cup \ldots. \cup F_N, *_1, *_2, \ldots, *_N, \circ_1, \circ_2, \ldots, \circ_N\}$ be a neutrosophic N -field. A proper subset $T = \{\mathrm{T}_1 \cup T_2 \cup \ldots. \cup \mathrm{T}_N, *_1, *_2, \ldots, *_N, \circ_1, \circ_2, \ldots, \circ_N\}$ of $N(\mathrm{F})$ is said to be a neutrosophic N -subfield if each $(T_i, *_i)$ is a neutrosophic subfield of $(\mathrm{F}_i, *_i, \circ_i)$ for $i = 1, 2, \ldots, N$ where $T_i = F_i \cap T$.

3 Soft Neutrosophic Birings

Definition 3.1: Let $(BN(\mathrm{R}), *, \circ)$ be a neutrosophic biring and (F, A) be a soft set over $(BN(\mathrm{R}), *, \circ)$. Then (F, A) is called soft neutrosophic biring if and only if $F(a)$ is a neutrosophic subbiring of $(BN(\mathrm{R}), *, \circ)$ for all $a \in A$.

Example 3.2: Let $BN(\mathrm{R}) = (\mathrm{R}_1, *, \circ) \cup (\mathrm{R}_2, *, \circ)$ be a neutrosophic biring, where $(\mathrm{R}_1, *, \circ) = (\langle \mathbb{Z} \cup I \rangle, +, \times)$ and $(\mathrm{R}_2, *, \circ) = (\mathbb{Q}, +, \times)$. Let $A = \{a_1, a_2, a_3, a_4\}$ be a set of parameters. Then clearly (F, A) is a soft neutrosophic biring over $BN(R)$, where

$$F(a_1) = \langle 2\mathbb{Z} \cup I \rangle \cup \mathbb{R}, F(a_2) = \langle 3\mathbb{Z} \cup I \rangle \cup \mathbb{Q},$$

$$F(a_3) = \langle 5\mathbb{Z} \cup I \rangle \cup \mathbb{Z}, F(a_4) = \langle 6\mathbb{Z} \cup I \rangle \cup 2\mathbb{Z}.$$

Theorem 3.3: Let F, A and (H, A) be two soft neutrosophic birings over $BN(R)$. Then their intersection $F, A \cap H, A$ is again a soft neutrosophic biring over $BN(R)$.

Proof. The proof is straightforward.

Theorem 3.4: Let F, A and H, B be two soft neutrosophic birings over $BN(R)$. If $A \cap B = \phi$, then $F, A \cup H, B$ is a soft neutrosophic biring over $BN(R)$.

Proof. This is straightforward.

Remark 3.5: The extended union of two soft neutrosophic birings F, A and K, B over $BN(R)$ is not a soft neutrosophic ring over $BN(R)$.

We check this by the help of Examples.

Remark 3.6: The restricted union of two soft neutrosophic rings F, A and K, B over $\langle R \cup I \rangle$ is not a soft neutrosophic ring over $\langle R \cup I \rangle$.

Theorem 3.7: The OR operation of two soft neutrosophic rings over $\langle R \cup I \rangle$ may not be a soft neutrosophic ring over $\langle R \cup I \rangle$.

One can easily check these remarks with the help of Examples.

Theorem 3.8: The extended intersection of two soft neu trosophic birings over $BN(R)$ is soft neutrosophic biring over $BN(R)$.

Proof. The proof is straightforward.

Theorem 3.9: The restricted intersection of two soft neutrosophic birings over $BN(R)$ is soft neutrosophic biring over $BN(R)$.

Theorem 3.10: The AND operation of two soft neutrosophic birings over $BN(R)$ is soft neutrosophic biring over $BN(R)$.

Definition 3.11: Let F, A be a soft set over a neutrosophic biring over $BN(R)$. Then (F, A) is called an absolute soft neutrosophic biring if $F(a) = BN(R)$ for all $a \in A$.

Definition 3.12: Let (F, A) be a soft set over a neutrosophic ring $BN(R)$. Then (F, A) is called soft neutrosophic biideal over $BN(R)$ if and only if $F(a)$ is a neutrosophic biideal of $BN(R)$.

Theorem 3.1.3: Every soft neutrosophic biideal (F, A) over a neutrosophic biring $BN(R)$ is trivially a soft neutrosophic biring but the converse may not be true.

Proposition 3.14: Let (F, A) and (K, B) be two soft neutosophic biideals over a neutrosophic biring

$BN(R)$. Then

1. Their extended union $(F, A) \cup_E (K, B)$ is again a soft neutrosophic biideal over $BN(R)$.
2. Their extended intersection $(F, A) \cap_E (K, B)$ is again a soft neutrosophic biideal over $BN(R)$.
3. Their restricted union $(F, A) \cup_R (K, B)$ is again a soft neutrosophic biideal over $BN(R)$.
4. Their restricted intersection $(F, A) \cap_R (K, B)$ is again a soft neutrosophic biideal over $BN(R)$.
5. Their OR operation $(F, A) \vee (K, B)$ is again a soft neutrosophic biideal over $BN(R)$.
6. Their AND operation $(F, A) \vee (K, B)$ is again a soft neutrosophic biideal over $BN(R)$.

Definition 3.15: Let (F, A) and (K, B) be two soft neutrosophic birings over $BN(R)$. Then (K, B) is called soft neutrosophic subbiring of (F, A), if

1. $B \subseteq A$, and
2. $K(a)$ is a neutrosophic subbiring of $F(a)$ for all $a \in A$.

Theorem 3.16: Every soft biring over a biring is a soft neutrosophic subbiring of a soft

neutrosophic biring over the corresponding neutrosophic biring if $B \subseteq A$.

Definition 3.16: Let (F, A) and (K, B) be two soft neutrosophic birings over $BN(R)$. Then (K, B) is called a soft neutrosophic biideal of (F, A), if

1. $B \subseteq A$, and
2. $K(a)$ is a neutrosophic biideal of $F(a)$ for all $a \in A$.

Proposition 3.17: All soft neutrosophic biideals are trivially soft neutrosophic subbirings.

4 Soft Neutrosophic N-Ring

Definition 4.1: Let $(N(\mathrm{R}), *_1, *_2, ..., *_N)$ be a neutrosophic N-ring and (F, A) be a soft set over $N(R)$ Then (F, A) is called soft neutrosophic N-ring if and only if $F(a)$ is a neutrosophic sub N-ring of $N(R)$ for all $a \in A$.

Example 4.2: Let $N(\mathrm{R}) = (\mathrm{R}_1, *, \circ) \cup (\mathrm{R}_2, *, \circ) \cup (\mathrm{R}_3, *, \circ)$ be aneutrosophic 3-ring, where $(\mathrm{R}_1, *, \circ) = (\langle \mathbb{Z} \cup I \rangle, +, \times)$, $(\mathrm{R}_2, *, \circ) = (\mathbb{C}, +, \times)$ and $(\mathrm{R}_3, *, \circ) = (\mathbb{R}, +, \times)$. Let $A = \{a_1, a_2, a_3, a_4\}$ be a set of parameters. Then clearly (F, A) is a soft neutrosophic N-ring over $N(R)$, where

$$F(a_1) = \langle 2\mathbb{Z} \cup I \rangle \cup \mathbb{R} \cup \mathbb{Q}, F(a_2) = \langle 3\mathbb{Z} \cup I \rangle \cup \mathbb{Q} \cup \mathbb{Z},$$

$$F(a_3) = \langle 5\mathbb{Z} \cup I \rangle \cup \mathbb{Z} \cup 2\mathbb{Z}, F(a_4) = \langle 6\mathbb{Z} \cup I \rangle \cup 2\mathbb{Z} \cup \mathbb{R}$$

.

Theorem 4.3: Let F, A and (H,A) be two soft neutrosophic N-rings over $N(R)$. Then their intersection $F, A \cap H, A$ is again a soft neutrosophic N-ring over $N(R)$.

Proof. The proof is straightforward.

Theorem 4.4: Let F, A and H, B be two soft neutrosophic N-rings over $N(R)$. If $A \cap B = \phi$, then $F, A \cup H, B$ is a soft neutrosophic N-ring over $N(R)$.

Proof. This is straightforward.

Remark 4.5: The extended union of two soft neutrosophic N-rings F, A and K, B over $BN(R)$ is not a soft neutrosophic ring over $N(R)$.

We can check this by the help of Examples.

Remark 4.6: The restricted union of two soft neutrosophic N-rings F, A and K, B over $N(R)$ is not a soft neutrosophic N-ring over $BN(R)$

Theorem 4.7: The OR operation of two soft neutrosophic N-rings over $N(R)$ may not be a soft neutrosophic N-ring over $N(R)$.

One can easily check these remarks with the help of Examples.

Theorem 4.8: The extended intersection of two soft neutrosophic N-rings over $N(R)$ is soft neutrosophic Nring over $N(R)$.

Proof. The proof is straightforward.

Theorem. The restricted intersection of two soft neutrosophic N-rings over $N(R)$ is soft neutrosophic N-ring over $N(\mathrm{R})$.

Proof. It is obvious.

Theorem 4.9: The AND operation of two soft neutrosophic N-rings over $N(R)$ is soft neutrosophic N-ring over $N(R)$.

Definition 4.10: Let F, A be a soft set over a neutrosophic N-ring over $N(R)$. Then (F, A) is called an absolute soft neutrosophic N-ring if $F(a) = N(R)$ for all $a \in A$.

Definition 4.11: Let (F, A) be a soft set over a neutrosophic N-ring $N(R)$. Then (F, A) is called soft neutrosophic N-ideal over $N(R)$ if and only if $F(a)$ is a neutosophic N-ideal of $N(R)$.

Theorem 4.12: Every soft neutrosophic N-ideal (F, A) over a neutrosophic N-ring $N(R)$ is trivially a soft neutrosophic N-ring but the converse may not be true.

Proposition 4.13: Let (F, A) and (K, B) be two soft neutosophic N-ideals over a neutrosophic N-ring $N(R)$. Then

1. Their extended intersection $(F, A) \cap_E (K, B)$ is again a soft neutrosophic N-ideal over $N(R)$.
2. Their restricted intersection $(F, A) \cap_R (K, B)$ is again a soft neutrosophic N-ideal over $N(R)$.
3. Their AND operation $(F, A) \vee (K, B)$ is again a soft neutrosophic N-ideal over $N(R)$.

Remark 4.14: Let (F, A) and (K, B) be two soft neutosophic N-ideals over a neutrosophic N-ring $N(R)$. Then

1. Their extended union $(F, A) \cup_E (K, B)$ is not a soft neutrosophic N-ideal over $N(R)$.
2. Their restricted union $(F, A) \cup_R (K, B)$ is not a soft neutrosophic N-ideal over $N(R)$.
3. Their OR operation $(F, A) \vee (K, B)$ is not a soft neutrosophic N-ideal over $N(R)$.

One can easily see these by the help of examples.

Definition. 4.15: Let (F, A) and (K, B) be two soft neutrosophic N-rings over $N(R)$. Then (K, B) is called soft neutrosophic sub N-ring of (F, A), if

1. $B \subseteq A$, and
2. $K(a)$ is a neutrosophic sub N-ring of $F(a)$ for

all $a \in A$.

Theorem 4.16: Every soft N-ring over a N-ring is a soft neutrosophic sub N-ring of a soft

neutrosophic N-ring over the corresponding neutrosophic N-ring if $B \subseteq A$.

Proof. Straightforward.

Definition 4.17: Let (F, A) and (K, B) be two soft neutrosophic N-rings over $N(R)$. Then (K, B) is called a soft neutrosophic N-ideal of (F, A), if

1. $B \subseteq A$, and
2. $K(a)$ is a neutrosophic N-ideal of $F(a)$ for all $a \in A$.

Proposition 4.18: All soft neutrosophic N-ideals are trivially soft neutrosophic sub N-rings.

5 Soft Neutrosophic Bifield

Defintion 5.1: Let $BN(K)$ be a neutrosophic bifield and let (F, A) be a soft set over $BN(K)$. Then (F, A) is said to be soft neutrosophic bifield if and only if $F(a)$ is a neutrosophic subbifield of $BN(K)$ for all $a \in A$.

Example 5.2: Let $BN(K) = \left\langle \mathbb{C} \cup I \right\rangle \cup \mathbb{R}$ be a neutrosophic bifield of complex numbers. Let $A = \{a_1, a_2\}$ be a set of parameters and let (F, A) be a soft set of $BN(K)$. Then (F, A) is a soft neutrosophic bifield over $BN(K)$, where

$$F(a_1) = \left\langle \mathbb{R} \cup I \right\rangle \cup \mathbb{Q}, F(a_2) = \left\langle \mathbb{Q} \cup I \right\rangle \cup \mathbb{Q}.$$

Where $\left\langle \mathbb{R} \cup I \right\rangle$ and $\left\langle \mathbb{Q} \cup I \right\rangle$ are the neutosophic fields of real numbers and rational numbers.

Proposition 5.3: Every soft neutrosophic bifield is trivially a soft neutrosophic biring.

Proof. The proof is trivial.

Definition 5.4: Let (F, A) be a soft neutrosophic bifield over a neutrosophic bifield $BN(K)$. Then (F, A) is called an absolute soft neutrosophic bifield if $F(a) = BN(K)$, for all $a \in A$.

Soft Neutrosophic N-field

Defintion 5.4: Let $N(K)$ be a neutrosophic N-field and let (F, A) be a soft set over $N(K)$. Then (F, A) is said to be soft neutrosophic N-field if and only if $F(a)$ is a neutrosophic sub N-field of $N(K)$ for all $a \in A$.

Proposition 5.5: Every soft neutrosophic N-field is trivially a soft neutrosophic N-ring.

Proof. The proof is trivial.

Definition 5.6: Let (F, A) be a soft neutrosophic N-field over a neutrosophic N-field $N(K)$. Then (F, A) is called an absolute soft neutrosophic N-field if $F(a) = N(K)$, for all $a \in A$.

Conclusion

In this paper we extend neutrosophicb rings, neutrosophic N-rings, Neutrosophic bifields and neutrosophic N-fields to soft neutrosophic birings, soft neutrosophic N-rings and soft neutrosophic bifields and soft neutrosophic N-fields respectively. The neutrosophic ideal theory is extend to soft neutrosophic biideal and soft neutrosophic N-ideal. Some new types of soft neutrosophic ideals are discovered which is strongly neutrosophic or purely neutrosophic. Related examples are given to illustrate soft neutrosophic biring, soft neutrosophic N-ring, soft neutrosophic bifield and soft neutrosophic N-field and many theorems and properties are discussed.

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Neutrosophic Refined Similarity Measure Based on Cosine Function

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Abstract: In this paper, the cosine similarity measure of neutrosophic refined (multi-) sets is proposed and its properties are studied. The concept of this cosine similarity measure of neutrosophic refined sets is the extension of improved cosine similarity measure of single valued neutrosophic. Finally, using this cosine similarity measure of neutrosophic refined set, the application of medical diagnosis is presented.

Keywords: Neutrosophic set, neutrosophic refined set, cosine similarity measure.

1.Introduction:

The neutrsophic sets (NS), proposed by F. Smarandache [7], has been studied and applied in different fields, including decision making problems [1,15], databases [21,22], medical diagnosis problems [2], topology [6], control theory [40], image processing [9,22,44] and so on. The concept of neutrosophic sets generalizes the following concepts: the classic set, fuzzy set [20],intuitionistic fuzzy set [19], and interval valued intuitionistic fuzzy set [18] and so on. The character of NSs is that the values of its membership function, non-membership function and indeterminacy function are subsets. Therefore, H.Wang et al [10] introduced an instance of neutrosophic sets known as single valued neutrosophic sets (SVNS), which were motivated from the practical point of view and that can be used in real scientific and engineering application, and provide the set theoretic operators and various properties of SVNSs. However, in many applications, due to lack of knowledge or data about the problem domains, the decision information may be provided with intervals, instead of real numbers. Thus, interval valued neutrosophic sets (IVNS), as a useful generation of NS, was introduced by H.Wang et al [11], which is characterized by a membership function, non-membership function and an indeterminacy function, whose values are intervals rather than real numbers. Also, the interval valued neutrosophic set can represent uncertain, imprecise, incomplete and inconsistent information which exist in the real world. As an important extension of NS, SVNS and IVNS has many applications in real life [13,14,15,16, 17,25,32,33,34,35,36,37,38,39]

Several similarity measures have been proposed by some researchers. Broumi and Smarandache [35] defined the Hausdorff distance between neutrosophic sets and some similarity measures based on the distance, set theoretic approach, and matching function to calculate the similarity degree between neutrosophic sets. In the same year, Broumi and Smarandache [32] also proposed the correlation coefficient between interval neutrosphic sets. Majumdar and Smanta [24] introduced several similarity measures of single valued neutrosophic sets(SVNs) based on distances, a maching function, memebership grades, and then proposed an entropy measure for a SVNS. J.Ye[13] also presented the Hamming and Euclidean distances between interval neutrosophic sets(INSs) an their similarity measures and applied them to multiple attribute decision –making problems with interval neutrosophic information. J.Ye [15] further proposed the distance-based similarity measure of SVNSs and applied it to the group decision making problems with single valued neutrosophic information. In other research, J.Ye [16] proposed three vector similarity measure for SNSs,an instance of SVNS and INS, including the Jaccard, Dice, and cosine similarity measures for SVNS and INSs, and applied them to multicriteria decision-making problems with simplified neutrosophic information. Recently, A.Salama [4], introduced and studied the concepts of correlation and correlation coefficient of neutrosophic data in probability spaces and study some of their properties.

The cosine similarity measure, based on Bhttacharya's distance [3] is the inner product of the two vectors divided by the product of their lengths. As the cosine similarity measure is the cosine of the angle between the vector representations of fuzzy sets, it is extended to cosine similarity measures between SVNSs by J.Ye [15,17] and also to cosine similarity measures between INSs by Broumi and Smarandache [36].

The notion of multisets was formulated first in [31] by Yager as generalization of the concept of set theory. Several authors from time to time made a number of generalization of set theory. For example, Sebastian and Ramakrishnan [42] introduced a new notion called multifuzzy sets, which is a generalization of multiset. Since then, Sebastian and Ramakrishnan [41,42] discussed more properties on multi fuzzy set. Later on, T. K. Shinoj and S. J. John [43] made an extension of the concept of fuzzy multisets by an intuitionistic fuzzy set, which called intuitionistic fuzzy multisets(IFMS). Since then in the study on IFMS, a lot of excellent results have been achieved by researchers [26,27,28,29,30]. An element of a multi fuzzy sets can occur more than once with possibly the same or different membership values, whereas an element of intuitionistic fuzzy multisets allows the repeated occurrences of membership and non--membership values. The concepts of FMS and IFMS fails to deal with indeterminatcy. In 2013, Smarandache [8] extended the classical neutrosophic logic to n-valued refined neutrosophic logic, by refining each neutrosophic component T, I, F into respectively, $T_1, T_2, ..., T_m$ and $I_1, I_2, ..., I_p$ and $F_1, F_2, ..., F_r$. Recently, I.Deli et al .[12] introduced the concept of neutrosophic refined sets and studied some of their basic properties. The concept of neutrosophic refined set (NRS) is a generalization of fuzzy multisets and intuitionistic fuzzy multisets.

In this paper, motivated by the cosine similarity measure based on Bhattacharya's distance and the improved cosine similarity measure of single valued neutrosophic proposed by J.Ye [17]. we propose a new method called "cosine similarity measure for neutrosophic refined sets. The proposed cosine similarity measure is applied to medical diagnosis problems. The paper is structured as follows. In Section 2, we first recall the necessary background on cosine similarity measure and neutrosophic refined sets. In Section 3,we present cosine similarity measure for neutrosophic refined sets and examines their respective properties. In section 4, we present a medical diagnosis using NRS –cosine similarity measure. Finally we conclude the paper.

2.Preliminaries

This section gives a brief overview of the concepts of neutrosophic set, single valued neutrosophic set, cosine similarity measure and neutrosophic refined sets.

2.1 Neutrosophic Sets

Definition 2.1 *[7]*

Let U be an universe of discourse then the neutrosophic set A is an object having the form

$A = \{< x: T_A(x), I_A(x), F_A(x)>, x \in U\}$, where the functions T, I, F : U→]−0, 1+[define respectively the degree of membership (or Truth) , the degree of indeterminacy, and the degree of non-membership (or Falsehood) of the element x ∈ U to the set A with the condition.

$$^{-}0 \leq sup\ (x) + supI\ (x) + supF\ (x) \leq 3^{+}. \quad \textbf{(1)}$$

From philosophical point of view, the neutrosophic set takes the value from real standard or non-standard subsets of $]^{-}0, 1^{+}[$. So instead of$]-0, 1^{+}[$ we need to take the interval [0, 1] for technical applications, because $]^{-}0, 1^{+}[$will be difficult to apply in the real applications such as in scientific and engineering problems.

For two NS, $A_{NS} = \{<x, T_A(x), I_A(x), F\ (x)> \mid x \in X\}$

And $B_{NS} = \{<x, T_B(x), I_B(x), F_B(x)> \mid x \in X\}$ the two relations are defined as follows:

(1) $A_{NS} \subseteq B_{NS}$ If and only if $(x) \leq {}_B(x)$, $I\ (x) \geq I_B(x)$, $F\ (x) \geq F_B(x)$

(2) $A_{NS} = B_{NS}$ if and only if, $(x)=T_B(x)$, $I\ (x) =I_B(x)$, $F\ (x) =F_B(x)$

2.2Single Valued Neutrosophic Sets

Definition 2.2 [10]

Let X be a space of points (objects) with generic elements in X denoted by x. An SVNS A in X is characterized by a truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$, and a falsity-membership function $F_A(x)$, for each point x in X, $T_A(x)$, $I\ (x)$, $F_A(x) \in [0, 1]$.

When X is continuous, an SVNS A can be written as

$$A=\int_X \frac{<T_A(x), I_A(x), F_A(x),>}{x}, x \in X. \quad \textbf{(2)}$$

When X is discrete, an SVNS A can be written as

$$A= \sum_1^n \frac{<T_A(x_i), I_A(x_i), F_A(x_i),>}{x_i}, x_i \in X$$

(3)

For two SVNS, A_{SVNS} = {<x,$T_A(x)$, $I_A(x)$, F (x)> | x ∈ X}

And B_{SVNS} ={ <x, $T_B(x)$, $I_B(x)$, $F_B(x)$> | x ∈ X} the two relations are defined as follows:

(1) $A_{SVNS} \subseteq B_{SVNS}$ if and only if $T_A(x) \leq$ $_B(x)$, $I_A(x) \geq I_B(x)$, $F_A(x) \geq F_B(x)$

(2) A_{SVNS} = B_{SVNS} if and only if , (x) =$T_B(x)$, I (x) =$I_B(x)$, $F_A(x)$ =$F_B(x)$ for any x ∈ X.

2.3 Cosine Similarity

Definition 2.3 [5]

Cosine similarity is a fundamental angle-based measure of similarity between two vectors of n dimensions using the cosine of the angle between them. It measures the similarity between two vectors based only on the direction, ignoring the impact of the distance between them. Given two vectors of attributes, X = ($x_1, x_2, \dots, x_n$) and Y= ($y_1, y_2, \dots, y_n$), the cosine similarity, cosθ, is represented using a dot product and magnitude as

$$\textbf{Cos}\theta = \frac{\sum_{i=1}^{n} x_i y_i}{\sqrt{\sum_{i=1}^{n} x_i^2}\sqrt{\sum_{i=1}^{n} y_i^2}} \qquad \textbf{(4)}$$

In vector space, a cosine similarity measure based on Bhattacharya's distance [3] between two fuzzy set $\mu_A(x_i)$ and $\mu_B(x_i)$ defined as follows:

$$C_F(A,B) = \frac{\sum_{i=1}^{n} \mu_A(x_i)\ \mu_B(x_i)}{\sqrt{\sum_{i=1}^{n} \mu_A(x_i)^2}\sqrt{\sum_{i=1}^{n} \mu_B(x_i)^2}} \qquad \textbf{(5)}$$

The cosine of the angle between the vectors is within the values between 0 and 1.

In 3-D vector space, J. Ye [15] defines cosine similarity measure between SVNS as follows:

$$C_{SVNS}(A,B) = \frac{\sum_{i=1}^{n} T_A(x_i)\ T_B(x_i)+I_A(x_i)I_B(x_i)+F_A(x_i)F_B(x_i)}{\sqrt{\sum_{i=1}^{n} T_A(x_i)^2+I_A(x_i)^2+F_A(x_i)^2}\sqrt{\sum_{i=1}^{n} T_B(x_i)^2+I_B(x_i)^2+F_A(x_i)^2}}$$

(6)

2.4. Neutrosophic Refined Sets.

Definition 2.4 [12]

Let A and B be two neutrosophic refined sets.

A = {<x,($T_A^1(x), T_A^2(x),..., T_A^p(x)$), ($I_A^1(x), I_A^2(x),..., I_A^p(x)$), ($F_A^1(x), F_A^2(x),\dots, F_A^p(x)$)>: x ∈ X}

where $T_A^1(x), T_A^2(x),..., T_A^p(x)$: E→ [0 ,1],

, $I_A^1(x), I_A^2(x),..., I_A^p(x)$: E→ [0 ,1], and

$F^1(x), F_A^2(x),..., F_A^p(x)$: E→ [0 ,1] such that 0≤ $sup T_A^i(x)$ +$\sup I_A^i(x)$ +$sup F_A^i(x)$ ≤3 for i=1,2,…,p for any x∈ X ,($T_A^1(x), T_A^2(x),\dots, T_A^p(x)$), ($I_A^1(x), I_A^2(x),\dots, I_A^p(x)$) and ($F_A^1(x), F_A^2(x),\dots, F_A^p(x)$) is the truth-membership sequence, indeterminacy-membership sequence and falsity-membership sequence of the element x, respectively. Also, P is called the dimension of neutrosophic refined sets (NRS) A.

3.Cosine similarity measure for Neutrosophic refined Sets.

Based on the improved cosine similarity measure of single valued neutrosophic sets proposed by J.Ye [17] which consists of membership, indeterminacy and non membership functions defined as follow:

$$C_{SVNS}(A,B)= \frac{1}{n}\sum_{i=1}^{n} cos\left[\frac{\pi(|T_A(x_i)-T_B(x_i)|+|I_A(x_i)-I_B(x_i)|+|F_A(x_i)-F_B(x_i)|)}{6}\right] \qquad \textbf{(7)}$$

And the cosine similarity measure of neutrosophic refined sets consisting of the multiple membership, indetrrminacy, and non-membership function is

$$C_{NRS}(A,B)= \frac{1}{p}\sum_{j=1}^{p}\left\{\frac{1}{n}\sum_{i=1}^{n} cos\left[\frac{\pi\left(\left|T_A^j(x_i)-T_B^j(x_i)\right|+\left|I_A^j(x_i)-I_B^j(x_i)\right|+\left|F_A^j(x_i)-F_B^j(x_i)\right|\right)}{6}\right]\right\} \qquad \textbf{(8)}$$

Proposition 3.1. The defined cosine similarity measure C_{NRS}(A,B) between NRS A and B satisfies the following properties

1. 0 ≤ C_{NRS}(A, B ≤1
2. C_{NRS}(A,B=1 if and only if A= B
3. C_{NRS}(A,B)= C_{NRS}(B,A)
4. If C is a NRS in X and A⊂B⊂C ,then C_{NRS}(A,C) ≤ C_{NRS}(A,B) and C_{NRS}(A,C) ≤ C_{NRS}(B,C)

Proof:

(1)

As the membership, indeterminacy and non-membership functions of the NRSs and the value of the cosine function are within [0 ,1],the similarity measure based on cosine function also is within [0.1]. Hence 0 ≤ C_{NRS}(A,B) ≤1.

(2)

For any two NRSs A and B , if A= B , this implies $T_A^j(x_i)$ = $T_B^j(x_i)$, $I_A^j(x_i)$ = $I_B^j(x_i)$, $F_A^j(x_i)$ = $F_B^j(x_i)$ for i= 1,2,…,n and j=1,2….,p and $x_i \in$ X. Hence $\left|T_A^j(x_i) - T_B^j(x_i)\right|$ = 0, $\left|I_A^j(x_i) - I_B^j(x_i)\right|$ =0, and $\left|F_A^j(x_i) - F_B^j(x_i)\right|$=0 .Thus C_{NRS}(A,B)=1.

If C_{NRS}A,B)=1 this refers that $\left|T_A^j(x_i) - T_B^j(x_i)\right|$ = 0, $\left|I_A^j(x_i) - I_B^j(x_i)\right|$ =0, and $\left|F_A^j(x_i) - F_B^j(x_i)\right|$=0 since cos(0)=1.Then ,these equalities indicates $T_A^j(x_i)$ = $T_B^j(x_i)$, $I_A^j(x_i)$ = $I_B^j(x_i)$, $F_A^j(x_i)$ = $F_B^j(x_i)$ for all i,j values and $x_i \in$ X. Hence A= B

(3)

Proof is straightforward

(4)

If A⊂B⊂C. then there are $T_A^j(x_i) \le T_B^j(x_i) \le T_C^j(x_i)$, $I_A^j(x_i) \ge I_B^j(x_i) \ge I_C^j(x_i)$, and $F_A^j(x_i) \ge F_B^j(x_i) \ge F_C^j(x_i)$ for all i,j values and $x_i \in$ X.Then we have the following inequalities

$\left|T_A^j(x_i) - T_B^j(x_i)\right| \le \left|T_A^j(x_i) - T_C^j(x_i)\right|$, $\left|T_B^j(x_i) - T_C^j(x_i)\right| \le \left|T_A^j(x_i) - T_C^j(x_i)\right|$,

$\left|I_A^j(x_i) - I_B^j(x_i)\right| \le \left|I_A^j(x_i) - T_C^j(x_i)\right|$, $\left|I_B^j(x_i) - I_C^j(x_i)\right| \le \left|I_A^j(x_i) - I_C^j(x_i)\right|$,

$\left|F_A^j(x_i) - F_B^j(x_i)\right| \le \left|F_A^j(x_i) - F_C^j(x_i)\right|$, $\left|F_B^j(x_i) - F_C^j(x_i)\right| \le \left|F_A^j(x_i) - F_C^j(x_i)\right|$,

Hence, C_{NRS}(A,C) $\le C_{NRS}$(A,B) and C_{NRS}(A,C) $\le$ C_{NRS}(B,C) for k=1,2, since the cosine function is a decreasing function within the interval [0, $\frac{\pi}{2}$].

4 Application

In this section, we give some applications of NRS in medical diagnosis problems using the cosine similarity measure. Some of it is quoted from [29,30,41].

From now on, we use

A = {<x,(T_A^1(x), I_A^1(x), F_A^1(x)),(T_A^2(x), I_A^2(x), F_A^2(x)),.., ,(T_A^p(x), I_A^p(x), F_A^p(x))>: x ∈ X}

Instead of

A = {<x,(T_A^1(x), T_A^2(x),..., T_A^p(x)), (I_A^1(x), I_A^2(x),..., I_A^p(x)), (F_A^1(x), F_A^2(x),…, F_A^p(x))>: x ∈ X}

4.1. Medical Diagnosis using NRS –cosine similarity measure

In what follows, let us consider an illustrative example adopted from Rajarajeswari and Uma [29] with minor changes and typically considered in [30,43]. Obviously, the application is an extension of intuitionistic fuzzy multi sets [29].

"As Medical diagnosis contains lots of uncertainties and increased volume of information available to physicians from new medical technologies, the process of classifying different set of symptoms under a single name of disease becomes difficult. In some practical situations, there is the possibility of each element having different truth membership, indeterminate and false membership functions. The proposed similarity measure among the patients Vs symptoms and symptoms Vs diseases gives the proper medical diagnosis. The unique feature of this proposed method is that it considers multi truth membership, indeterminate and false membership. By taking one time inspection, there may be error in diagnosis. Hence, this multi time inspection, by taking the samples of the same patient at different times gives best diagnosis" [29].

Now, an example of a medical diagnosis will be presented.

Example: Let P={P_1,P_2,P_3} be a set of patients, D={Viral Fever, Tuberculosis, Typhoid, Throat disease} be a set of diseases and S={Temperature, cough, throat pain, headache, body pain} be a set of symptoms. Our solution is to examine the patient at different time intervals (three times a day), which in turn give arise to different truth membership, indeterminate and false membership function for each patient.

Table I: Q (the relation Between Patient and Symptoms)

	Temperature	Cough	Throat pain	Headache	Body Pain
P_1	(0.4,0.3,0.4) (0.3,0.4,0.6) (0.2,0.5,0.5)	(0.5,0.4,0.4) (0.4,0.1,0.3) (0.3,0.4,0.5)	(0.3,0.5,0.5) (0.2,0.6,0.4) (0.1,0.6,0.3)	(0.5,0.3,0.4) (0.5,0.4,0.7) (0.3,0.3,0.6)	(0.5,0.2,0.4) (0.2,0.3,0.5) (0.1,0.4,0.3)
P_2	(0.6,0.3,0.5) (0.5,0.5,0.2) (0.4,0.4,0.5)	(0.6,0.3,0.7) (0.4,0.4,0.2) (0.2,0.4,0.5)	(0.6,0.3,0.3) (0.3,0.5,0.4) (0.1,0.4,0.5)	(0.6,0.3,0.1) (0.4,0.5,0.8) (0.2,0.4,0.3)	(0.4,0.4,0.5) (0.3,0.2,0.7) (0.1,0.5,0.5)
P_3	(0.8,0.3,0.5) (0.7,0.5,0.4) (0.6,0.4,0.4)	(0.5,0.5,0.3) (0.1,0.6,0.4) (0.3,0.4,0.3)	(0.3,0.3,0.6) (0.2,0.5,0.7) (0.1,0.4,0.5)	(0.6,0.2,0.5) (0.5,0.3,0.6) (0.2,0.2,0.6)	(0.6,0.4,0.5) (0.3,0.3,0.4) (0.2,0.2,0.6)

Let the samples be taken at three different timings in a day (in 08:00,16:00,24:00)

Remark :At three different timings in a day (in 08:00,16:00,24:00)

P_1 upon the Temperature may have the disease 1 with chance (0.4, 0.3 , 0.4) at 08:00
P_1 upon the Temperature may have the disease 2 with chance (0.3, 0.4 , 0.6) at 16:00
P_1 upon the Temperature may have the disease 3 with chance (0.2, 0.5 , 0.5) at 24:00

Table II: R (the relation among Symptoms and Diseases)

R	Viral Fever	Tuberculosis	Typhoid	Throat disease
Temperature	(0.2,0.5,0.6)	(0.4,0.6,0.5)	(0.6,0.4,0.5)	(0.3,0.7,0.8)
Cough	(0.6,0.4,0.6)	(0.8,0.2,0.3)	(0.3,0.2,0.6)	(0.2,0.4,0.1)
Throat Pain	(0.5,0.2,0.3)	(0.4,0.5,0.3)	(0.4,0.5,0.5)	(0.2,0.6,0.2)
Headache	(0.6,0.8,0.2)	(0.2,0.3,0.6)	(0.1,0.6,0.3)	(0.2,0.5,0.5)
Body Pain	(0.7,0.4,0.4)	(0.2,0.3,0.4)	(0.2,0.3,0.4)	(0.2,0.2,0.3)

Table III: The Correlation Measure between NRS Q and R

Cosine similarity measure	Viral Fever	Tuberculosis	Typhoid	Throat diseas
P_1	0.9793	0.9915	**0.9896**	0.9794
P_2	0.9831	0.9900	**0.9870**	0.9723
P_3	0.9811	0.9931	**0.9917**	0.9822

The highest correlation measure from the Table III gives the proper medical diagnosis. Therefore, patient P_1, P_2 and P_3 suffers from Tuberculosis

5.Conclusion

In this paper, we have extended the improved cosine similarity of single valued neutrosophic set proposed by J.Ye [17] to the case of neutrosophic refined sets and proved some of their basic properties. We have present an application of cosine similarity measure of neutrosophic refined sets in medical diagnosis problems. In The future work, we will extend this cosine similarity measure to the case of interval neutrosophic refined sets.

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Dice Similarity Measure between Single Valued Neutrosophic Multisets and Its Application in Medical Diagnosis

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Abstract. This paper introduces the concept of a single valued neutrosophic multiset (SVNM) as a generalization of an intuitionistic fuzzy multiset (IFM) and some basic operational relations of SVNMs, and then proposes the Dice similarity measure and the weighted Dice similarity measure for SVNMs and investigates their properties. Finally, the Dice similarity measure is applied to a medical diagnosis problem with SVNM information. This diagnosis method can deal with the medical diagnosis problem with indeterminate and inconsistent information which cannot be handled by the diagnosis method based on IFMs.

Keywords: Single valued neutrosophic set, multiset, single valued neutrosophic multiset, Dice similarity measure, medical diagnosis.

1 Introduction

In medical diagnosis problems, physicians can obtain a lot of information from modern medical technologies, which is often incomplete and indeterminate information due to the complexity of various diseases. Therefore, real medical diagnosis contains lots of incomplete and uncertainty information, which is a usual phenomenon of medical diagnosis problems. To represent incomplete and uncertainty information, Atanassov [1] introduced intuitionistic fuzzy sets (IFSs) as a generalization of fuzzy sets [2]. The prominent characteristic of IFS is that a membership degree and a non-membership degree are assigned to each element in the set. Then, various medical diagnosis methods have been presented under intuitionistic fuzzy environments [3, 4]. Recently, Ye [5] proposed a cosine similarity measure between IFSs and applied it to pattern recognition and medical diagnosis. Hung [6] introduced an intuitionistic fuzzy likelihood-based measurement and applied it to the medical diagnosis and bacteria classification problems. Further, Tian [7] developed the cotangent similarity measure of IFSs and applied it to medical diagnosis.

As a generalization of fuzzy sets and IFSs, Wang et al. [8] introduced a single valued neutrosophic set (SVNS) as a subclass of the neutrosophic set proposed by Smarandache [9]. SVNS consists of the three terms like the truth-membership, indeterminacy-membership and falsity-membership functions and can be better to express indeterminate and inconsistent information, but fuzzy sets and IFSs cannot handle indeterminate and inconsistent information. However, similarity measures play an important role in the analysis and research of medical diagnosis, pattern recognition, machine learning, decision making, and clustering analysis in uncertainty environment. Therefore, various similarity measures of SVNSs have been proposed and mainly applied them to decision making and clustering analysis. For instance, Majumdar and Samanta [10] introduced several similarity measures of SVNSs based on distances, a matching function, membership grades, and then proposed an entropy measure for a SVNS. Ye [11] proposed three vector similarity measures for simplified neutrosophic sets (SNSs), including the Jaccard, Dice, and cosine similarity measures for SVNSs and interval neutrosophic sets (INSs), and applied them to multicriteria decision-making problems with simplified neutrosophic information. Ye [12] and Ye and Zhang [13] further proposed the similarity measures of SVNSs for decision making problems. Furthermore, Ye [14] put forward distance-based similarity measures of SVNSs and applied them to clustering analysis.

In real medical diagnosis problems, however, by only taking one time inspection, we wonder whether one can obtain a conclusion from a particular person with a particular decease or not. Sometimes he/she may also show the symptoms of different diseases. Then, how can we give a proper conclusion? One solution is to examine the patient at different time intervals (e.g. two or three times a day). In this case, a fuzzy multiset concept introduced by Yager [15] is very suitable for expressing this information at different time intervals, which allows the repeated occurrences of any element. Thus, the fuzzy multiset can occur more than once with the possibility of the same or different membership values. Then, Shinoj and Sunil [16] extended the fuzzy multiset to the intuitionistic fuzzy multiset (IFM) and presented some basic operations and a distance measure for IFMs, and then applied the distance measure to medical diagnosis problem. Rajarajeswari and Uma [17] presented the Hamming distance-based similarity measure for IFMs and its application in medical diagnosis. However, existing IFMs cannot represent and deal with the indeterminacy and inconsistent information which exists in real situations (e.g. medicine diagnosis problems). To handle the medical diagnosis problems with indeterminacy and inconsistent information, the aims of this paper are: (1) to introduce a single valued neutrosophic multiset (SVNM) as a generalization of IFMs and some operational relations for SVNMs, (2) to propose the Dice similarity measure of SVNMs, (3) to apply the Dice similarity measure to medical diagnosis.

The rest of the article is organized as follows. Section 2 introduces some basic concepts of IFSs, IFMs, and SVNSs. Sections 3 introduces a concept of SVNM and some operational relations of SVNMs. In Section 4, we present the Dice similarity measure and the weighted Dice similarity measure for SVNMs and investigate their properties. In Section 5, we apply the proposed similarity measure to a medical diagnosis problem. Conclusions and further research are contained in Section 6.

2 Preliminaries

2.1 Some basic concepts of IFSs and IFMs

Atanassov [1] introduced IFSs as an extension of fuzzy sets [2] and gave the following definition.

Definition 1 [1]. An IFS A in the universe of discourse X is defined as $A=\{\langle x, \mu_A(x), \nu_A(x)\rangle \mid x \in X\}$, where $\mu_A(x)$: $X \to [0, 1]$ and $\nu_A(x)$: $X \to [0, 1]$ are the membership degree and non-membership degree of the element x to the set A with the condition $0 \le \mu_A(x) + \nu_A(x) \le 1$ for $x \in X$.

Then, $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$ is called Atanassov's intuitionistic index or a hesitancy degree of the element x in the set A. obviously there is $0 \le \pi_A(x) \le 1$ for $x \in X$.

Further, Shinoj and Sunil [16] introduced an IFM concept by combining the two concepts for IFSs and fuzzy multisets together and gave the following definition.

Definition 2 [16]. Let X be a nonempty set. Then, an IFM drawn from X is characterized by the two functions: count membership of CM_A and count non-membership of CN_A such that $CM_A(x)$: $X \to R$ and $CN_A(x)$: $X \to R$ for $x \in X$, where R is the set of all real number multisets drawn from the unit interval [0, 1]. Thus, an IFM A is denoted by

$$A = \left\{\left\langle x, (\mu_A^1(x), \mu_A^2(x), \ldots, \mu_A^q(x)), (\nu_A^1(x), \nu_A^2(x), \ldots, \nu_A^q(x))\right\rangle \mid x \in X\right\},$$

where the membership sequence $(\mu_A^1(x), \mu_A^2(x), \ldots, \mu_A^q(x))$ is a decreasingly ordered sequence $\mu_A^1(x) \ge \mu_A^2(x) \ge, \ldots \ge, \mu_A^q(x)$, the corresponding non-membership sequence $(\nu_A^1(x), \nu_A^2(x), \ldots, \nu_A^q(x))$ may not be in decreasing or increasing order, and the sum of $\mu_A^i(x)$ and $\nu_A^i(x)$ satisfies the condition $0 \le \mu_A^i(x) + \nu_A^i(x) \le 1$ for $x \in X$ and $i = 1, 2, \ldots, q$.

For convenience, an IFM A can be denoted by the following simplified form:

$$A = \left\{\left\langle x, \mu_A^i(x), \nu_A^i(x)\right\rangle \mid x \in X, i = 1,2,\ldots,q\right\}.$$

Let $A = \left\{\left\langle x, \mu_A^i(x), \nu_A^i(x)\right\rangle \mid x \in X, i = 1,2,\ldots,q\right\}$ and $B = \left\{\left\langle x, \mu_B^i(x), \nu_B^i(x)\right\rangle \mid x \in X, i = 1,2,\ldots,q\right\}$ be two IFMs. Then there are the following relations [16]:

(1) Complement: $A^c = \left\{\left\langle x, \nu_A^i(x), \mu_A^i(x)\right\rangle \mid x \in X, i = 1,2,\ldots,q\right\}$;

(2) Inclusion: $A \subseteq B$ if and only if $\mu_A^i(x) \le \mu_B^i(x)$, $\nu_A^i(x) \ge \nu_B^i(x)$ for $i = 1, 2, \ldots, q$ and $x \in X$;

(3) Equality: $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$;

(4) Union:

$$A \cup B = \left\{\left\langle x, \mu_A^i(x) \vee \mu_B^i(x), \nu_A^i(x) \wedge \nu_B^i(x)\right\rangle \mid x \in X, i = 1,2,\ldots,q\right\};$$

(5) Intersection:

$$A \cap B = \left\{\left\langle x, \mu_A^i(x) \wedge \mu_B^i(x), \nu_A^i(x) \vee \nu_B^i(x)\right\rangle \mid x \in X, i = 1,2,\ldots,q\right\};$$

(6) Addition:

$$A + B = \left\{\begin{matrix}\left\langle x, \mu_A^i(x) + \mu_B^i(x) - \mu_A^i(x)\mu_B^i(x), \nu_A^i(x)\nu_B^i(x)\right\rangle \\ \mid x \in X, i = 1,2,\ldots,q\end{matrix}\right\};$$

(7) Multiplication:

$$A \times B = \left\{\begin{matrix}\left\langle x, \mu_A^i(x)\mu_B^i(x), \nu_A^i(x) + \nu_B^i(x) - \nu_A^i(x)\nu_B^i(x),\right\rangle \\ \mid x \in X, i = 1,2,\ldots,q\end{matrix}\right\}.$$

2.2 Some concepts of SVNSs

Smarandache [9] originally presented the concept of a neutrosophic set from philosophical point of view. A neutrosophic set A in a universal set X is characterized by a truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$, and a falsity-membership function $F_A(x)$. The functions $T_A(x)$, $I_A(x)$, $F_A(x)$ in X are real standard or nonstandard subsets of $]^-0, 1^+[$, such that $T_A(x)$: $X \to\ ]^-0, 1^+[$, $I_A(x)$: $X \to\ ]^-0, 1^+[$, and $F_A(x)$: $X \to\ ]^-0, 1^+[$. Then, the sum of $T_A(x)$, $I_A(x)$ and $F_A(x)$ satisfies $^-0 \le \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \le 3^+$.

However, the neutrosophic set introduced from philosophical point of view is difficult to apply it to practical applications. Thus, Wang et al. [8] introduced a SVNS as a subclass of the neutrosophic set and the following definition of SVNS.

Definition 3 [8]. Let X be a universal set. A SVNS A in X is characterized by a truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$, and a falsity-membership function $F_A(x)$. Then, a SVNS A can be denoted as

$$A = \left\{ \left\langle x, T_A(x), I_A(x), F_A(x) \right\rangle \mid x \in X \right\},$$

where the sum of $T_A(x)$, $I_A(x)$, $F_A(x) \in [0, 1]$ satisfies $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$ for each x in X.

For two SVNSs $A = \left\{ \left\langle x, T_A(x), I_A(x), F_A(x) \right\rangle \mid x \in X \right\}$ and $B = \left\{ \left\langle x, T_B(x), I_B(x), F_B(x) \right\rangle \mid x \in X \right\}$, there are the following relations [8]:

(1) Complement: $A^c = \left\{ \left\langle x, F_A(x), 1 - I_A(x), T_A(x) \right\rangle \mid x \in X \right\}$;

(2) Inclusion: $A \subseteq B$ if and only if $T_A(x) \leq T_B(x)$, $I_A(x) \geq I_B(x)$, $F_A(x) \geq F_B(x)$ for any x in X;

(3) Equality: $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$;

(4) Union:
$$A \bigcup B = \left\{ \left\langle x, T_A(x) \vee T_B(x), I_A(x) \wedge I_B(x), F_A(x) \wedge F_B(x) \right\rangle \mid x \in X \right\};$$

(5) Intersection:
$$A \bigcap B = \left\{ \left\langle x, T_A(x) \wedge T_B(x), I_A(x) \vee I_B(x), F_A(x) \vee F_B(x) \right\rangle \mid x \in X \right\}.$$

3 Single valued neutrosophic multisets

This section introduces SVNMs as a generalization of SVNSs and IFMs and some operational relations for SVNMs.

Definition 4. Let X be a nonempty set with generic elements in X denoted by x. A SVNM A drawn from X is characterized by the three functions: count truth-membership of CT_A, count indeterminacy-membership of CI_A, and count falsity-membership of CF_A such that $CT_A(x)$: $X \to R$, $CI_A(x)$: $X \to R$, $CF_A(x)$: $X \to R$ for $x \in X$, where R is the set of all real number multisets in the real unit interval [0, 1]. Then, a SVNM A is denoted by

$$A = \left\{ \left\langle \begin{array}{l} x, (T_A^1(x), T_A^2(x), ..., T_A^q(x)), (I_A^1(x), I_A^2(x), ..., \\ I_A^q(x)), (F_A^1(x), F_A^2(x), F_A^q(x)) \end{array} \right\rangle \mid x \in X \right\},$$

where the truth-membership sequence $(T_A^1(x), T_A^2(x), ..., T_A^q(x))$, the indeterminacy-membership sequence $(I_A^1(x), I_A^2(x), ..., I_A^q(x))$, and the falsity-membership sequence $(F_A^1(x), F_A^2(x), ..., F_A^q(x))$ may be in decreasing or increasing order, and the sum of $T_A^i(x)$, $I_A^i(x)$, $F_A^i(x) \in [0, 1]$ satisfies the condition $0 \leq \sup T_A^i(x) + \sup I_A^i(x) + \sup F_A^i(x) \leq 3$ for $x \in X$ and $i = 1, 2, \ldots, q$.

For convenience, a SVNM A can be denoted by the simplified form:

$$A = \left\{ \left\langle x, T_A^i(x), I_A^i(x), F_A^i(x) \right\rangle \mid x \in X, i = 1, 2, ..., q \right\}.$$

Definition 5. The length of an element x in a SVNM is defined as the cardinality of $CT_A(x)$ or $CI_A(x)$, or $CF_A(x)$ and is denoted by $L(x$: $A)$. Then $L(x: A) = |CT_A(x)| = |CI_A(x)| = |CF_A(x)|$.

Definition 6. Let A and B be two SVNMs in X, then the length of an element x in A and B is denoted by $l_x = L(x: A, B) = \max\{L(x: A), L(x: B)\}$.

For example, we consider SVNMs in the set $X = \{x_1, x_2, x_3\}$ as

$A = \{<x_1$, (0.1, 0.2), (0.2, 0.3), (0.6, 0.8)>, < x_2, (0.3, 0.4, 0.5), (0.2, 0.3, 0.4), (0.5, 0.6, 0.7)>},

$B = \{<x_1$, (0.2), (0.2), (0.4) >, < x_3, (0.3, 0.4, 0.5, 0.6), (0.1, 0.2, 0.3, 0.4), (0.1, 0.2, 0.3, 0.5)>}.

Thus, there are $L(x_1: A) = 2$, $L(x_2: A) = 3$, $L(x_3: A) = 0$; $L(x_1: B) = 1$, $L(x_2: B) = 0$, $L(x_3: B) = 4$, $l_{x1} = L(x_1: A, B) = 2$, $l_{x2} = L(x_2: A, B) = 3$, and $l_{x3} = L(x_3: A, B) = 4$.

For convenient operation between SVNMs A and B in X, one can make $L(x: A) = L(x: B)$ by appending sufficient minimal numbers for the truth-membership degree and sufficient maximum numbers for the indeterminacy-membership and falsity-membership degrees as pessimists or sufficient maximum numbers for the truth-membership value and sufficient minimal numbers for the indeterminacy-membership and falsity-membership values as optimists.

Definition 7. Let $A = \{\langle x, T_A^i(x), I_A^i(x), F_A^i(x) \mid x \in X$, $i = 1, 2, \ldots, q\}$ and $B = \{\langle x, T_B^i(x), I_B^i(x), F_B^i(x) \mid x \in X$, $i = 1, 2, \ldots, q\}$ be two SVNMs in X. Then, there are the following relations:

(1) Inclusion: $A \subseteq B$ if and only if $T_A^i(x) \leq T_B^i(x)$, $I_A^i(x) \geq I_B^i(x)$, $F_A^i(x) \geq F_B^i(x)$ for $i = 1, 2, \ldots, q$ and $x \in X$;

(2) Equality: $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$;

(3) Complement:
$$A^c = \left\{ \left\langle x, F_A^i(x), 1 - I_A^i(x), T_A^i(x) \right\rangle \mid x \in X, i = 1, 2, ..., q \right\};$$

(4) Union:
$$A \bigcup B = \left\{ \begin{array}{l} \left\langle x, T_A^i(x) \vee T_B^i(x), I_A^i(x) \wedge I_B^i(x), F_A^i(x) \wedge F_B^i(x) \right\rangle \\ \mid x \in X, i = 1, 2, ..., q \end{array} \right\};$$

(5) Intersection:
$$A \bigcap B = \left\{ \begin{array}{l} \left\langle x, T_A^i(x) \wedge T_B^i(x), I_A^i(x) \vee I_B^i(x), F_A^i(x) \vee F_B^i(x) \right\rangle \\ \mid x \in X, i = 1, 2, ..., q \end{array} \right\}.$$

4 Dice similarity measure of SVNMs

In this section, we propose the Dice similarity measure and the weighted Dice similarity measure for SVNMs and investigate their properties.

Definition 8. Let $A = \{\langle x_j, T_A^i(x_j), I_A^i(x_j), F_A^i(x_j) \mid x_j \in X,\ i = 1, 2, \ldots, q\}$ and $B = \{\langle x_j, T_B^i(x_j), I_B^i(x_j), F_B^i(x_j) \mid x_j \in X,\ i = 1, 2, \ldots, q\}$ be any two SVNMs in $X = \{x_1, x_2, \ldots, x_n\}$. Then, we define the following Dice similarity measure between A and B:

$$S_D(A,B) = \frac{1}{n}\sum_{j=1}^{n} \frac{\frac{2}{l_j}\sum_{i=1}^{l_j}\left[\begin{array}{l} T_A^i(x_j)T_B^i(x_j) + I_A^i(x_j)I_B^i(x_j) \\ + F_A^i(x_j)F_B^i(x_j) \end{array}\right]}{\left(\begin{array}{l} \frac{1}{l_j}\sum_{i=1}^{l_j}\left[\left(T_A^i(x_j)\right)^2 + \left(I_A^i(x_j)\right)^2 + \left(F_A^i(x_j)\right)^2\right] \\ + \frac{1}{l_j}\sum_{i=1}^{l_j}\left[\left(T_B^i(x_j)\right)^2 + \left(I_B^i(x_j)\right)^2 + \left(F_B^i(x_j)\right)^2\right] \end{array}\right)}, \tag{1}$$

where $l_j = L(x_j: A, B) = \max\{L(x_j: A), L(x_j: B)\}$ for $j = 1, 2, \ldots, n$.

Then, the Dice similarity measure has the following Proposition 1:

Proposition 1. For two SVNMs A and B in $X = \{x_1, x_2, \ldots, x_n\}$, the Dice similarity measure $S_D(A, B)$ should satisfy the following properties (P1)-(P3):

(P1) $0 \le S_D(A, B) \le 1$;

(P2) $S_D(A, B) = S_D(B, A)$;

(P3) $S_D(A, B) = 1$ if $A = B$, i.e., $T_A^i(x_j) = T_B^i(x_j)$, $I_A^i(x_j) = I_B^i(x_j)$, $F_A^i(x_j) = F_B^i(x_j)$ for every $x_j \in X$, $j = 1, 2, \ldots, n$, and $i = 1, 2, \ldots, q$.

Proof:

(P1) It is obvious that the property is true according to the inequality $a^2 + b^2 \ge 2ab$ for Eq. (1).

(P2) It is straightforward.

(P3) If $A = B$, then there are $T_A^i(x_j) = T_B^i(x_j)$, $I_A^i(x_j) = I_B^i(x_j)$, $F_A^i(x_j) = F_B^i(x_j)$ for every $x_j \in X$, $j = 1, 2, \ldots, n$ and $i = 1, 2, \ldots, q$. Hence there is $S_D(A, B) = 1$. □

Taking the weight w_j of each element x_j ($j = 1, 2, \ldots, n$) into account with $w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$, we introduce the following weighted Dice similarity measure between SVNMs A and B:

$$W_D(A,B) = \sum_{j=1}^{n} w_j \frac{\frac{2}{l_j}\sum_{i=1}^{l_j}\left[\begin{array}{l} T_A^i(x_j)T_B^i(x_j) + I_A^i(x_j)I_B^i(x_j) \\ + F_A^i(x_j)F_B^i(x_j) \end{array}\right]}{\left(\begin{array}{l} \frac{1}{l_j}\sum_{i=1}^{l_j}\left[\left(T_A^i(x_j)\right)^2 + \left(I_A^i(x_j)\right)^2 + \left(F_A^i(x_j)\right)^2\right] \\ + \frac{1}{l_j}\sum_{i=1}^{l_j}\left[\left(T_B^i(x_j)\right)^2 + \left(I_B^i(x_j)\right)^2 + \left(F_B^i(x_j)\right)^2\right] \end{array}\right)}, \tag{2}$$

where $l_j = L(x_j: A, B) = \max\{L(x_j: A), L(x_j: B)\}$ for $j = 1, 2, \ldots, n$. If $\mathbf{W} = (1/n, 1/n, \ldots, 1/n)^T$, then Eq. (2) reduces to Eq. (1).

Then, the weighted Dice similarity measure has the following Proposition 2:

Proposition 2. For two SVNMs A and B in $X = \{x_1, x_2, \ldots, x_n\}$, the weighted Dice similarity measure $W_D(A, B)$ should satisfy the following properties (P1)-(P3):

(P1) $0 \le W_D(A, B) \le 1$;

(P2) $W_D(A, B) = W_D(B, A)$;

(P3) $W_D(A, B) = 1$ if $A = B$, i.e., $T_A^i(x_j) = T_B^i(x_j)$, $I_A^i(x_j) = I_B^i(x_j)$, $F_A^i(x_j) = F_B^i(x_j)$ for every $x_j \in X$, $j = 1, 2, \ldots, n$ and $i = 1, 2, \ldots, q$.

By a similar proof method of Proposition 1, we can prove that the properties (P1)–(P3).

5 Medical diagnosis using the Dice similarity measure

In this section, we apply the Dice similarity measure to the medical diagnosis problem with SVNM information. The details of a typical example adapted from [16] are given below.

Let $P = \{P_1, P_2, P_3, P_4\}$ be a set of four patients, $D = \{D_1, D_2, D_3, D_4\}$ = {Viral fever, Tuberculosis, Typhoid, Throat disease} be a set of diseases, and $S = \{S_1, S_2, S_3, S_4, S_5\}$ = {Temperature, Cough, Throat pain, Headache, Body pain} be a set of symptoms. In the medical diagnosis problem, when we have to take three different samples in three different times in a day (e.g. morning, noon and night), the characteristic values between patients and the indicated symptoms are represented by the following SVNMs:

P_1 ={<S_1, (0.8, 0.6, 0.5), (0.3, 0.2, 0.1), (0.4, 0.2, 0.1)>, <S_2, (0.5, 0.4, 0.3), (0.4, 0.4, 0.3), (0.6, 0.3, 0.4)>, <S_3, (0.2, 0.1, 0.0), (0.3, 0.2, 0.2), (0.8, 0.7, 0.7)>, <S_4, (0.7, 0.6, 0.5), (0.3, 0.2, 0.1), (0.4, 0.3, 0.2)>, <S_5, (0.4, 0.3, 0.2), (0.6, 0.5, 0.5), (0.6, 0.4, 0.4)>};

P_2 ={<S_1, (0.5, 0.4, 0.3), (0.3, 0.3, 0.2),(0.5, 0.4, 0.4)>, <S_2, (0.9, 0.8, 0.7), (0.2, 0.1, 0.1), (0.2, 0.1, 0.0)>, <S_3, (0.6, 0.5, 0.4), (0.3, 0.2, 0.2), (0.4, 0.3, 0.3)>, <S_4, (0.6, 0.4, 0.3), (0.3, 0.1, 0.1), (0.7, 0.7, 0.3)>, <S_5, (0.8, 0.7, 0.5), (0.4, 0.3, 0.1), (0.3, 0.2, 0.1)>;

$P_3 = \{\langle S_1, (0.2, 0.1, 0.1), (0.3, 0.2, 0.2), (0.8, 0.7, 0.6)\rangle, \langle S_2, (0.3, 0.2, 0.2), (0.4, 0.2, 0.2), (0.7, 0.6, 0.5)\rangle, \langle S_3, (0.8, 0.8, 0.7), (0.2, 0.2, 0.2), (0.1, 0.1, 0.0)\rangle, \langle S_4, (0.3, 0.2, 0.2), (0.3, 0.3, 0.3), (0.7, 0.6, 0.6)\rangle, \langle S_5, (0.4, 0.4, 0.3), (0.4, 0.3, 0.2), (0.7, 0.7, 0.5)\rangle\}$;

$P_4 = \{\langle S_1, (0.5, 0.5, 0.4), (0.3, 0.2, 0.2), (0.4, 0.4, 0.3)\rangle, \langle S_2, (0.4, 0.3, 0.1), (0.4, 0.3, 0.2), (0.7, 0.5, 0.3)\rangle, \langle S_3, (0.7, 0.1, 0.0), (0.4, 0.3, 0.3), (0.7, 0.7, 0.6)\rangle, \langle S_4, (0.6, 0.5, 0.3), (0.6, 0.2, 0.1), (0.6, 0.4, 0.3)\rangle, \langle S_5, (0.5, 0.1, 0.1), (0.3, 0.3, 0.2), (0.6, 0.5, 0.4)\rangle\}$.

Then, the characteristic values between symptoms and the considered diseases are represented by the form of SVNSs:

D_1 (Viral fever) = $\{\langle S_1, 0.8, 0.1, 0.1\rangle, \langle S_2, 0.2, 0.7, 0.1\rangle, \langle S_3, 0.3, 0.5, 0.2\rangle, \langle S_4, 0.5, 0.3, 0.2\rangle, \langle S_5, 0.5, 0.4, 0.1\rangle\}$;

D_2 (Tuberculosis) = $\{\langle S_1, 0.2, 0.7, 0.1\rangle, \langle S_2, 0.9, 0.0, 0.1\rangle, \langle S_3, 0.7, 0.2, 0.1\rangle, \langle S_4, 0.6, 0.3, 0.1\rangle, \langle S_5, 0.7, 0.2, 0.1\rangle\}$;

D_3 (Typhoid) = $\{\langle S_1, 0.5, 0.3, 0.2\rangle, \langle S_2, 0.3, 0.5, 0.2\rangle, \langle S_3, 0.2, 0.7, 0.1\rangle, \langle S_4, 0.2, 0.6, 0.2\rangle, \langle S_5, 0.4, 0.4, 0.2\rangle\}$;

D_4 (Throat disease) = $\{\langle S_1, 0.1, 0.7, 0.2\rangle, \langle S_2, 0.3, 0.6, 0.1\rangle, \langle S_3, 0.8, 0.1, 0.1\rangle, \langle S_4, 0.1, 0.8, 0.1\rangle, \langle S_5, 0.1, 0.8, 0.1\rangle\}$.

Then, by using Eq. (1), we can obtain the Dice similarity measure between each patient P_i (i = 1, 2, 3, 4) and the considered disease D_j (j = 1, 2, 3, 4), which are shown in Table 1.

Table 1 Measure values of $S_D(P_i, D_j)$

	D_1 (Viral fever)	D_2 (Tuberculosis)	D_3 (Typhoid)	D_4 (Throat disease)
P_1	0.7810	0.7753	**0.8007**	0.6946
P_2	**0.7978**	0.7656	0.7969	0.6826
P_3	0.7576	0.7063	**0.7807**	0.6492
P_4	0.8188	**0.8278**	0.8266	0.7139

In Tables 1, the largest similarity measure indicates the proper diagnosis. Hence, Patient P_1 suffers from typhoid, Patient P_2 suffers from viral fever, Patient P_3 also suffers from typhoid, and Patient P_4 suffers from tuberculosis.

6 Conclusion

This paper introduced a concept of SVNM and some basic operational relations of SVNMs, and then proposed the Dice similarity measure and the weighted Dice similarity measure for SVNMs and investigated their properties. Finally, the Dice similarity measure of SVNMs was applied to medicine diagnosis under the SVNM environment. The Dice similarity measure of SVNMs is effective in handling the medical diagnosis problems with indeterminate and inconsistent information which the similarity measures of IFMSs cannot handle, because IFMSs cannot express and deal with indeterminate and inconsistent information.

In further work, it is necessary and meaningful to extend SVNMs to interval neutrosophic multisets and their operations and measures and to investigate their applications such as decision making, pattern recognition, and medical diagnosis.

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Several Similarity Measures of Interval Valued Neutrosophic Soft Sets and Their Application in Pattern Recognition Problems

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Abstract. Interval valued neutrosophic soft set introduced by Irfan Deli in 2014[8] is a generalization of neutrosophic set introduced by F. Smarandache in 1995[19], which can be used in real scientific and engineering applications. In this paper the Hamming and Euclidean distances between two interval valued neutrosophic soft sets (IVNS sets) are defined and similarity measures based on distances between two interval valued neutrosophic soft sets are proposed. Similarity measure based on set theoretic approach is also proposed. Some basic properties of similarity measures between two interval valued neutrosophic soft sets is also studied. A decision making method is established for interval valued neutrosophic soft set setting using similarity measures between IVNS sets. Finally an example is given to demonstrate the possible application of similarity measures in pattern recognition problems.

Keywords: Soft set, Neutrosophic soft set, Interval valued neutrosophic soft set, Hamming distance, Euclidean distance, Similarity measure, pattern recognition.

1 Introduction

After the introduction of Fuzzy Set Theory by by Prof. L. A. Zadeh in 1965[27], several researchers have extended this concept in many directions. The traditional fuzzy sets is characterized by the membership value or the grade of membership value. Some times it may be very difficult to assign the membership value for a fuzzy set. Consequently the concept of interval valued fuzzy sets[28] was proposed to capture the uncertainty of grade of membership value. In some real life problems in expert system, belief system, information fusion and so on, we must consider the truth-membership as well as the falsity-membership for proper description of an object in uncertain, ambiguous environment. Neither the fuzzy sets nor the interval valued fuzzy sets is appropriate for such a situation. Intuitionistic fuzzy sets[1] introduced by Atanassov in 1986 and interval valued intuitionistic fuzzy sets[2] introduced by K. Atanassov and G. Gargov in 1989 are appropriate for such a situation. The intuitionistic fuzzy sets can only handle the incomplete information considering both the truth-membership (or simply membership) and falsity-membership (or non-membership) values. But it does not handle the indeterminate and inconsistent information which exists in belief system. F. Smarandache in 1995 introduced the concept of neutrosophic set[19], which is a mathematical tool for handling problems involving imprecise, indeterminacy and inconsistent data. Soft set theory[11,14] has enriched its potentiality since its introduction by Molodtsov in 1999. Using the concept of soft set theory P. K. Maji in 2013 introduced neutrosophic soft set[15] and Irfan Deli in 2014 introduced the concept of interval valued neutrosophic soft sets[8] . Neutrosophic sets and neutrosophic soft sets now become the most useful mathematical tools to deal with the problems which involves the indeterminate and inconsistent informations.

Similarity measure is an important topic in the fuzzy set theory. The similarity measure indicates the similar degree between two fuzzy sets. In [23] P. Z.Wang first introduced the concept of similarity measure of fuzzy sets and gave a computational formula. Science then, similarity measure of fuzzy sets has attracted several researchers ([3],[4],[5],[6],[7],[9],[10],[12],[13],[16],[17],[18],[22],[24],[25],[26]) interest and has been investigated more. Similarity measure of fuzzy sets is now being extensively applied in many research fields such as fuzzy clustering, image processing, fuzzy reasoning, fuzzy neural network, pattern recognition, medical diagnosis , game theory, coding theory and several problems that contain uncertainties.

Similarity measure of fuzzy values[5], vague sets[6], between vague sets and between elements[7], similarity measure of soft sets[12], similarity measure of intuitionistic fuzzy soft sets[4], similarity measures of interval-valued fuzzy soft sets have been studied by several researchers. Recently Said Broumi and Florentin Smarandache introduced the concept of several similarity measures of neutrosophic sets[3] , Jun Ye introduced the concept of similarity measures between interval neutrosophic sets[26] and A. Mukherjee and S. Sarkar intoduced similarity measures for neutrosophic soft sets [18]. In this paper the Hamming and Euclidean distances between two interval valued neutrosophic soft sets(IVNS sets) are defined and similarity measures between two IVNS sets based on distances are proposed. Similarity measures between two IVNS sets based on set theoretic approach also proposed in this paper. A decision making method is established based on the proposed similarity measures. An illustrative example demonstrates the application of proposed decision making method in pattern recognition problem.

The rest of the paper is organized as --- section 2: some preliminary basic definitions are given in this section. In section 3 similarity measures between two IVNS sets is defined with example. In section 4 similarity measures between two IVNS sets based on set theoretic approach is defined with example, weighted distances , similarity measures based on weighted distances is defined. Also some properties of similarity measures are studied. In section 5 a decision making method is established with an with an example in pattern recognition problem. In Section 6 a comparative study of similarity measures is given. Finally in section 7 some conclusions of the similarity measures between IVNS sets and the proposed decision making method are given.

2 Preliminaries

In this section we briefly review some basic definitions related to interval-valued neutrosophic soft sets which will be used in the rest of the paper.

Definition 2.1[27] Let X be a non empty collection of objects denoted by x. Then a fuzzy set (FS for short) α in X is a set of ordered pairs having the form $\alpha = \{(x, \mu_\alpha(x)) : x \in X\}$, where the function $\mu_\alpha : X \to [0,1]$ is called the membership function or grade of membership (also degree of compatibility or degree of truth) of x in α .The interval M = [0,1] is called membership space.

Definition 2.2[28] Let D[0, 1] be the set of closed subintervals of the interval [0, 1]. An interval-valued fuzzy set in X is an expression A given by

$A = \{(x, M_A(x)) : x \in X\}$,where M_A:X→D[0,1].

Definition 2.3[1] Let X be a non empty set. Then an intuitionistic fuzzy set (IFS for short) A is a set having the form A={(x, μ_A(x), γ_A(x)): x∈X} where the functions μ_A: X→[0,1] and γ_A: X→[0,1] represents the degree of membership and the degree of non-membership respectively of each element x∈X and 0≤μ_A(x)+γ_A(x)≤1 for each x∈X.

Definition 2.4[2] An interval valued intuitionistic fuzzy set A over a universe set U is defined as the object of the form A={<x, μ_A(x), γ_A(x)>: x∈U)}, where μ_A(x): U→D[0,1] and γ_A(x): U→D[0,1] are functions such that the condition: ∀x∈U, supμ_A(x)+supγ_A(x)≤1 is satisfied(where D[0,1] is the set of all closed subintervals of [0,1]).

Definition 2.5[11,14] Let U be an initial universe and E be a set of parameters. Let P(U) denotes the power set of U and A⊆ E. Then the pair (F,A) is called a soft set over U, where F is a mapping given by F:A→P(U).

Definition 2.6[19,20] A neutrosophic set A on the universe of discourse X is defined as $A = \{(x, T_A(x), I_A(x), F_A(x)), x \in X\}$ where T, I, F : $X \to]^-0,1^+[$ and $^-0 \le T_A(x) + I_A(x) + F_A(x) \le 3^+$.

From philosophical point of view, the neutrosophic set takes the value from real standard or non-standard subsets of $]^-0,1^+[$. But in real life application in scientific and engineering problems it is difficult to use neutrosophic set with value from real standard or non-standard subset of $]^-0,1^+[$. Hence we consider the neutrosophic set which takes the value from the subset of [0,1] that is

$$0 \le T_A(x) + I_A(x) + F_A(x) \le 3.$$

Where T_A(x) is called truth-membership function, I_A(x) is called an indeterminacy-membership function and F_A(x) is called a falsity membership function

Definition 2.7[15] Let U be the universe set and E be the set of parameters. Also let A ⊆ E and P(U) be the set of all neutrosophic sets of U. Then the collection (F, A) is called neutrosophic soft set over U, where F is a mapping given by F: A → P(U).

Definition 2.8[21] Let U be a space of points (objects), with a generic element in U. An interval value neutrosophic set (IVN-set) A in U is characterized by truth membership function T_A, a indeterminacy-membership function I_A and a falsity- membership function F_A. For each point u∈ U; T_A, I_A and F_A ⊆ $[0,1]$.
Thus a IVN-set A over U is represented as

$$A = \{(T_A(u), I_A(u), F_A(u)) : u \in U\}$$

Where $0 \le \sup(T_A(u) + \sup I_A(u) + \sup F_A(u) \le 3$ and $(T_A(u), I_A(u), F_A(u))$ is called interval value neutrosophic number for all u ∈ U .

Definition 2.9[8] Let U be an initial universe set, E be a set of parameters and A ⊆ E. Let IVNS(U) denotes the set of all interval value neutrosophic subsets of U. The collection (F,A) is termed to be the interval valued neutrosophic soft set over U, where F is a mapping given by F: A → IVNS(U).

3 Similarity measure between two IVNS sets based on distances

In this section we define Hamming and Euclidean distances between two interval valued neutrosophic soft sets and proposed similarity measures based on theses distances.

Definition 3.1 Let $U=\{x_1,x_2,x_3,.......,x_n\}$ be an initial universe and $E=\{e_1,e_2,e_3,.......,x_m\}$ be a set of parameters. Let IVNS(U) denotes the set of all interval valued neutrosophic subsets of U. Also let (N_1,E) and (N_2,E) be two interval valued neutrosophic soft sets over U, where N_1 and N_2 are mappings given by N_1,N_2: E → IVNS(U). We define the following distances between (N_1, E) and (N_2, E) as follows:

1. Hamming Distance:

$$D_H(N_1,N_2)=\frac{1}{6}\sum_{i=1}^{n}\sum_{j=1}^{m}\Big\{\left|\overline{T}_{N_1}(x_i)(e_j)-\overline{T}_{N_2}(x_i)(e_j)\right|+\left|\overline{I}_{N_1}(x_i)(e_j)-\overline{I}_{N_2}(x_i)(e_j)\right|+\left|\overline{F}_{N_1}(x_i)(e_j)-\overline{F}_{N_2}(x_i)(e_j)\right|\Big\}$$

2. Normalized Hamming distance:

$$D_H(N_1,N_2)=\frac{1}{6n}\sum_{i=1}^{n}\sum_{j=1}^{m}\Big\{\left|\overline{T}_{N_1}(x_i)(e_j)-\overline{T}_{N_2}(x_i)(e_j)\right|+\left|\overline{I}_{N_1}(x_i)(e_j)-\overline{I}_{N_2}(x_i)(e_j)\right|+\left|\overline{F}_{N_1}(x_i)(e_j)-\overline{F}_{N_2}(x_i)(e_j)\right|\Big\}$$

3. Euclidean distance:

$$D_E(N_1,N_2)=\left[\frac{1}{6}\sum_{i=1}^{n}\sum_{j=1}^{m}\Big\{\left(\overline{T}_{N_1}(x_i)(e_j)-\overline{T}_{N_2}(x_i)(e_j)\right)^2+\left(\overline{I}_{N_1}(x_i)(e_j)-\overline{I}_{N_2}(x_i)(e_j)\right)^2+\left(\overline{F}_{N_1}(x_i)(e_j)-\overline{F}_{N_2}(x_i)(e_j)\right)^2\right]^{\frac{1}{2}}$$

4. Normalized Euclidean distance:

$$D_E(N_1,N_2)=\left[\frac{1}{6n}\sum_{i=1}^{n}\sum_{j=1}^{m}\Big\{\left(\overline{T}_{N_1}(x_i)(e_j)-\overline{T}_{N_2}(x_i)(e_j)\right)^2+\left(\overline{I}_{N_1}(x_i)(e_j)-\overline{I}_{N_2}(x_i)(e_j)\right)^2+\left(\overline{F}_{N_1}(x_i)(e_j)-\overline{F}_{N_2}(x_i)(e_j)\right)^2\Big\}\right]^{\frac{1}{2}}$$

Where

$$\overline{T}_{N_1}(x_i)(e_j)=\frac{1}{2}\left\{\inf T_{N_1}(x_i)(e_j)+\sup T_{N_1}(x_i)(e_j)\right\}$$

$$\overline{I}_{N_1}(x_i)(e_j)=\frac{1}{2}\left\{\inf I_{N_1}(x_i)(e_j)+\sup I_{N_1}(x_i)(e_j)\right\}$$

$$\overline{F}_{N_1}(x_i)(e_j)=\frac{1}{2}\left\{\inf F_{N_1}(x_i)(e_j)+\sup F_{N_1}(x_i)(e_j)\right\}\quad etc.$$

Definition 3.2 Let U be universe and E be the set of parameters and (N_1,E) , (N_2,E) be two interval valued neutrosophic soft sets over U. Then based on the distances defined in definition 3.1 similarity measure between (N_1,E) and (N_2,E) is defined as

$$SM(\mathrm{N_1,N_2})=\frac{1}{1+D(N_1,N_2)}\ \text{…………} \ (3.1)$$

Another similarity measure of (N_1,E) and (N_2,E) can also be defined as

$$SM(\mathrm{N_1,N_2})=\mathrm{e}^{-\alpha \mathrm{D(N_1,N_2)}}\ \text{……………….} \ (3.2)$$

Where D(N_1,N_2) is the distance between the interval valued neutrosophic soft sets (N_1,E) and (N_2,E) and α is a positive real number, called steepness measure.

Definition 3.3 Let U be universe and E be the set of parameters and (N_1,E) , (N_2,E) be two interval valued neutrosophic soft sets over U. Then we define the following distances between (N_1,E) , (N_2,E) as follows:

$$D(N_1,N_2)=\left[\frac{1}{6}\sum_{i=1}^{n}\sum_{j=1}^{m}\Big\{\left|\overline{T}_{N_1}(x_i)(e_j)-\overline{T}_{N_2}(x_i)(e_j)\right|^p+\left|\overline{I}_{N_1}(x_i)(e_j)-\overline{I}_{N_2}(x_i)(e_j)\right|^p+\left|\overline{F}_{N_1}(x_i)(e_j)-\overline{F}_{N_2}(x_i)(e_j)\right|^p\Big\}\right]^{\frac{1}{p}}$$

……………… (3.3)

and

$$D(N_1,N_2)=\left[\frac{1}{6n}\sum_{i=1}^{n}\sum_{j=1}^{m}\Big\{\left|\overline{T}_{N_1}(x_i)(e_j)-\overline{T}_{N_2}(x_i)(e_j)\right|^p+\left|\overline{I}_{N_1}(x_i)(e_j)-\overline{I}_{N_2}(x_i)(e_j)\right|^p+\left|\overline{F}_{N_1}(x_i)(e_j)-\overline{F}_{N_2}(x_i)(e_j)\right|^p\Big\}\right]^{\frac{1}{p}}$$

……………… (3.4)

Where p > 0. If p = 1 then equation (3.3) and (3.4) are respectively reduced to Hamming distance and Normalized Hamming distance. Again if p = 2 then equation (3.3) and (3.4) are respectively reduced to Euclidean distance and Normalized Euclidean distance.

The weighted distance is defined as

$$D^w(N_1,N_2)=\left[\frac{1}{6}\sum_{i=1}^{n}\sum_{j=1}^{m}w_i\left\{\left|\overline{T}_{N_1}(x_i)(e_j)-\overline{T}_{N_2}(x_i)(e_j)\right|^p+\left|\overline{I}_{N_1}(x_i)(e_j)-\overline{I}_{N_2}(x_i)(e_j)\right|^p+\left|\overline{F}_{N_1}(x_i)(e_j)-\overline{F}_{N_2}(x_i)(e_j)\right|^p\right\}\right]^{\frac{1}{p}} \quad \text{………………(3.5)}$$

Where $w=(w_1,w_2,w_3,\ldots,w_n)^T$ is the weight vector of x_i (i = 1,2,3, …. ,n) and p > 0. Especially, if p = 1 then (3.5) is reduced to the weighted Hamming distance and if p = 2, then (3.5) is reduced to the weighted Euclidean distance.

Definition 3.4 Based on the weighted distance between two interval valued neutrosophic soft sets (N_1,E) and (N_2,E) given by equation (3.6) , the similarity measure between (N_1,E) and (N_2,E)) is defined as

$$\mathrm{SM}(N_1,N_2)=\frac{1}{1+D^w(N_1,N_2)} \quad \text{………..(3.6)}$$

Example 3.5 Let U={x_1,x_2,x_3} be the universal set and E={e_1,e_2} be the set of parameters. Let (N_1,E) and (N_2,E) be two interval-valued neutrosophic soft sets over U such that their tabular representations are as follows:

N_1	e_1	e_2
x_1	[0.1,0.3],[0.3,0.6],[0.8,0.9]	[0.7,0.8],[0.6,0.7],[0.4,0.5]
x_2	[0.4,0.5],[0.2,0.3],[0.1,0.2]	[0.6,0.8],[0.4,0.5],[0.5,0.6]
x_3	[0.3,0.5],[0.3,0.4],[0.2,0.4]	[0.9,1.0],[0.4,0.5],[0.6,0.7]

Table 1: tabular representation of (N_1,E)

N_2	e_1	e_2
x_1	[0.2,0.3],[0.4,0.5],[0.7,0.9]	[0.7,0.8],[0.5,0.7],[0.3,0.5]
x_2	[0.3,0.5],[0.2,0.4],[0.4,0.6]	[0.6,0.7],[0.3,0.5],[0.4,0.6]
x_3	[0.4,0.5],[0.3,0.4],[0.7,0.8]	[0.8,0.9],[0.2,0.5],[0.5,0.8]

Table 2: tabular representation of (N_2,E)

Now by definition 3.1 the Hamming distance between (N_1,E) and (N_2,E) is given by $D_H(N_1,N_2)$ = 0.25 and hence by equation (3.1) similarity measure between (N_1,E) and (N_2,E) is given by SM(N_1,N_2) = 0.80 .

4. Similarity measure between two ivns sets based on set theoretic approach

Definition 4.1 Let $U=\{x_1,x_2,x_3,\ldots\ldots,x_n\}$ be an initial universe and $E=\{e_1,e_2,e_3,\ldots\ldots,x_m\}$ be a set of parameters. Let IVNS(U) denotes the set of all interval valued neutrosophic subsets of U. Also let (N_1,E) and (N_2,E) be two interval valued neutrosophic soft sets over U, where N_1 and N_2 are mappings given by N_1,N_2: E → IVNS(U). We define similarity measure *SM*(N_1,N_2) between (N_1,E) and (N_2,E) based on set theoretic approach as follows:

$$\mathrm{SM}(N_1,N_2)=\left[\sum_{i=1}^{n}\sum_{j=1}^{m}\left\{\left(\overline{T}_{N_1}(x_i)(e_j)\wedge\overline{T}_{N_2}(x_i)(e_j)\right)+\left(\overline{I}_{N_1}(x_i)(e_j)\wedge\overline{I}_{N_2}(x_i)(e_j)\right)+\left(\overline{F}_{N_1}(x_i)(e_j)\wedge\overline{F}_{N_2}(x_i)(e_j)\right)\right\}\right]$$
$$\div\left[\sum_{i=1}^{n}\sum_{j=1}^{m}\left\{\left(\overline{T}_{N_1}(x_i)(e_j)\vee\overline{T}_{N_2}(x_i)(e_j)\right)+\left(\overline{I}_{N_1}(x_i)(e_j)\vee\overline{I}_{N_2}(x_i)(e_j)\right)+\left(\overline{F}_{N_1}(x_i)(e_j)\vee\overline{F}_{N_2}(x_i)(e_j)\right)\right\}\right]$$

Example 4.2 Here we consider example 3.5. Then by definition 4.1 similarity measure measure between (N_1,E) and (N_2,E) is given by

SM(N_1,N_2) = 0.86 .

Theorem 4.3 If *SM*(N_1,N_2) be the similarity measure between two IVNSS (N_1,E) and (N_2,E) then

(i) *SM*(N_1,N_2) = *SM*(N_2,N_1)
(ii) $0\leq SM(N_1,N_2)\leq 1$
(iii) *SM*(N_1,N_2) = 1 if and only if (N_1,E) = (N_2,E)

Proof: Immediately follows from definitions 3.2 and 4.1.

Definition 4.4 Let (N_1,E) and (N_2,E) be two IVNSS over U. Then (N_1,E) and (N_2,E) are said be α -similar , denoted if by $(N_1,E)\overset{\alpha}{\simeq}(N_2,E)$ and only if *SM*((N_1,E),(N_2,E)) > α for α ∈ (0,1). We call the two IVNSS significantly similar if *SM*((N_1,E),(N_2,E)) > $\frac{1}{2}$.

Example 4.5 In example 3.5 *SM*(N_1,N_2) = 0.80 > 0.5 . Therefore the IVNSS (N_1,E) and (N_2,E) are significantly similar

5 Application in pattern recognition problem

In this section we developed an algorithm based on similarity measures of two interval valued neutrosophic soft sets based on distances for possible application in pattern recognition problems. In this method we assume that if similarity between the ideal pattern and sample pattern is greater than or equal to 0.7(which may vary for different problem) then the sample pattern belongs to the family of ideal pattern in consideration.

The algorithm of this method is as follows:
Step 1: construct an ideal IVNSS (A, E) over the universe U.
Step 2: construct IVNS Sets (A_i, E), i = 1, 2, 3, … , n, over the universe U for the sample patterns which are to recognized.
Step 3: calculate the distances of (A, E) and (A_i, E).
Step 4: calculate similarity measure $SM(A, A_i)$ between (A, E) and (A_i, E).
Step 5: If $SM(A, A_i) \geq 0.7$ then the pattern A_i is to be recognized to belong to the ideal Pattern A and if $SM(A, A_i) < 0.7$ then the pattern A_i is to be recognized not to belong to the ideal Pattern A.

Example 5.1 Here a fictitious numerical example is given to illustrate the application of similarity measures between two interval valued neutrosophic soft sets in pattern recognition problem. In this example we take three sample patterns which are to be recognized.
Let U = $\{x_1,x_2,x_3\}$ be the universe and E = $\{e_1,e_2,e_3\}$ be the set of parameters. Also let (A,E) be IVNS set of the ideal pattern and (A_1,E), (A_2,E), (A_3,E) be the IVNS sets of three sample patterns.

Step 1: Construct an ideal IVNS Set (A,E) over the universe U.

A	e_1	e_2	e_3
x_1	[0.6,0.7],[0.1,0.2], [0.4,0.5]	[0.8,0.9],[0.2,0.3], [0.5,0.6]	[0.5,0.6],[0.1,0.3], [0.6,0.8]
x_2	[0.5,0.6],[0.0,0.1], [0.3,0.4]	[0.2,0.4],[0.1,0.2], [0.6,0.7]	[0.4,0.5],[0.2,0.3], [0.5,0.6]
x_3	[0.7,0.8],[0.3,0.4], [0.2,0.3]	[0.7,0.8],[0.4,0.5], [0.3,0.5]	[0.7,0.8],[0.0,0.2], [0.5,0.7]

Table 3: tabular representation of (A,E)

Step 2: Construct IVNS Sets (A_1,E), (A_2,E), (A_3,E) over the universe U for the sample patterns which are to recognized.

A_1	e_1	e_2	e_3
x_1	[0.2,0.3],[0.4,0.5], [0.6,0.7]	[0.2,0.3],[0.6,0.7], [0.8,1.0]	[0.8,1.0],[0.5,0.6], [0.1,0.2]
x_2	[0.1,0.2],[0.6,0.7], [0.7,0.9]	[0.8,0.9],[0.4,0.5], [0.2,0.3]	[0.0,0.1],[0.6,0.7], [0.8,0.9]
x_3	[0.3,0.4],[0.0,0.1], [0.7,0.8]	[0.1,0.2],[0.2,0.3], [0.7,0.8]	[0.1,0.2],[0.3,0.4], [0.2,0.3]

Table 4: tabular representation of (A_1,E)

A_2	e_1	e_2	e_3
x_1	[0.6,0.8],[0.15,0.25], [0.3,0.5]	[0.75,0.85],[0.1,0.2], [0.4,0.5]	[0.4,0.55],[0.2,0.3], [0.7,0.9]
x_2	[0.4,0.6],[0.0,0.2], [0.4,0.5]	[0.3,0.4],[0.0,0.2], [0.5,0.7]	[0.4,0.5],[0.15,0.25], [0.4,0.6]
x_3	[0.6,0.8],[0.2,0.3], [0.2,0.3]	[0.6,0.75],[0.3,0.4], [0.4,0.5]	[0.65,0.85],[0.1,0.2], [0.4,0.6]

Table 5: tabular representation of (A_2,E)

A_2	e_1	e_2	e_3
x_1	[0.5,0.7],[0.1,0.3], [0.45,0.6]	[0.7,1.0],[0.1,0.25], [0.5,0.7]	[0.55,0.7],[0.2,0.3], [0.6,0.8]
x_2	[0.5,0.6],[0.0,0.2], [0.2,0.4]	[0.3,0.5],[0.1,0.3], [0.6,0.8]	[0.4,0.6],[0..2,0.4], [0.5,0.7]
x_3	[0.7,0.9],[0.1,0.35], [0.1,0.35]	[0.75,0.9],[0.2,0.4], [0.35,0.6]	[0.6,0.7],[0.1,0.3], [0.4,0.6]

Table 6: tabular representation of (A_3,E)

Step 3: Calculate the Hamming distances of (A, E) and (A_i, E) for i = 1, 2, 3.
By definition 3.1 the Hamming distances between (A,E) and (A_i,E) for i = 1,2,3 are given by

$$D_H(A,A_1) = 1.825$$

$$D_H(A,A_2) = 0.254$$

$$D_H(A,A_3) = 0.279$$

Step 4: Calculate similarity measures $SM(A,A_i)$ between (A, E) and (A_i, E) for i = 1, 2, 3.
By equation 3.1 similarity measures between (A,E) and (A_i,E) for i = 1,2,3 using Hamming distance are given by

$$SM(A,A_1) = 0.35$$
$$SM(A,A_2) = 0.80$$
$$SM(A,A_3) = 0.78$$

Again by definition 4.1 similarity measures between (A,E) and (A_i,E) for i = 1,2,3 are given by

$$SM(A,A_1) = 0.39$$
$$SM(A,A_2) = 0.87$$
$$SM(A,A_3) = 0.86$$

Step 5: Here we see that $SM(A,A_1) < 0.7$, $SM(A,A_2) > 0.7$ and $SM(A,A_3) > 0.7$.

Hence the sample patterns whose corresponding IVNS sets are represented by (A_2,E) and (A_3,E) are recognized as similar patterns of the family of ideal pattern whose IVNS set is represented by (A,E) and the pattern whose IVNS set is represented by (A_1,E) does not belong to the family of ideal pattern (A_1,E). Here we see that if we use similarity measures based on set theoretic approach then also we get the same results.

6 Comparison of different similarity measures

In this section we make comparative study among similarity measures proposed in this paper. Table 7 shows the comparison of similarity measures between two IVNS sets based on distance (Hamming distance) and similarity measure based on set theoretic approach as obtained in example 3.5, 4.2 and 5.1 .

Similarity measure based on	(N_1,N_2)	(A,A_1)	(A,A_2)	(A,A_3)
Hamming distance	0.80	0.35	0.80	0.78
Set theoretic approach	0.86	0.39	0.87	0.86

Table 7: comparison of similarity measures

Table 7 shows that each method has its own measuring but the results are almost same. So any method can be applied to evaluate the similarity measures between two interval valued neutrosophic soft sets.

Conclusions

In this paper we have defined several distances between two interval valued neutrosophoic soft sets and based on these distances we proposed similarity measure between two interval valued neutrosophic soft sets. We also proposed similarity measure between two interval valued neutrosophic soft sets based on set theoretic approach. A decision making method based on similarity measure is developed and a numerical example is illustrated to show the possible application of similarity measures between two interval valued neutrosophic soft sets for a pattern recognition problem. Thus we can use the method to solve the problem that contain uncertainty such as problem in social, economic system, medical diagnosis, game theory, coding theory and so on. A comparative study of different similarity measures also done .

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Soft Neutrosophic Groupoids and Their Generalization

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Abstract. Soft set theory is a general mathematical tool for dealing with uncertain, fuzzy, not clearly defined objects. In this paper we introduced soft neutrosophic groupoid and their generalization with the discuissionf of some of their characteristics. We also introduced a new type of soft neutrophic groupoid, the so called soft strong neutrosophic goupoid which is of pure neutrosophic character. This notion also found in all the other corresponding notions of soft neutrosophic thoery. We also given some of their properties of this newly born soft structure related to the strong part of neutrosophic theory.

Keywords: Neutrosophic groupoid, neutrosophic bigroupoid, neutrosophic N -groupoid, soft set, soft neutrosophic groupoid, soft neutrosophic bigroupoid, soft neutrosophic N -groupoid.

1 Introduction

Florentine Smarandache for the first time introduced the concept of neutrosophy in $1995,$ which is basically a new branch of philosophy which actually studies the origin, nature, and scope of neutralities. The neutrosophic logic came into being by neutrosophy. In neutrosophic logic each proposition is approximated to have the percentage of truth in a subset T , the percentage of indeterminacy in a subset I , and the percentage of falsity in a subset F . Neutrosophic logic is an extension of fuzzy logic. In fact the neutrosophic set is the generalization of classical set, fuzzy conventional set, intuitionistic fuzzy set, and interval valued fuzzy set. Neutrosophic logic is used to overcome the problems of impreciseness, indeterminate, and inconsistencies of date etc. The theory of neutrosophy is so applicable to every field of algebra. W.B. Vasantha Kandasamy and Florentin Smarandache introduced neutrosophic fields, neutrosophic rings, neutrosophic vector spaces, neutrosophic groups, neutrosophic bigroups and neutrosophic N -groups, neutrosophic semigroups, neutrosophic bisemigroups, and neutrosophic N -semigroups, neutrosophic loops, nuetrosophic biloops, and neutrosophic N -loops, and so on. Mumtaz ali et al. introduced nuetrosophic LA -semigroups.

Molodtsov introduced the theory of soft set. This mathematical tool is free from parameterization inadequacy, syndrome of fuzzy set theory, rough set theory, probability theory and so on. This theory has been applied successfully in many fields such as smoothness of functions, game theory, operation research, Riemann integration, Perron integration, and probability. Recently soft set theory attained much attention of the researchers since its appearance and the work based on several operations of soft set introduced in $[2,9,10]$. Some properties and algebra may be found in $[1]$. Feng et al. introduced soft semirings in $[5]$. By means of level soft sets an adjustable approach to fuzzy soft set can be seen in $[6]$. Some other concepts together with fuzzy set and rough set were shown in $[7,8]$.

This paper is about to introduced soft nuetrosophic groupoid, soft neutrosophic bigroupoid, and soft neutrosophic N -groupoid and the related strong or pure part of neutrosophy with the notions of soft set theory. In the proceeding section, we define soft neutrosophic groupoid, soft neutrosophic strong groupoid, and some of their properties are discussed. In the next section, soft neutrosophic bigroupoid are presented with their strong neutrosophic part. Also in this section some of their characterization have been made. In the last section soft neutrosophic N -groupoid and their corresponding strong theory have been constructed with some of their properties.

2 Fundamental Concepts

2.1 Neutrosophic Groupoid

Definition 2.1.1. Let G be a groupoid, the groupoid generated by G and I i.e. $G \cup I$ is denoted by $\langle G \cup I \rangle$ is defined to be a neutrosophic groupoid where I is the indeterminacy element and termed as neutrosophic element.

Definition 2.1.2. Let $\langle G \cup I \rangle$ be a neutrosophic groupoid. A proper subset P of $\langle G \cup I \rangle$ is said to be a neutrosophic subgroupoid, if P is a neutrosophic groupoid under the operations of $\langle G \cup I \rangle$. A neutrosophic groupoid $\langle G \cup I \rangle$ is said to have a subgroupoid if $\langle G \cup I \rangle$ has a proper subset which is a groupoid under the operations of $\langle G \cup I \rangle$.

Theorem 2.1.3. Let $\langle G \cup I \rangle$ be a neutrosophic groupoid. Suppose P_1 and P_2 be any two neutrosophic subgroupoids of $\langle G \cup I \rangle$, then $P_1 \cup P_2$, the union of two neutrosophic subgroupoids in general need not be a neutrosophic subgroupoid.

Definition 2.1.4. Let $\langle G \cup I \rangle$ be a neutrosophic groupoid under a binary operation $*$. P be a proper subset of $\langle G \cup I \rangle$. P is said to be a neutrosophic ideal of $\langle G \cup I \rangle$ if the following conditions are satisfied.

1. P is a neutrosophic groupoid.
2. For all $p \in P$ and for all $s \in \langle G \cup I \rangle$ we have $p * s$ and $s * p$ are in P.

2.2 Neutrosophic Bigroupoid

Definition 2.2.1. Let $(BN(\mathrm{G}), *, \circ)$ be a non-empty set with two binary operations $*$ and $\circ$. $(BN(\mathrm{G}), *, \circ)$ is said to be a neutrosophic bigroupoid if $BN(\mathrm{G}) = P_1 \cup P_2$ where atleast one of $(P_1, *)$ or $(P_2, \circ)$ is a neutrosophic groupoid and other is just a groupoid. P_1 and P_2 are proper subsets of $BN(\mathrm{G})$.

If both $(P_1, *)$ and $(P_2, \circ)$ in the above definition are neutrosophic groupoids then we call $(BN(\mathrm{G}), *, \circ)$ a strong neutrosophic bigroupoid. All strong neutrosophic bigroupoids are trivially neutrosophic bigroupoids.

Definition 2.2.2. Let $(BN(\mathrm{G}) = \mathrm{P}_1 \cup P;: *, \circ)$ be a neutrosophic bigroupoid. A proper subset $(T, \circ, *)$ is said to be a neutrosophic subbigroupoid of $BN(\mathrm{G})$ if

1) $T = T_1 \cup T_2$ where $T_1 = P_1 \cap T$ and $T_2 = P_2 \cap T$ and

2) At least one of $(T_1, \circ)$ or $(T_2, *)$ is a neutrosophic groupoid.

Definition 2.2.3. Let $(BN(\mathrm{G}) = \mathrm{P}_1 \cup P, *, \circ)$ be a neutrosophic strong bigroupoid. A proper subset T of $BN(S)$ is called the strong neutrosophic subbigroupoid if $T = T_1 \cup T_2$ with $T_1 = P_1 \cap T$ and $T_2 = P_2 \cap T$ and if both $(T_1, *)$ and $(T_2, \circ)$ are neutrosophic subgroupoids of $(P_1, *)$ and $(P_2, \circ)$ respectively. We call $T = T_1 \cup T_2$ to be a neutrosophic strong subbigroupoid, if atleast one of $(T_1, *)$ or $(T_2, \circ)$ is a groupoid then $T = T_1 \cup T_2$ is only a neutrosophic subgroupoid.

Definition 2.2.4. Let $(BN(\mathrm{G}) = \mathrm{P}_1 \cup P_2, *, \circ)$ be any neutrosophic bigroupoid. Let J be a proper subset of $BN(\mathrm{J})$ such that $J_1 = J \cap P_1$ and $J_2 = J \cap P_2$ are ideals of P_1 and P_2 respectively. Then J is called the neutrosophic biideal of $BN(\mathrm{G})$.

Definition 2.2.5. Let $(BN(\mathrm{G}), *, \circ)$ be a strong neutrosophic bigroupoid where $BN(S) = \mathrm{P}_1 \cup P_2$ with $(P_1, *)$ and $(P_2, \circ)$ be any two neutrosophic groupoids. Let J be a proper subset of $BN(\mathrm{G})$ where $I = I_1 \cup I_2$ with $I_1 = I \cap P_1$ and $I_2 = I \cap P_2$ are neutrosophic ideals of the neutrosophic groupoids P_1 and P_2 respectively. Then I is called or defined as the strong neutrosophic biideal of $BN(\mathrm{G})$.

Union of any two neutrosophic biideals in general is not a neutrosophic biideal. This is true of neutrosophic strong biideals.

2.3 Neutrosophic N -groupoid

Definition 2.3.1. Let $\{\mathrm{N}(\mathrm{G}), *_1, \ldots, *_2\}$ be a non-empty set with N -binary operations defined on it. We call $N(G)$ a neutrosophic N -groupoid (N a positive integer) if the following conditions are satisfied.

1) $\mathrm{N}(\mathrm{G}) = G_1 \cup \ldots \cup \mathrm{G}_N$ where each G_i is a proper subset of $N(G)$ i.e. $G_i \subset G_j$ or $G_j \subset G_i$ if $i \neq j$.

2) $(\mathrm{G}_i, *_i)$ is either a neutrosophic groupoid or a groupoid for $i = 1, 2, 3, \ldots, N$.

If all the N -groupoids $(G_i, *_i)$ are neutrosophic groupoids (i.e. for $i = 1, 2, 3, ..., N$) then we call $N(G)$ to be a neutrosophic strong N -groupoid.

Definition 2.3.2. Let $N(G) = \{G_1 \cup G_2 \cup .... \cup G_N, *_1, *_2, ..., *_N\}$ be a neutrosophic N -groupoid. A proper subset $P = \{P_1 \cup P_2 \cup .... P_N, *_1, *_2, ..., *_N\}$ of $N(G)$ is said to be a neutrosophic N -subgroupoid if $P_i = P \cap G_i, i = 1, 2, ..., N$ are subgroupiids of G_i in which atleast some of the subgroupoids are neutrosophic subgroupoids.

Definition 2.3.3. Let $N(G) = \{G_1 \cup G_2 \cup .... \cup G_N, *_1, *_2, ..., *_N\}$ be a neutrosophic strong N -groupoid. A proper subset $T = \{T_1 \cup T_2 \cup .... \cup T_N, *_1, *_2, ..., *_N\}$ of $N(G)$ is said to be a neutrosophic strong sub N -groupoid if each $(T_i, *_i)$ is a neutrosophic subgroupoid of $(G_i, *_i)$ for $i = 1, 2, ..., N$ where $T_i = G_i \cap T$.

If only a few of the $(T_i, *_i)$ in T are just subgroupoids of $(G_i, *_i)$, (i.e. $(T_i, *_i)$ are not neutrosophic subgroupoids then we call T to be a sub N -groupoid of $N(G)$.

Definition 2.3.4. Let $N(G) = \{G_1 \cup G_2 \cup .... \cup G_N, *_1, *_2, ..., *_N\}$ be a neutrosophic N -groupoid. A proper subset $P = \{P_1 \cup P_2 \cup .... \cup P_N, *_1, *_2, ..., *_N\}$ of $N(G)$ is said to be a neutrosophic N -subgroupoid, if the following conditions are true,

1. P is a neutrosophic sub N -groupoid of $N(G)$.
2. Each $P_i = G \cap P_i, i = 1, 2, ..., N$ is an ideal of G_i.

Then P is called or defined as the neutrosophic N -ideal of the neutrosophic N -groupoid $N(G)$.

Definition 2.3.5. *Let* $N(G) = \{G_1 \cup G_2 \cup .... G_N, *_1, *_2, ..., *_N\}$ *be a neutrosophic strong* N *-groupoid. A proper subset* $J = \{J_1 \cup J_2 \cup .... J_N, *_1, *_2, ..., *_N\}$ *where* $J_t = J \cap G_t$ *for* $t = 1, 2, ..., N$ *is said to be a neutrosophic strong* N *-ideal of* $N(G)$ *if the following conditions are satisfied.*

1) Each it is a neutrosophic subgroupoid of $G_t, t = 1, 2, ..., N$ i.e. It is a neutrosophic strong N-subgroupoid of $N(G)$.
2) Each it is a two sided ideal of G_t for $t = 1, 2, ..., N$.

Similarly one can define neutrosophic strong N -left ideal or neutrosophic strong right ideal of $N(G)$.

A neutrosophic strong N -ideal is one which is both a neutrosophic strong N -left ideal and N -right ideal of $S(N)$.

2.4 Soft Sets

Throughout this subsection U refers to an initial universe, E is a set of parameters, $P(U)$ is the power set of U , and $A, B \subset E$. Molodtsov defined the soft set in the following manner:

Definition 2.4.1. A pair (F, A) is called a soft set over U where F is a mapping given by $F : A \to P(U)$. In other words, a soft set over U is a parameterized family of subsets of the universe U . For $a \in A$, $F(a)$ may be considered as the set of a -elements of the soft set (F, A) , or as the set of a -approximate elements of the soft set.

Definition 2.4.2. For two soft sets (F, A) and (H, B) over U , (F, A) is called a soft subset of (H, B) if

1. $A \subseteq B$ and
2. $F(a) \subseteq H(a)$, for all $x \in A$.

This relationship is denoted by $(F, A) \subset (H, B)$. Similarly (F, A) is called a soft superset of (H, B) if (H, B) is a soft subset of (F, A) which is denoted by $(F, A) \supset (H, B)$.

Definition 2.4.3. Two soft sets (F, A) and (H, B) over U are called soft equal if (F, A) is a soft subset of (H, B) and (H, B) is a soft subset of (F, A).

Definition 2.4.4. Let (F, A) and (K, B) be two soft sets over a common universe U such that $A \cap B \neq \phi$. Then their restricted intersection is denoted by $(F, A) \cap_R (K, B) = (H, C)$ where (H, C) is defined as $H(c) = F(c) \cap K(c)$ for all $c \in C = A \cap B$.

Definition 2.4.5. The extended intersection of two soft sets (F, A) and (K, B) over a common universe U is the soft set (H, C) , where $C = A \cup B$, and for all $c \in C$, $H(c)$ is defined as

$$H(c)=\begin{cases} F(c) & \text{if } c\in A-B,\\ G(c) & \text{if } c\in B-A,\\ F(c)\cap G(c) & \text{if } c\in A\cap B.\end{cases}$$

We write $(F,A)\cap_{\varepsilon}(K,B)=(H,C)$.

Definition 2.4.6. The restricted union of two soft sets (F,A) and (K,B) over a common universe U is the soft set (H,C), where $C=A\cup B$, and for all $c\in C$, $H(c)$ is defined as $H(c)=F(c)\cup G(c)$ for all $c\in C$. We write it as $(F,A)\cup_R(K,B)=(H,C)$.

Definition 2.4.7. The extended union of two soft sets (F,A) and (K,B) over a common universe U is the soft set (H,C), where $C=A\cup B$, and for all $c\in C$, $H(c)$ is defined as

$$H(c)=\begin{cases} F(c) & \text{if } c\in A-B,\\ G(c) & \text{if } c\in B-A,\\ F(c)\cup G(c) & \text{if } c\in A\cap B.\end{cases}$$

We write $(F,A)\cup_{\varepsilon}(K,B)=(H,C)$.

3 Soft Neutrosophic Groupoid and Their Properties

3.1 Soft Neutrosophic Groupoid

Definition 3.1.1. Let $\{\langle G\cup I\rangle,*\}$ be a neutrosophic groupoid and (F,A) be a soft set over $\{\langle G\cup I\rangle,*\}$. Then (F,A) is called soft neutrosophic groupoid if and only if $F(a)$ is neutrosophic subgroupoid of $\{\langle G\cup I\rangle,*\}$ for all $a\in A$.

Example 3.1.2. Let

$$\langle Z_{10}\cup I\rangle=\begin{Bmatrix}0,\ 1,\ 2,\ 3,\ \ldots,\ 9,\ I,\ 2I,\ \ldots,\ 9I,\\ 1+I,\ 2+I,\ \ldots,\ 9+9I\end{Bmatrix}$$

be a neutrosophic groupoid where $*$ is defined on $\langle Z_{10}\cup I\rangle$ by $a*b=3a+2b(\mathrm{mod}10)$ for all $a,b\in\langle Z10\cup I\rangle$. Let $A=\{a_1,a_2\}$ be a set of parameters. Then (F,A) is a soft neutrosophic groupoid over $\{\langle Z10\cup I\rangle,*\}$, where

$$F(a_1)=\{0,5,5\mathrm{I},5+5\mathrm{I}\},$$

$$F(a_2)=(\mathrm{Z}_{10},*).$$

Theorem 3.1.3. A soft neutrosophic groupoid over $\{\langle G\cup I\rangle,*\}$ always contain a soft groupoid over $(G,*)$.

Proof. The proof of this theorem is straightforward.

Theorem 3.1.4. Let (F,A) and (H,A) be two soft neutrosophic groupoids over $\{\langle G\cup I\rangle,*\}$. Then their intersection $(F,A)\cap(H,A)$ is again a soft neutrosophic groupoid over $\{\langle G\cup I\rangle,*\}$.

Proof. The proof is straightforward.

Theorem 3.1.5. Let (F,A) and (H,B) be two soft neutrosophic groupoids over $\{\langle G\cup I\rangle,*\}$. If $A\cap B=\phi$, then $(F,A)\cup(H,B)$ is a soft neutrosophic groupoid over $\{\langle G\cup I\rangle,*\}$.

Remark 3.1.6. The extended union of two soft neutrosophic groupoids (F,A) and (K,B) over a neutrosophic groupoid $\{\langle G\cup I\rangle,*\}$ is not a soft neutrosophic groupoid over $\{\langle G\cup I\rangle,*\}$.

Proposition 3.1.7. The extended intersection of two soft neutrosophic groupoids over a neutrosophic groupoid $\{\langle G\cup I\rangle,*\}$ is a soft neutrosophic groupoid over $\{\langle G\cup I\rangle,*\}$.

Remark 3.1.8. The restricted union of two soft neutrosophic groupoids (F,A) and (K,B) over $\{\langle G\cup I\rangle,*\}$ is not a soft neutrosophic groupoid over $\{\langle G\cup I\rangle,*\}$.

Proposition 3.1.9. The restricted intersection of two soft neutrosophic groupoids over $\{\langle G\cup I\rangle,*\}$ is a soft neutrosophic groupoid over $\{\langle G\cup I\rangle,*\}$.

Proposition 3.1.10. The AND operation of two soft neutrosophic groupoids over $\{\langle G\cup I\rangle,*\}$ is a soft neutrosophic groupoid over $\{\langle G\cup I\rangle,*\}$.

Remark 3.1.11. The OR operation of two soft neutosophic groupoids over $\{\langle G\cup I\rangle,*\}$ is not a soft nuetrosophic groupoid over $\{\langle G\cup I\rangle,*\}$.

Definition 3.1.12. Let (F,A) be a soft neutrosophic groupoid over $\{\langle G\cup I\rangle,*\}$. Then (F,A) is called an absolute-soft neutrosophic groupoid over $\{\langle G\cup I\rangle,*\}$ if $F(a)=\{\langle G\cup I\rangle,*\}$, for all $a\in A$.

Theorem 3.1.13. Every absolute-soft neutrosophic groupoid over $\{\langle G\cup I\rangle,*\}$ always contain absolute soft groupoid over $\{G,*\}$.

Definition 3.1.14. Let (F,A) and (H,B) be two soft neutrosophic groupoids over $\{\langle G\cup I\rangle,*\}$. Then (H,B) is a soft neutrosophic subgroupoid of (F,A), if

1. $B\subseteq A$.
2. $H(a)$ is neutrosophic subgroupoid of $F(a)$, for all $a\in B$.

Example 3.1.15. Let

$$\langle Z_4\cup I\rangle=\left\{\begin{array}{l}0,1,2,3,I,2I,3I,1+I,1+2I,1+3I\\2+I,2+2I,2+3I,3+I,3+2I,3+3I\end{array}\right\}$$

be a neutrosophic groupoid with respect to the operation $*$ where $*$ is defined as $a*b=2a+b(\mathrm{mod}\,4)$ for all $a,b\in\langle Z_4\cup I\rangle$. Let $A=\{a_1,a_2,a_3\}$ be a set of parameters. Then (F,A) is a soft neutrosophic groupoid over $\langle Z_4\cup I\rangle$, where

$$F(a_1)=\{0,2,2I,2+2I\},$$

$$F(a_2)=\{0,2,2+2\mathrm{I}\},$$

$$F(a_3)=\{0,2+2I\}.$$

Let $B=\{a_1,a_2\}\subseteq \mathrm{A}$. Then (H,B) is a soft neutrosophic subgroupoid of (F,A), where

$$H(a_1)=\{0,2+2I\},$$
$$H(a_2)=\{0,2+2I\}.$$

Definition 3.1.16. Let $\{\langle G\cup I\rangle,*\}$ be a neutrosophic groupoid and (F,A) be a soft neutrosophic groupoid over $\{\langle G\cup I\rangle,*\}$. Then (F,A) is called soft Lagrange neutrosophic groupoid if and only if $F(a)$ is a Lagrange neutrosophic subgroupoid of $\{\langle G\cup I\rangle,*\}$ for all $a\in A$.

Example 3.1.17. Let

$$\langle Z_4\cup I\rangle=\left\{\begin{array}{l}0,1,2,3,I,2I,3I,1+I,1+2I,1+3I\\2+I,2+2I,2+3I,3+I,3+2I,3+3I\end{array}\right\}$$

be a neutrosophic groupoid of order 16 with respect to the operation $*$ where $*$ is defined as $a*b=2a+b(\mathrm{mod}\,4)$ for all $a,b\in\langle Z_4\cup I\rangle$. Let $A=\{a_1,a_2\}$ be a set of parameters. Then (F,A) is a soft Lagrange neutrosophic groupoid over $\langle Z_4\cup I\rangle$, where

$$F(a_1)=\{0,2,2I,2+2I\},$$
$$F(a_2)=\{0,2+2I\}.$$

Theorem 3.1.18. Every soft Lagrange neutrosophic groupoid over $\{\langle G\cup I\rangle,*\}$ is a soft neutrosophic groupoid over $\{\langle G\cup I\rangle,*\}$ but the converse is not true. We can easily show the converse by the help of example.

Theorem 3.1.19. If $\{\langle G\cup I\rangle,*\}$ is a Lagrange neutrosophic groupoid, then (F,A) over $\{\langle G\cup I\rangle,*\}$ is a soft Lagrange neutrosophic groupoid but the converse is not true.

Remark 3.1.20. Let (F,A) and (K,C) be two soft Lagrange neutrosophic groupoids over $\{\langle G\cup I\rangle,*\}$. Then

1. Their extended intersection $(F,A)\cap_E(K,\mathrm{C})$ may not be a soft Lagrange neutrosophic groupoid over $\{\langle G\cup I\rangle,*\}$.
2. Their restricted intersection $(F,A)\cap_R(K,\mathrm{C})$ may not be a soft Lagrange neutrosophic groupoid over $\{\langle G\cup I\rangle,*\}$.
3. Their AND operation $(F,A)\wedge(K,\mathrm{C})$ may not be a soft Lagrange neutrosophic groipoid over $\{\langle G\cup I\rangle,*\}$.
4. Their extended union $(F,A)\cup_E(K,\mathrm{C})$ may not be a soft Lagrange neutrosophic groupoid over $\{\langle G\cup I\rangle,*\}$.
5. Their restricted union $(F,A)\cup_R(K,\mathrm{C})$ may not be a soft Lagrange neutrosophic groupoid over $\{\langle G\cup I\rangle,*\}$.
6. Their OR operation $(F,A)\vee(K,\mathrm{C})$ may not be a soft Lagrange neutrosophic groupoid over $\{\langle G\cup I\rangle,*\}$.

One can easily verify $(1),(2),(3),(4),(5)$ and (6) by the help of examples.

Definition 3.1.21. Let $\{\langle G\cup I\rangle,*\}$ be a neutrosophic groipoid and (F,A) be a soft neutrosophic groupoid over $\{\langle G\cup I\rangle,*\}$. Then (F,A) is called soft weak Lagrange neutrosophic groupoid if atleast one $F(a)$ is not a Lagrange neutrosophic subgroupoid of $\{\langle G \cup I \rangle, *\}$ for some $a \in A$.

Example 3.1.22. Let

$$\langle Z_4 \cup I \rangle = \begin{Bmatrix} 0,1,2,3,I,2I,3I,1+I,1+2I,1+3I \\ 2+I,2+2I,2+3I,3+I,3+2I,3+3I \end{Bmatrix}$$

be a neutrosophic groupoid of order 16 with respect to the operation $*$ where $*$ is defined as $a*b = 2a + b(\bmod 4)$ for all $a,b \in \langle Z_4 \cup I \rangle$. Let $A = \{a_1, a_2, a_3\}$ be a set of parameters. Then (F, A) is a soft weak Lagrange neutrosophic groupoid over $\langle Z_4 \cup I \rangle$, where

$$F(a_1) = \{0,2,2I,2+2I\},$$

$$F(a_2) = \{0,2,2+2\mathrm{I}\},$$

$$F(a_3) = \{0,2+2I\}.$$

Theorem 3.1.23. Every soft weak Lagrange neutrosophic groupoid over $\{\langle G \cup I \rangle, *\}$ is a soft neutrosophic groupoid over $\{\langle G \cup I \rangle, *\}$ but the converse is not true.

Theorem 3.1.24. If $\{\langle G \cup I \rangle, *\}$ is weak Lagrange neutrosophic groupoid, then (F, A) over $\{\langle G \cup I \rangle, *\}$ is also soft weak Lagrange neutrosophic groupoid but the converse is not true.

Remark 3.1.25. Let (F, A) and (K, C) be two soft weak Lagrange neutrosophic groupoids over $\{\langle G \cup I \rangle, *\}$. Then

1. Their extended intersection $(F, A) \cap_E (K, \mathrm{C})$ is not a soft weak Lagrange neutrosophic groupoid over $\{\langle G \cup I \rangle, *\}$.
2. Their restricted intersection $(F, A) \cap_R (K, \mathrm{C})$ is not a soft weak Lagrange neutrosophic groupoid over $\{\langle G \cup I \rangle, *\}$.
3. Their AND operation $(F, A) \wedge (K, \mathrm{C})$ is not a soft weak Lagrange neutrosophic groupoid over $\{\langle G \cup I \rangle, *\}$.
4. Their extended union $(F, A) \cup_E (K, \mathrm{C})$ is not a soft weak Lagrnage neutrosophic groupoid over $\{\langle G \cup I \rangle, *\}$.
5. Their restricted union $(F, A) \cup_R (K, \mathrm{C})$ is not a soft weak Lagrange neutrosophic groupoid over $\{\langle G \cup I \rangle, *\}$.
6. Their OR operation $(F, A) \vee (K, \mathrm{C})$ is not a soft weak Lagrange neutrosophic groupoid over $\{\langle G \cup I \rangle, *\}$.

One can easily verify $(1),(2),(3),(4),(5)$ and (6) by the help of examples.

Definition 3.126. Let $\{\langle G \cup I \rangle, *\}$ be a neutrosophic groupoid and (F, A) be a soft neutrosophic groupoid over $\{\langle G \cup I \rangle, *\}$. Then (F, A) is called soft Lagrange free neutrosophic groupoid if $F(a)$ is not a lagrange neutrosophic subgroupoid of $\{\langle G \cup I \rangle, *\}$ for all $a \in A$.

Example 3.1.27. Let

$$\langle Z_4 \cup I \rangle = \begin{Bmatrix} 0,1,2,3,I,2I,3I,1+I,1+2I,1+3I \\ 2+I,2+2I,2+3I,3+I,3+2I,3+3I \end{Bmatrix}$$

be a neutrosophic groupoid of order 16 with respect to the operation $*$ where $*$ is defined as $a*b = 2a + b(\bmod 4)$ for all $a,b \in \langle Z_4 \cup I \rangle$. Let $A = \{a_1, a_2, a_3\}$ be a set of parameters. Then (F, A) is a soft Lagrange free neutrosophic groupoid over $\langle Z_4 \cup I \rangle$, where

$$F(a_1) = \{0,2I,2+2I\},$$
$$F(a_2) = \{0,2,2+2\mathrm{I}\}.$$

Theorem 3.1.28. Every soft Lagrange free neutrosophic groupoid over $\{\langle G \cup I \rangle, *\}$ is trivially a soft neutrosophic groupoid over $\{\langle G \cup I \rangle, *\}$ but the converse is not true.

Theorem 3.1.29. If $\{\langle G \cup I \rangle, *\}$ is a Lagrange free neutrosophic groupoid, then (F, A) over $\{\langle G \cup I \rangle, *\}$ is also a soft Lagrange free neutrosophic groupoid but the converse is not true.

Remark 3.1.30. Let (F, A) and (K, C) be two soft Lagrange free neutrosophic groupoids over $\{\langle G \cup I \rangle, *\}$. Then

1. Their extended intersection $(F, A) \cap_E (K, \mathrm{C})$ is not a soft Lagrange free neutrosophic groupoid

over $\{\langle G \cup I \rangle, *\}$.

2. Their restricted intersection $(F, A) \cap_R (K, C)$ is not a soft Lagrange free neutrosophic groupoid over $\{\langle G \cup I \rangle, *\}$.
3. Their AND operation $(F, A) \wedge (K, C)$ is not a soft Lagrange free neutrosophic groupoid over $\{\langle G \cup I \rangle, *\}$.
4. Their extended union $(F, A) \cup_E (K, C)$ is not a soft Lagrnage free neutrosophic groupoid over $\{\langle G \cup I \rangle, *\}$.
5. Their restricted union $(F, A) \cup_R (K, C)$ is not a soft Lagrange free neutrosophic groupoid over $\{\langle G \cup I \rangle, *\}$.
6. Their OR operation $(F, A) \vee (K, C)$ is not a soft Lagrange free neutrosophic groupoid over $\{\langle G \cup I \rangle, *\}$.

One can easily verify $(1),(2),(3),(4),(5)$ and (6) by the help of examples.

Definition 3.1.31. (F, A) is called soft neutrosophic ideal over $\{\langle G \cup I \rangle, *\}$ if $F(a)$ is a neutrosophic ideal of $\{\langle G \cup I \rangle, *\}$, for all $a \in A$.

Theorem 3.1.32. Every soft neutrosophic ideal (F, A) over $\{\langle G \cup I \rangle, *\}$ is trivially a soft neutrosophic subgroupid but the converse may not be true.

Proposition 3.1.33. Let (F, A) and (K, B) be two soft neutrosophic ideals over $\{\langle G \cup I \rangle, *\}$. Then

1) Their extended intersection $(F, A) \cap_E (K, B)$ is soft neutrosophic ideal over $\{\langle G \cup I \rangle, *\}$.
2) Their restricted intersection $(F, A) \cap_R (K, B)$ is soft neutrosophic ideal over $\{\langle G \cup I \rangle, *\}$.
3) Their AND operation $(F, A) \wedge (K, B)$ is soft neutrosophic ideal over $\{\langle G \cup I \rangle, *\}$.

Remark 3.1.34. Let (F, A) and (K, B) be two soft neutrosophic ideal over $\{\langle G \cup I \rangle, *\}$. Then

1) Their extended union $(F, A) \cup_E (K, B)$ is not soft neutrosophic ideal over $\{\langle G \cup I \rangle, *\}$.
2) Their restricted union $(F, A) \cup_R (K, B)$ is not soft neutrosophic ideal over $\{\langle G \cup I \rangle, *\}$.
3) Their OR operation $(F, A) \vee (K, B)$ is not soft neutrosophic ideal over $\{\langle G \cup I \rangle, *\}$.

One can easily proved $(1),(2),$ and (3) by the help of examples.

Theorem 3.1.35. Let (F, A) be a soft neutrosophic ideal over $\{\langle G \cup I \rangle, *\}$ and $\{(H_i, B_i) : \mathrm{i} \in \mathrm{J}\}$ is a non-empty family of soft neutrosophic ideals of (F, A). Then

1. $\underset{i \in J}{\cap}(H_i, B_i)$ is a soft neutrosophic ideal of (F, A).
2. $\underset{i \in J}{\wedge}(H_i, B_i)$ is a soft neutrosophic ideal of $\underset{i \in J}{\wedge}(F, A)$.

3.2 Soft Neutrosophic Strong Groupoid

Definition 3.2.1. Let $\{\langle G \cup I \rangle, *\}$ be a neutrosophic groupoid and (F, A) be a soft set over $\{\langle G \cup I \rangle, *\}$. Then (F, A) is called soft neutrosophic strong groupoid if and only if $F(a)$ is a neutrosophic strong subgroupoid of $\{\langle G \cup I \rangle, *\}$ for all $a \in A$.

Example 3.2.2. Let

$$\langle Z_4 \cup I \rangle = \begin{Bmatrix} 0,1,2,3,I,2I,3I,1+I,1+2I,1+3I \\ 2+I,2+2I,2+3I,3+I,3+2I,3+3I \end{Bmatrix}$$

be a neutrosophic groupoid with respect to the operation $*$ where $*$ is defined as $a * b = 2a + b (\mathrm{mod}\, 4)$ for all $a, b \in \langle Z_4 \cup I \rangle$. Let $A = \{a_1, a_2, a_3\}$ be a set of parameters. Then (F, A) is a soft neutrosophic strong groupoid over $\langle Z_4 \cup I \rangle$, where

$$F(a_1) = \{0, 2I, 2+2I\},$$
$$F(a_2) = \{0, 2+2\mathrm{I}\}.$$

Proposition 3.2.3. Let (F, A) and (K, C) be two soft neutrosophic strong groupoids over $\{\langle G \cup I \rangle, *\}$. Then

1. Their extended intersection $(F, A) \cap_E (K, C)$ is a soft neutrosophic strong groupoid over $\{\langle G \cup I \rangle, *\}$.

2. Their restricted intersection $(F,A)\cap_R (K,C)$ is a soft neutrosophic strong groupoid over $\{\langle G\cup I\rangle,*\}$.
3. Their AND operation $(F,A)\wedge(K,C)$ is a soft neutrosophic strong groupoid over $\{\langle G\cup I\rangle,*\}$.

Remark 3.2.4. Let (F,A) and (K,C) be two soft neutrosophic strong groupoids over $\{\langle G\cup I\rangle,*\}$. Then

1. Their extended union $(F,A)\cup_E (K,C)$ is a soft neutrosophic strong groupoid over $\{\langle G\cup I\rangle,*\}$.
2. Their restricted union $(F,A)\cup_R (K,C)$ is a soft neutrosophic strong groupoid over $\{\langle G\cup I\rangle,*\}$.
3. Their OR operation $(F,A)\vee(K,C)$ is a soft neutrosophic strong groupoid over $\{\langle G\cup I\rangle,*\}$.

Definition 3.2.5. Let (F,A) and (H,C) be two soft neutrosophic strong groupoids over $\{\langle G\cup I\rangle,*\}$. Then (H,C) is called soft neutrosophic strong sublgroupoid of (F,A), if

1. $C\subseteq A$.
2. $H(a)$ is a neutrosophic strong subgroupoid of $F(a)$ for all $a\in A$.

Definition 3.2.6. Let $\{\langle G\cup I\rangle,*\}$ be a neutrosophic strong groupoid and (F,A) be a soft neutrosophic groupoid over $\{\langle G\cup I\rangle,*\}$. Then (F,A) is called soft Lagrange neutrosophic strong groupoid if and only if $F(a)$ is a Lagrange neutrosophic strong subgroupoid of $\{\langle G\cup I\rangle,*\}$ for all $a\in A$.

Theorem 3.2.7. Every soft Lagrange neutrosophic strong groupoid over $\{\langle G\cup I\rangle,*\}$ is a soft neutrosophic groupoid over $\{\langle G\cup I\rangle,*\}$ but the converse is not true.

Theorem 3.2.8. If $\{\langle G\cup I\rangle,*\}$ is a Lagrange neutrosophic strong groupoid, then (F,A) over $\{\langle G\cup I\rangle,*\}$ is a soft Lagrange neutrosophic groupoid but the converse is not true.

Remark 3.2.9. Let (F,A) and (K,C) be two soft Lagrange neutrosophic strong groupoids over $\{\langle G\cup I\rangle,*\}$. Then

1. Their extended intersection $(F,A)\cap_E (K,C)$ may not be a soft Lagrange neutrosophic strong groupoid over $\{\langle G\cup I\rangle,*\}$.
2. Their restricted intersection $(F,A)\cap_R (K,C)$ may not be a soft Lagrange strong neutrosophic groupoid over $\{\langle G\cup I\rangle,*\}$.
3. Their AND operation $(F,A)\wedge(K,C)$ may not be a soft Lagrange neutrosophic strong groupoid over $\{\langle G\cup I\rangle,*\}$.
4. Their extended union $(F,A)\cup_E (K,C)$ may not be a soft Lagrange neutrosophic strong groupoid over $\{\langle G\cup I\rangle,*\}$.
5. Their restricted union $(F,A)\cup_R (K,C)$ may not be a soft Lagrange neutrosophic strong groupoid over $\{\langle G\cup I\rangle,*\}$.
6. Their OR operation $(F,A)\vee(K,C)$ may not be a soft Lagrange neutrosophic strong groupoid over $\{\langle G\cup I\rangle,*\}$.

One can easily verify $(1),(2),(3),(4),(5)$ and (6) by the help of examples.

Definition 3.2.10. Let $\{\langle G\cup I\rangle,*\}$ be a neutrosophic strong groupoid and (F,A) be a soft neutrosophic groupoid over $\{\langle G\cup I\rangle,*\}$. Then (F,A) is called soft weak Lagrange neutrosophic strong groupoid if atleast one $F(a)$ is not a Lagrange neutrosophic strong subgroupoid of $\{\langle G\cup I\rangle,*\}$ for some $a\in A$.

Theorem 3.2.11. Every soft weak Lagrange neutrosophic strong groupoid over $\{\langle G\cup I\rangle,*\}$ is a soft neutrosophic groupoid over $\{\langle G\cup I\rangle,*\}$ but the converse is not true.

Theorem 3.2.12. If $\{\langle G\cup I\rangle,*\}$ is weak Lagrange neutrosophic strong groupoid, then (F,A) over $\{\langle G\cup I\rangle,*\}$ is also soft weak Lagrange neutrosophic strong groupoid but the converse is not true.

Remark 3.2.13. Let (F,A) and (K,C) be two soft weak Lagrange neutrosophic strong groupoids over $\{\langle G \cup I \rangle, *\}$. Then

1. Their extended intersection $(F, A) \cap_E (K, C)$ is not a soft weak Lagrange neutrosophic strong groupoid over $\{\langle G \cup I \rangle, *\}$.
2. Their restricted intersection $(F, A) \cap_R (K, C)$ is not a soft weak Lagrange neutrosophic strong groupoid over $\{\langle G \cup I \rangle, *\}$.
3. Their AND operation $(F, A) \wedge (K, C)$ is not a soft weak Lagrange neutrosophic strong groupoid over $\{\langle G \cup I \rangle, *\}$.
4. Their extended union $(F, A) \cup_E (K, C)$ is not a soft weak Lagrnage neutrosophic strong groupoid over $\{\langle G \cup I \rangle, *\}$.
5. Their restricted union $(F, A) \cup_R (K, C)$ is not a soft weak Lagrange neutrosophic strong groupoid over $\{\langle G \cup I \rangle, *\}$.
6. Their OR operation $(F, A) \vee (K, C)$ is not a soft weak Lagrange neutrosophic strong groupoid over $\{\langle G \cup I \rangle, *\}$.

One can easily verify $(1),(2),(3),(4),(5)$ and (6) by the help of examples.

Definition 3.2.14. Let $\langle L \cup I \rangle$ be a neutrosophic strong groupoid and (F, A) be a soft neutrosophic groupoid over $\langle L \cup I \rangle$. Then (F, A) is called soft Lagrange free neutrosophic strong groupoid if $F(a)$ is not a Lagrange neutrosophic strong subgroupoid of $\{\langle G \cup I \rangle, *\}$ for all $a \in A$.

Theorem 3.2.14. Every soft Lagrange free neutrosophic strong groupoid over $\langle L \cup I \rangle$ is a soft neutrosophic groupoid over $\{\langle G \cup I \rangle, *\}$ but the converse is not true.

Theorem 3.2.15. If $\{\langle G \cup I \rangle, *\}$ is a Lagrange free neutrosophic strong groupoid, then (F, A) over $\{\langle G \cup I \rangle, *\}$ is also a soft Lagrange free neutrosophic strong groupoid but the converse is not true.

Remark 3.2.16. Let (F, A) and (K, C) be two soft Lagrange free neutrosophic strong groupoids over $\langle L \cup I \rangle$. Then

1. Their extended intersection $(F, A) \cap_E (K, C)$ is not a soft Lagrange free neutrosophic strong groupoid over $\{\langle G \cup I \rangle, *\}$.
2. Their restricted intersection $(F, A) \cap_R (K, C)$ is not a soft Lagrange free neutrosophic strong groupoid over $\{\langle G \cup I \rangle, *\}$.
3. Their AND operation $(F, A) \wedge (K, C)$ is not a soft Lagrange free neutrosophic strong groupoid over $\{\langle G \cup I \rangle, *\}$.
4. Their extended union $(F, A) \cup_E (K, C)$ is not a soft Lagrange free neutrosophic strong groupoid over $\{\langle G \cup I \rangle, *\}$.
5. Their restricted union $(F, A) \cup_R (K, C)$ is not a soft Lagrange free neutrosophic groupoid over $\{\langle G \cup I \rangle, *\}$.
6. Their OR operation $(F, A) \vee (K, C)$ is not a soft Lagrange free neutrosophic strong groupoid over $\{\langle G \cup I \rangle, *\}$.

One can easily verify $(1),(2),(3),(4),(5)$ and (6) by the help of examples.

Definition 3.2.17. (F, A) is called soft neutrosophic strong ideal over $\{\langle G \cup I \rangle, *\}$ if $F(a)$ is a neutrosophic strong ideal of $\{\langle G \cup I \rangle, *\}$, for all $a \in A$.

Theorem 3.2.18. Every soft neutrosophic strong ideal (F, A) over $\{\langle G \cup I \rangle, *\}$ is trivially a soft neutrosophic strong groupoid.

Theorem 3.2.19. Every soft neutrosophic strong ideal (F, A) over $\{\langle G \cup I \rangle, *\}$ is trivially a soft neutrosophic ideal.

Proposition 3.2.20. Let (F, A) and (K, B) be two soft neutrosophic strong ideals over $\{\langle G \cup I \rangle, *\}$. Then

1. Their extended intersection $(F, A) \cap_E (K, B)$ is soft neutrosophic strong ideal over $\{\langle G \cup I \rangle, *\}$.
2. Their restricted intersection $(F, A) \cap_R (K, B)$ is soft neutrosophic strong ideal over $\{\langle G \cup I \rangle, *\}$.
3. Their AND operation $(F, A) \wedge (K, B)$ is soft

neutrosophic strong ideal over $\{\langle G\cup I\rangle,*\}$.

Remark 3.2.21. Let (F,A) and (K,B) be two soft neutrosophic strong ideal over $\{\langle G\cup I\rangle,*\}$. Then

1. Their extended union $(F,A)\cup_E (K,B)$ is not soft neutrosophic strong ideal over $\{\langle G\cup I\rangle,*\}$.
2. Their restricted union $(F,A)\cup_R (K,B)$ is not soft neutrosophic strong ideal over $\{\langle G\cup I\rangle,*\}$.
3. Their OR operation $(F,A)\vee(K,B)$ is not soft neutrosophic strong ideal over $\{\langle G\cup I\rangle,*\}$.

One can easily proved $(1),(2),$ and (3) by the help of examples.

Theorem 3.2.22. Let (F,A) be a soft neutrosophic strong ideal over $\{\langle G\cup I\rangle,*\}$ and $\{(H_i,B_i):\mathrm{i}\in\mathrm{J}\}$ is a non-empty family of soft neutrosophic strong ideals of (F,A). Then

1. $\bigcap_{i\in J}(H_i,B_i)$ is a soft neutrosophic strong ideal of (F,A).
2. $\bigwedge_{i\in J}(H_i,B_i)$ is a soft neutrosophic strong ideal of $\bigwedge_{i\in J}(F,A)$.

4 Soft Neutrosophic Bigroupoid and Their Properties

4.1 Soft Neutrosophic Bigroupoid

Definition 4.1.1. Let $\{\mathrm{B}_N(\mathrm{G}),*,\circ\}$ be a neutrosophic bigroupoid and (F,A) be a soft set over $\{\mathrm{B}_N(\mathrm{G}),*,\circ\}$. Then (F,A) is called soft neutrosophic bigroupoid if and only if $F(a)$ is neutrosophic sub bigroupoid of $\{\mathrm{B}_N(\mathrm{G}),*,\circ\}$ for all $a\in A$.

Example 4.1.2. Let $\{\mathrm{B}_N(\mathrm{G}),*,\circ\}$ be a neutrosophic groupoid with $\mathrm{B}_N(\mathrm{G})=G_1\cup G_2$, where
$G_1=\{\langle Z_{10}\cup I\rangle \mid a*b=2a+3b(\mathrm{mod}\,10);a,b\in\langle Z_{10}\cup I\rangle\}$ and
$G_2=\{\langle Z_4\cup I\rangle \mid a\circ b=2a+b(\mathrm{mod}\,4);a,b\in\langle Z_4\cup I\rangle\}$.
Let $A=\{a_1,a_2\}$ be a set of parameters. Then (F,A) is a soft neutrosophic bigroupoid over $\{\mathrm{B}_N(\mathrm{G}),*,\circ\}$, where

$$F(a_1)=\{0,5,5I,5+5I\}\cup\{0,2,2I,2+2I\},$$
$$F(a_2)=(Z_{10},*)\cup\{0,2+2\mathrm{I}\}.$$

Theorem 4.1.3. Let (F,A) and (H,A) be two soft neutrosophic bigroupoids over $\{\mathrm{B}_N(\mathrm{G}),*,\circ\}$. Then their intersection $(F,A)\cap(H,A)$ is again a soft neutrosophic groupoid over $\{\mathrm{B}_N(\mathrm{G}),*,\circ\}$.

Proof. The proof is staightforward.

Theorem 4.1.4. Let (F,A) and (H,B) be two soft neutrosophic groupoids over $\{\langle G\cup I\rangle,*\}$. If $A\cap B=\phi$, then $(F,A)\cup(H,B)$ is a soft neutrosophic groupoid over $\{\langle G\cup I\rangle,*\}$.

Proposition 4.1.5. Let (F,A) and (K,C) be two soft neutrosophic bigroupoids over $\{\mathrm{B}_N(\mathrm{G}),*,\circ\}$. Then

1. Their extended intersection $(F,A)\cap_E (K,\mathrm{C})$ is a soft neutrosophic bigroupoid over $\{\mathrm{B}_N(\mathrm{G}),*,\circ\}$.
2. Their restricted intersection $(F,A)\cap_R (K,\mathrm{C})$ is a soft neutrosophic bigroupoid over $\{\mathrm{B}_N(\mathrm{G}),*,\circ\}$.
3. Their AND operation $(F,A)\wedge(K,\mathrm{C})$ is a soft neutrosophic bigroupoid over $\{\mathrm{B}_N(\mathrm{G}),*,\circ\}$.

Remark 4.1.6. Let (F,A) and (K,C) be two soft neutrosophic biloops over $\{\mathrm{B}_N(\mathrm{G}),*,\circ\}$. Then

1. Their extended union $(F,A)\cup_E (K,\mathrm{C})$ is not a soft neutrosophic bigroupoid over $\{\mathrm{B}_N(\mathrm{G}),*,\circ\}$.
2. Their restricted union $(F,A)\cup_R (K,\mathrm{C})$ is not a soft neutrosophic bigroupoid over $\{\mathrm{B}_N(\mathrm{G}),*,\circ\}$.
3. Their OR operation $(F,A)\vee(K,\mathrm{C})$ is not a soft neutrosophic bigroupoid over $\{\mathrm{B}_N(\mathrm{G}),*,\circ\}$.

One can easily verify $(1),(2),$ and (3) by the help of examples.

Definition 4.1.7. Let (F,A) be a soft neutrosophic bigroupoid over $\{\mathrm{B}_N(\mathrm{G}),*,\circ\}$. Then (F,A) is called an absolute soft neutrosophic bigroupoid over $\{\mathrm{B}_N(\mathrm{G}),*,\circ\}$ if $F(a)=\{\mathrm{B}_N(\mathrm{G}),*,\circ\}$ for all $a\in A$.

Definition 4.1.8. Let (F,A) and (H,C) be two soft neutrosophic bigroupoids over $\{\mathrm{B}_N(\mathrm{G}),*,\circ\}$. Then (H,C) is called soft neutrosophic sub bigroupoid of

(F,A), if

1. $C \subseteq A$.
2. $H(a)$ is a neutrosophic sub bigroupoid of $F(a)$ for all $a \in A$.

Example 4.1.9. Let $\{B_N(G),*,\circ\}$ be a neutrosophic groupoid with $B_N(G) = G_1 \cup G_2$, where
$G_1 = \{\langle Z_{10} \cup I\rangle \mid a*b = 2a+3b(\text{mod}10); a,b \in \langle Z_{10} \cup I\rangle\}$
and
$G_2 = \{\langle Z_4 \cup I\rangle \mid a\circ b = 2a+b(\text{mod}4); a,b \in \langle Z_4 \cup I\rangle\}$.
Let $A = \{a_1, a_2\}$ be a set of parameters. Let (F,A) is a soft neutrosophic bigroupoid over $\{B_N(G),*,\circ\}$, where

$$F(a_1) = \{0,5,5I,5+5I\} \cup \{0,2,2I,2+2I\},$$
$$F(a_2) = (Z_{10},*) \cup \{0,2+2I\}.$$

Let $B = \{a_1\} \subseteq A$. Then (H,B) is a soft neutrosophic sub bigroupoid of (F,A), where

$$H(a_1) = \{0,5\} \cup \{0,2+2I\}.$$

Definition 4.1.10. Let $\{B_N(G),*,\circ\}$ be a neutrosophic strong bigroupoid and (F,A) be a soft neutrosophic bigroupoid over $\{B_N(G),*,\circ\}$. Then (F,A) is called soft Lagrange neutrosophic bigroupoid if and only if $F(a)$ is a Lagrange neutrosophic sub bigroupoid of $\{B_N(G),*,\circ\}$ for all $a \in A$.

Theorem 4.1.11. Every soft Lagrange neutrosophic bigroupoid over $\{B_N(G),*,\circ\}$ is a soft neutrosophic bigroupoid over $\{B_N(G),*,\circ\}$ but the converse is not true.

One can easily see the converse by the help of examples.

Theorem 4.1.12. If $\{B_N(G),*,\circ\}$ is a Lagrange neutrosophic bigroupoid, then (F,A) over $\{B_N(G),*,\circ\}$ is a soft Lagrange neutrosophic bigroupoid but the converse is not true.

Remark 4.1.13. Let (F,A) and (K,C) be two soft Lagrange neutrosophic bigroupoids over $\{B_N(G),*,\circ\}$. Then

1. Their extended intersection $(F,A)\cap_E(K,C)$ may not be a soft Lagrange neutrosophic bigroupoid over $\{B_N(G),*,\circ\}$.
2. Their restricted intersection $(F,A)\cap_R(K,C)$ may not be a soft Lagrange neutrosophic bigroupoid over $\{B_N(G),*,\circ\}$.
3. Their *AND* operation $(F,A)\wedge(K,C)$ may not be a soft Lagrange neutrosophic bigroupoid over $\{B_N(G),*,\circ\}$.
4. Their extended union $(F,A)\cup_E(K,C)$ may not be a soft Lagrange neutrosophic bigroupoid over $\{B_N(G),*,\circ\}$.
5. Their restricted union $(F,A)\cup_R(K,C)$ may not be a soft Lagrange neutrosophic bigroupoid over $\{B_N(G),*,\circ\}$.
6. Their *OR* operation $(F,A)\vee(K,C)$ may not be a soft Lagrange neutrosophic bigroupoid over $\{B_N(G),*,\circ\}$.

One can easily verify $(1),(2),(3),(4),(5)$ and (6) by the help of examples.

Definition 4.1.14. Let $\{B_N(G),*,\circ\}$ be a neutrosophic bigroupoid and (F,A) be a soft neutrosophic bigroupoid over $\{B_N(G),*,\circ\}$. Then (F,A) is called soft weak Lagrange neutrosophic bigroupoid if atleast one $F(a)$ is not a Lagrange neutrosophic sub bigroupoid of $\{B_N(G),*,\circ\}$ for some $a \in A$.

Theorem 4.1.15. Every soft weak Lagrange neutrosophic bigroupoid over $\{B_N(G),*,\circ\}$ is a soft neutrosophic groupoid over $\{B_N(G),*,\circ\}$ but the converse is not true.

Theorem 4.1.16. If $\{B_N(G),*,\circ\}$ is weak Lagrange neutrosophic bigroupoid, then (F,A) over $\{B_N(G),*,\circ\}$ is also soft weak Lagrange neutrosophic bigroupoid but the converse is not true.

Remark 4.1.17. Let (F,A) and (K,C) be two soft weak Lagrange neutrosophic bigroupoids over $\{B_N(G),*,\circ\}$. Then

1. Their extended intersection $(F,A)\cap_E(K,C)$ is not a soft weak Lagrange neutrosophic bigroupoid over $\{B_N(G),*,\circ\}$.
2. Their restricted intersection $(F,A)\cap_R(K,C)$ is not a soft weak Lagrange neutrosophic bigroupoid over $\{B_N(G),*,\circ\}$.
3. Their *AND* operation $(F,A)\wedge(K,C)$ is not a soft weak Lagrange neutrosophic bigroupoid over $\{B_N(G),*,\circ\}$.
4. Their extended union $(F,A)\cup_E(K,C)$ is not a soft weak Lagrnage neutrosophic bigroupoid over $\{B_N(G),*,\circ\}$.
5. Their restricted union $(F,A)\cup_R(K,C)$ is not a soft weak Lagrange neutrosophic bigroupoid over $\{B_N(G),*,\circ\}$.
6. Their *OR* operation $(F,A)\vee(K,C)$ is not a

soft weak Lagrange neutrosophic bigroupoid over $\{B_N(G),*,\circ\}$.

One can easily verify $(1),(2),(3),(4),(5)$ and (6) by the help of examples.

Definition 4.1.18. Let $\{B_N(G),*,\circ\}$ be a neutrosophic bigroupoid and (F,A) be a soft neutrosophic groupoid over $\{B_N(G),*,\circ\}$. Then (F,A) is called soft Lagrange free neutrosophic bigroupoid if $F(a)$ is not a Lagrange neutrosophic sub bigroupoid of $\{B_N(G),*,\circ\}$ for all $a\in A$.

Theorem 4.1.19. Every soft Lagrange free neutrosophic bigroupoid over $\{B_N(G),*,\circ\}$ is a soft neutrosophic bigroupoid over $\{B_N(G),*,\circ\}$ but the converse is not true.

Theorem 4.1.20. If $\{B_N(G),*,\circ\}$ is a Lagrange free neutrosophic bigroupoid, then (F,A) over $\{B_N(G),*,\circ\}$ is also a soft Lagrange free neutrosophic bigroupoid but the converse is not true.

Remark 4.1.21. Let (F,A) and (K,C) be two soft Lagrange free neutrosophic bigroupoids over $\{B_N(G),*,\circ\}$. Then

1. Their extended intersection $(F,A)\cap_E(K,C)$ is not a soft Lagrange free neutrosophic bigroupoid over $\{B_N(G),*,\circ\}$.
2. Their restricted intersection $(F,A)\cap_R(K,C)$ is not a soft Lagrange free neutrosophic bigroupoid over $\{B_N(G),*,\circ\}$.
3. Their AND operation $(F,A)\wedge(K,C)$ is not a soft Lagrange free neutrosophic bigroupoid over $\{B_N(G),*,\circ\}$.
4. Their extended union $(F,A)\cup_E(K,C)$ is not a soft Lagrange free neutrosophic bigroupoid over $\{B_N(G),*,\circ\}$.
5. Their restricted union $(F,A)\cup_R(K,C)$ is not a soft Lagrange free neutrosophic bigroupoid over $\{B_N(G),*,\circ\}$.
6. Their OR operation $(F,A)\vee(K,C)$ is not a soft Lagrange free neutrosophic bigroupoid over $\{B_N(G),*,\circ\}$.

One can easily verify $(1),(2),(3),(4),(5)$ and (6) by the help of examples.

Definition 4.1.22. (F,A) is called soft neutrosophic biideal over $\{B_N(G),*,\circ\}$ if $F(a)$ is a neutrosophic biideal of $\{B_N(G),*,\circ\}$, for all $a\in A$.

Theorem 4.1.23. Every soft neutrosophic biideal (F,A) over $\{B_N(G),*,\circ\}$ is a soft neutrosophic bigroupoid.

Proposition 4.1.24. Let (F,A) and (K,B) be two soft neutrosophic biideals over $\{B_N(G),*,\circ\}$. Then

1. Their extended intersection $(F,A)\cap_E(K,B)$ is soft neutrosophic biideal over $\{B_N(G),*,\circ\}$.
2. Their restricted intersection $(F,A)\cap_R(K,B)$ is soft neutrosophic biideal over $\{B_N(G),*,\circ\}$.
3. Their AND operation $(F,A)\wedge(K,B)$ is soft neutrosophic biideal over $\{B_N(G),*,\circ\}$.

Remark 4.1.25. Let (F,A) and (K,B) be two soft neutrosophic biideals over $\{B_N(G),*,\circ\}$. Then

1. Their extended union $(F,A)\cup_E(K,B)$ is not soft neutrosophic biideals over $\{B_N(G),*,\circ\}$.
2. Their restricted union $(F,A)\cup_R(K,B)$ is not soft neutrosophic biidleals over $\{B_N(G),*,\circ\}$.
3. Their OR operation $(F,A)\vee(K,B)$ is not soft neutrosophic biideals over $\{B_N(G),*,\circ\}$.

One can easily proved $(1),(2),$ and (3) by the help of examples

Theorem 4.1.26. Let (F,A) be a soft neutrosophic biideal over $\{B_N(G),*,\circ\}$ and $\{(H_i,B_i): i\in J\}$ is a non-empty family of soft neutrosophic biideals of (F,A). Then

1. $\bigcap_{i\in J}(H_i,B_i)$ is a soft neutrosophic biideal of (F,A).
2. $\bigwedge_{i\in J}(H_i,B_i)$ is a soft neutrosophic biideal of $\bigwedge_{i\in J}(F,A)$.

4.2 Soft Neutrosophic Strong Bigroupoid

Definition 4.2.1. Let $\{B_N(G),*,\circ\}$ be a neutrosophic bigroupoid and (F,A) be a soft set over $\{B_N(G),*,\circ\}$. Then (F,A) is called soft neutrosophic strong bigroupoid if and only if $F(a)$ is neutrosophic strong sub bigroupoid of $\{B_N(G),*,\circ\}$ for all $a\in A$.

Example 4.2.2. Let $\{B_N(G),*,\circ\}$ be a neutrosophic groupoid with $B_N(G)=G_1\cup G_2$, where
$G_1=\{\langle Z_{10}\cup I\rangle \mid a*b=2a+3b(\text{mod}10); a,b\in\langle Z_{10}\cup I\rangle\}$
and
$G_2=\{\langle Z_4\cup I\rangle \mid a\circ b=2a+b(\text{mod}4); a,b\in\langle Z_4\cup I\rangle\}$.
Let $A=\{a_1,a_2\}$ be a set of parameters. Then (F,A) is a soft neutrosophic strong bigroupoid over $\{B_N(G),*,\circ\}$, where

$$F(a_1)=\{0,5+5I\}\cup\{0,2+2I\},$$
$$F(a_2)=\{0,5I\}\cup\{0,2+2I\}.$$

Theorem 4.2.3. Let (F,A) and (H,A) be two soft neutrosophic strong bigroupoids over $\{B_N(G),*,\circ\}$. Then their intersection $(F,A)\cap(H,A)$ is again a soft neutrosophic strong bigroupoid over $\{B_N(G),*,\circ\}$.

Proof. The proof is staightforward.

Theorem 4.2.4. Let (F,A) and (H,B) be two soft neutrosophic strong bigroupoids over $\{B_N(G),*,\circ\}$. If $A\cap B=\phi$, then $(F,A)\cup(H,B)$ is a soft neutrosophic strong bigroupoid over $\{B_N(G),*,\circ\}$.

Proposition 4.2.5. Let (F,A) and (K,C) be two soft neutrosophic strong bigroupoids over $\{B_N(G),*,\circ\}$. Then

1. Their extended intersection $(F,A)\cap_E(K,C)$ is a soft neutrosophic strong bigroupoid over $\{B_N(G),*,\circ\}$.
2. Their restricted intersection $(F,A)\cap_R(K,C)$ is a soft neutrosophic strong bigroupoid over $\{B_N(G),*,\circ\}$.
3. Their AND operation $(F,A)\wedge(K,C)$ is a soft neutrosophic strong bigroupoid over $\{B_N(G),*,\circ\}$.

Remark 4.2.6. Let (F,A) and (K,C) be two soft neutrosophic strong bigroupoids over $\{B_N(G),*,\circ\}$. Then

1. Their extended union $(F,A)\cup_E(K,C)$ is not a soft neutrosophic strong bigroupoid over $\{B_N(G),*,\circ\}$.
2. Their restricted union $(F,A)\cup_R(K,C)$ is not a soft neutrosophic strong bigroupoid over $\{B_N(G),*,\circ\}$.
3. Their OR operation $(F,A)\vee(K,C)$ is not a soft neutrosophic strong bigroupoid over $\{B_N(G),*,\circ\}$.

One can easily verify $(1),(2),$ and (3) by the help of examples.

Definition 4.2.7. Let (F,A) and (H,C) be two soft neutrosophic strong bigroupoids over $\{B_N(G),*,\circ\}$. Then (H,C) is called soft neutrosophic strong sub bigroupoid of (F,A), if

1. $C\subseteq A$.
2. $H(a)$ is a neutrosophic strong sub bigroupoid of $F(a)$ for all $a\in A$.

Definition 4.2.8. Let $\{B_N(G),*,\circ\}$ be a neutrosophic strong bigroupoid and (F,A) be a soft neutrosophic strong bigroupoid over $\{B_N(G),*,\circ\}$. Then (F,A) is called soft Lagrange neutrosophic strong bigroupoid if and only if $F(a)$ is a Lagrange neutrosophic strong sub bigroupoid of $\{B_N(G),*,\circ\}$ for all $a\in A$.

Theorem 4.2.9. Every soft Lagrange neutrosophic strong bigroupoid over $\{B_N(G),*,\circ\}$ is a soft neutrosophic strong bigroupoid over $\{B_N(G),*,\circ\}$ but the converse is not true.

One can easily see the converse by the help of examples.

Theorem 4.2.10. If $\{B_N(G),*,\circ\}$ is a Lagrange neutrosophic strong bigroupoid, then (F,A) over $\{B_N(G),*,\circ\}$ is a soft Lagrange neutrosophic strong bigroupoid but the converse is not true.

Remark 4.2.11. Let (F,A) and (K,C) be two soft Lagrange neutrosophic strong bigroupoids over $\{B_N(G),*,\circ\}$. Then

1. Their extended intersection $(F,A)\cap_E(K,C)$ may not be a soft Lagrange neutrosophic strong bigroupoid over $\{B_N(G),*,\circ\}$.
2. Their restricted intersection $(F,A)\cap_R(K,C)$ may not be a soft Lagrange neutrosophic strong bigroupoid over $\{B_N(G),*,\circ\}$.
3. Their AND operation $(F,A)\wedge(K,C)$ may not be a soft Lagrange neutrosophic strong bigroupoid over $\{B_N(G),*,\circ\}$.Their extended union $(F,A)\cup_E(K,C)$ may not be a soft Lagrange neutrosophic strong bigroupoid over $\{B_N(G),*,\circ\}$.
4. Their restricted union $(F,A)\cup_R(K,C)$ may not be a soft Lagrange neutrosophic strong bigroupoid over $\{B_N(G),*,\circ\}$.

5. Their OR operation $(F,A)\vee(K,\mathrm{C})$ may not be a soft Lagrange neutrosophic strong bigroupoid over $\{\mathrm{B}_N(\mathrm{G}),*,\circ\}$.

One can easily verify $(1),(2),(3),(4),(5)$ and (6) by the help of examples.

Definition 4.2.12. Let $\{\mathrm{B}_N(\mathrm{G}),*,\circ\}$ be a neutrosophic strong bigroupoid and (F,A) be a soft neutrosophic strong bigroupoid over $\{\mathrm{B}_N(\mathrm{G}),*,\circ\}$. Then (F,A) is called soft weak Lagrange neutrosophic strong bigroupoid if atleast one $F(a)$ is not a Lagrange neutrosophic strong sub bigroupoid of $\{\mathrm{B}_N(\mathrm{G}),*,\circ\}$ for some $a\in A$.

Theorem 4.2.13. Every soft weak Lagrange neutrosophic strong bigroupoid over $\{\mathrm{B}_N(\mathrm{G}),*,\circ\}$ is a soft neutrosophic strong bigroupoid over $\{\mathrm{B}_N(\mathrm{G}),*,\circ\}$ but the converse is not true.

Theorem 4.2.14. If $\{\mathrm{B}_N(\mathrm{G}),*,\circ\}$ is weak Lagrange neutrosophic strong bigroupoid, then (F,A) over $\{\mathrm{B}_N(\mathrm{G}),*,\circ\}$ is also soft weak Lagrange neutrosophic strong bigroupoid but the converse is not true.

Remark 4.2.15. Let (F,A) and (K,C) be two soft weak Lagrange neutrosophic strong bigroupoids over $\{\mathrm{B}_N(\mathrm{G}),*,\circ\}$. Then

1. Their extended intersection $(F,A)\cap_E(K,\mathrm{C})$ is not a soft weak Lagrange neutrosophic strong bigroupoid over $\{\mathrm{B}_N(\mathrm{G}),*,\circ\}$.
2. Their restricted intersection $(F,A)\cap_R(K,\mathrm{C})$ is not a soft weak Lagrange neutrosophic strong bigroupoid over $\{\mathrm{B}_N(\mathrm{G}),*,\circ\}$.
3. Their AND operation $(F,A)\wedge(K,\mathrm{C})$ is not a soft weak Lagrange neutrosophic strong bigroupoid over $\{\mathrm{B}_N(\mathrm{G}),*,\circ\}$.
4. Their extended union $(F,A)\cup_E(K,\mathrm{C})$ is not a soft weak Lagrnage neutrosophic strong bigroupoid over $\{\mathrm{B}_N(\mathrm{G}),*,\circ\}$.
5. Their restricted union $(F,A)\cup_R(K,\mathrm{C})$ is not a soft weak Lagrange neutrosophic strong bigroupoid over $\{\mathrm{B}_N(\mathrm{G}),*,\circ\}$.
6. Their OR operation $(F,A)\vee(K,\mathrm{C})$ is not a soft weak Lagrange neutrosophi strong bigroupoid over $\{\mathrm{B}_N(\mathrm{G}),*,\circ\}$.

One can easily verify $(1),(2),(3),(4),(5)$ and (6) by the help of examples.

Definition 4.2.16. Let $\{\mathrm{B}_N(\mathrm{G}),*,\circ\}$ be a neutrosophic strong bigroupoid and (F,A) be a soft neutrosophic strong bigroupoid over $\{\mathrm{B}_N(\mathrm{G}),*,\circ\}$. Then (F,A) is called soft Lagrange free neutrosophic strong bigroupoid if $F(a)$ is not a Lagrange neutrosophic strong sub bigroupoid of $\{\mathrm{B}_N(\mathrm{G}),*,\circ\}$ for all $a\in A$.

Theorem 4.2.17. Every soft Lagrange free neutrosophic strong bigroupoid over $\{\mathrm{B}_N(\mathrm{G}),*,\circ\}$ is a soft neutrosophic strong bigroupoid over $\{\mathrm{B}_N(\mathrm{G}),*,\circ\}$ but the converse is not true.

Theorem 4.2.18. If $\{\mathrm{B}_N(\mathrm{G}),*,\circ\}$ is a Lagrange free neutrosophic strong bigroupoid, then (F,A) over $\{\mathrm{B}_N(\mathrm{G}),*,\circ\}$ is also a soft Lagrange free neutrosophic strong bigroupoid but the converse is not true.

Remark 4.2.19. Let (F,A) and (K,C) be two soft Lagrange free neutrosophic strong bigroupoids over $\{\mathrm{B}_N(\mathrm{G}),*,\circ\}$. Then

1. Their extended intersection $(F,A)\cap_E(K,\mathrm{C})$ is not a soft Lagrange free neutrosophic strong bigroupoid over $\{\mathrm{B}_N(\mathrm{G}),*,\circ\}$.
2. Their restricted intersection $(F,A)\cap_R(K,\mathrm{C})$ is not a soft Lagrange free neutrosophic strong bigroupoid over $\{\mathrm{B}_N(\mathrm{G}),*,\circ\}$.
3. Their AND operation $(F,A)\wedge(K,\mathrm{C})$ is not a soft Lagrange free neutrosophic strong bigroupoid over $\{\mathrm{B}_N(\mathrm{G}),*,\circ\}$.
4. Their extended union $(F,A)\cup_E(K,\mathrm{C})$ is not a soft Lagrange free neutrosophic strong bigroupoid over $\{\mathrm{B}_N(\mathrm{G}),*,\circ\}$.
5. Their restricted union $(F,A)\cup_R(K,\mathrm{C})$ is not a soft Lagrange free neutrosophic strong bigroupoid over $\{\mathrm{B}_N(\mathrm{G}),*,\circ\}$.
6. Their OR operation $(F,A)\vee(K,\mathrm{C})$ is not a soft Lagrange free neutrosophic strong bigroupoid over $\{\mathrm{B}_N(\mathrm{G}),*,\circ\}$.

One can easily verify $(1),(2),(3),(4),(5)$ and (6) by the help of examples.

Definition 4.2.20. (F,A) is called soft neutrosophic strong biideal over $\{\mathrm{B}_N(\mathrm{G}),*,\circ\}$ if $F(a)$ is a neutrosophic strong biideal of $\{B_N(G),*,\circ\}$, for all $a \in A$.

Theorem 4.2.21. Every soft neutrosophic strong biideal (F,A) over $\{B_N(G),*,\circ\}$ is a soft neutrosophic strong bigroupoid.

Proposition 4.2.22. Let (F,A) and (K,B) be two soft neutrosophic strong biideals over $\{B_N(G),*,\circ\}$. Then

1. Their extended intersection $(F,A)\cap_E (K,B)$ is soft neutrosophic strong biideal over $\{B_N(G),*,\circ\}$.
2. Their restricted intersection $(F,A)\cap_R (K,B)$ is soft neutrosophic strong biideal over $\{B_N(G),*,\circ\}$.
3. Their AND operation $(F,A)\wedge(K,B)$ is soft neutrosophic strong biideal over $\{B_N(G),*,\circ\}$.

Remark 4.2.23. Let (F,A) and (K,B) be two soft neutrosophic strong biideals over $\{B_N(G),*,\circ\}$. Then

1. Their extended union $(F,A)\cup_E (K,B)$ is not soft neutrosophic strong biideals over $\{B_N(G),*,\circ\}$.
2. Their restricted union $(F,A)\cup_R (K,B)$ is not soft neutrosophic strong biidleals over $\{B_N(G),*,\circ\}$.
3. Their OR operation $(F,A)\vee(K,B)$ is not soft neutrosophic strong biideals over $\{B_N(G),*,\circ\}$.

One can easily proved $(1),(2),$ and (3) by the help of examples

Theorem 4.2.24. Let (F,A) be a soft neutrosophic strong biideal over $\{B_N(G),*,\circ\}$ and $\{(H_i,B_i): i\in J\}$ is a non-empty family of soft neutrosophic strong biideals of (F,A). Then

1. $\underset{i\in J}{\cap}(H_i,B_i)$ is a soft neutrosophic strong biideal of (F,A).
2. $\underset{i\in J}{\wedge}(H_i,B_i)$ is a soft neutrosophic strong biideal of $\underset{i\in J}{\wedge}(F,A)$.

5 Soft Neutrosophic N-groupoid and Their Properties

5.1 Soft Neutrosophic N-groupoid

Definition 5.1.1. Let $N(G)=\{G_1\cup G_2\cup\ldots\cup G_N,*_1,*_2,\ldots,*_N\}$ be a neutrosophic N-groupoid and (F,A) be a soft set over $N(G)=\{G_1\cup G_2\cup\ldots\cup G_N,*_1,*_2,\ldots,*_N\}$. Then (F,A) is called soft neutrosophic N-groupoid if and only if $F(a)$ is neutrosophic sub N-groupoid of $N(G)=\{G_1\cup G_2\cup\ldots\cup G_N,*_1,*_2,\ldots,*_N\}$ for all $a\in A$.

Example 5.1.2. Let $N(G)=\{G_1\cup G_2\cup G_3,*_1,*_2,*_3\}$ be a neutrosophic 3-groupoid, where $G_1=\{\langle Z_{10}\cup I\rangle \mid a*b=2a+3b(\text{mod}10); a,b\in\langle Z_{10}\cup I\rangle\}$, $G_2=\{\langle Z_4\cup I\rangle \mid a\circ b=2a+b(\text{mod}\,4); a,b\in\langle Z_4\cup I\rangle\}$ and $G_3=\{\langle Z_{12}\cup I\rangle \mid a*b=8a+4b(\text{mod}12); a,b\in\langle Z_{12}\cup I\rangle\}$.

Let $A=\{a_1,a_2\}$ be a set of parameters. Then (F,A) is a soft neutrosophic N-groupoid over $N(G)=\{G_1\cup G_2\cup G_3,*_1,*_2,*_3\}$, where $F(a_1)=\{0,5,5I,5+5I\}\cup\{0,2,2I,2+2I\}\cup\{0,2\}$,

$$F(a_2)=(Z_{10},*)\cup\{0,2+2I\}\cup\{0,2I\}.$$

Theorem 5.1.3. Let (F,A) and (H,A) be two soft neutrosophic N-groupoids over $N(G)$. Then their intersection $(F,A)\cap(H,A)$ is again a soft neutrosophic N-groupoid over $N(G)$.

Theorem 5.1.4. Let (F,A) and (H,B) be two soft neutrosophic N-groupoids over $N(G)$. If $A\cap B=\phi$, then $(F,A)\cup(H,B)$ is a soft neutrosophic N-groupoid over $N(G)$.

Proposition 5.1.5. Let (F,A) and (K,C) be two soft neutrosophic N-groupoids over $N(G)$. Then

1. Their extended intersection $(F,A)\cap_E (K,C)$ is a soft neutrosophic N-groupoid over $N(G)$.
2. Their restricted intersection $(F,A)\cap_R (K,C)$ is a soft neutrosophic N-groupoid over $N(G)$.
3. Their AND operation $(F,A)\wedge(K,C)$ is a soft neutrosophic N-groupoid over $N(G)$.

Remark 5.1.4. Let (F,A) and (K,C) be two soft neutrosophic N-groupoids over $N(G)$. Then

1. Their extended union $(F,A)\cup_E (K,C)$ is not a soft neutrosophic N-groupoid over $N(G)$.
2. Their restricted union $(F,A)\cup_R (K,C)$ is not a soft neutrosophic N-groupoid over $N(G)$.
3. Their OR operation $(F,A)\vee(K,C)$ is not a soft neutrosophic N-groupoid over $N(G)$.

One can easily verify $(1),(2),$ and (3) by the help of examples.

Definition 5.1.5. Let (F,A) be a soft neutrosophic N-groupoid over $N(G)$. Then (F,A) is called an absolute soft neutrosophic N-groupoid over $N(G)$ if $F(a)=N(G)$ for all $a\in A$.

Definition 5.1.6. Let (F,A) and (H,C) be two soft neutrosophic N-groupoids over $N(G)$. Then (H,C) is called soft neutrosophic sub N-groupoid of (F,A), if

1. $C\subseteq A$.
2. $H(a)$ is a neutrosophic sub bigroupoid of $F(a)$ for all $a\in A$.

Example 5.1.7. Let $N(G)=\{G_1\cup G_2\cup G_3,*_1,*_2,*_3\}$ be a neutrosophic 3-groupoid, where $G_1=\{\langle Z_{10}\cup I\rangle \mid a*b=2a+3b(\text{mod}10);a,b\in\langle Z_{10}\cup I\rangle\}$, $G_2=\{\langle Z_4\cup I\rangle \mid a\circ b=2a+b(\text{mod}4);a,b\in\langle Z_4\cup I\rangle\}$ and $G_3=\{\langle Z_{12}\cup I\rangle \mid a*b=8a+4b(\text{mod}12);a,b\in\langle Z_{12}\cup I\rangle\}$.

Let $A=\{a_1,a_2\}$ be a set of parameters. Then (F,A) is a soft neutrosophic N-groupoid over $N(G)=\{G_1\cup G_2\cup G_3,*_1,*_2,*_3\}$, where

$$F(a_1)=\{0,5,5I,5+5I\}\cup\{0,2,2I,2+2I\}\cup\{0,2\},$$

$$F(a_2)=(Z_{10},*)\cup\{0,2+2I\}\cup\{0,2I\}.$$

Let $B=\{a_1\}\subseteq A$. Then (H,B) is a soft neutrosophic sub N-groupoid of (F,A), where

$$H(a_1)=\{0,5\}\cup\{0,2+2I\}\cup\{0,2\}.$$

Definition 5.1.8. Let $N(G)$ be a neutrosophic N-groupoid and (F,A) be a soft neutrosophic N-groupoid over $N(G)$. Then (F,A) is called soft Lagrange neutrosophic N-groupoid if and only if $F(a)$ is a Lagrange neutrosophic sub N-groupoid of $N(G)$ for all $a\in A$.

Theorem 5.1.9. Every soft Lagrange neutrosophic N-groupoid over $N(G)$ is a soft neutrosophic N-groupoid over $N(G)$ but the converse may not be true.

One can easily see the converse by the help of examples.

Theorem 5.1.10. If $N(G)$ is a Lagrange neutrosophic N-groupoid, then (F,A) over $N(G)$ is a soft Lagrange neutrosophic N-groupoid but the converse is not true.

Remark 5.1.11. Let (F,A) and (K,C) be two soft Lagrange neutrosophic N-groupoids over $N(G)$. Then

1. Their extended intersection $(F,A)\cap_E(K,C)$ may not be a soft Lagrange neutrosophic N-groupoid over $N(G)$.
2. Their restricted intersection $(F,A)\cap_R(K,C)$ may not be a soft Lagrange neutrosophic N-groupoid over $N(G)$.
3. Their AND operation $(F,A)\wedge(K,C)$ may not be a soft Lagrange neutrosophic N-groupoid over $N(G)$.
4. Their extended union $(F,A)\cup_E(K,C)$ may not be a soft Lagrange neutrosophic N-groupoid over $N(G)$.
7. Their restricted union $(F,A)\cup_R(K,C)$ may not be a soft Lagrange neutrosophic N-groupoid over $N(G)$.
8. Their OR operation $(F,A)\vee(K,C)$ may not be a soft Lagrange neutrosophic N-groupoid over $N(G)$.

One can easily verify $(1),(2),(3),(4),(5)$ and (6) by the help of examples.

Definition 5.1.12. Let $N(G)$ be a neutrosophic N-groupoid and (F,A) be a soft neutrosophic N-groupoid over $N(G)$. Then (F,A) is called soft weak Lagrange neutrosophic N-groupoid if atleast one $F(a)$ is not a Lagrange neutrosophic sub N-groupoid of $N(G)$ for some $a\in A$.

Theorem 5.1.13. Every soft weak Lagrange neutrosophic N-groupoid over $N(G)$ is a soft neutrosophic N-groupoid over $N(G)$ but the converse is not true.

Theorem 5.1.14. If $N(G)$ is weak Lagrange neutrosophic N-groupoid, then (F,A) over $N(G)$ is also a soft weak Lagrange neutrosophic bigroupoid but the converse is not true.

Remark 5.1.15. Let (F,A) and (K,C) be two soft weak Lagrange neutrosophic N-groupoids over $N(G)$. Then

1. Their extended intersection $(F,A)\cap_E(K,C)$ may not be a soft weak Lagrange neutrosophic N-groupoid over $N(G)$.
2. Their restricted intersection $(F,A)\cap_R(K,C)$ may not be a soft weak Lagrange neutrosophic N-groupoid over $N(G)$.
3. Their AND operation $(F,A)\wedge(K,C)$ may not be a soft weak Lagrange neutrosophic N-groupoid over $N(G)$.

4. Their extended union $(F,A)\cup_E(K,C)$ may not be a soft weak Lagrnage neutrosophic N-groupoid over $N(G)$.
5. Their restricted union $(F,A)\cup_R(K,C)$ may not be a soft weak Lagrange neutrosophic N-groupoid over $N(G)$.
6. Their OR operation $(F,A)\vee(K,C)$ may not be a soft weak Lagrange neutrosophic N-groupoid over $N(G)$.

One can easily verify $(1),(2),(3),(4),(5)$ and (6) by the help of examples.

Definition 5.1.16. Let $N(G)$ be a neutrosophic N-groupoid and (F,A) be a soft neutrosophic N-groupoid over $N(G)$. Then (F,A) is called soft Lagrange free neutrosophic N-groupoid if $F(a)$ is not a Lagrange neutrosophic sub N-groupoid of $N(G)$ for all $a\in A$.

Theorem 5.1.17. Every soft Lagrange free neutrosophic N-groupoid over $N(G)$ is a soft neutrosophic N-groupoid over $N(G)$ but the converse is not true.

Theorem 5.1.18. If $N(G)$ is a Lagrange free neutrosophic N-groupoid, then (F,A) over $N(G)$ is also a soft Lagrange free neutrosophic N-groupoid but the converse is not true.

Remark 5.1.19. Let (F,A) and (K,C) be two soft Lagrange free neutrosophic N-groupoids over $N(G)$. Then

1. Their extended intersection $(F,A)\cap_E(K,C)$ is not a soft Lagrange free neutrosophic N-groupoid over $N(G)$.
2. Their restricted intersection $(F,A)\cap_R(K,C)$ is not a soft Lagrange free neutrosophic N-groupoid over $N(G)$.
3. Their AND operation $(F,A)\wedge(K,C)$ is not a soft Lagrange free neutrosophic N-groupoid over $N(G)$.
4. Their extended union $(F,A)\cup_E(K,C)$ is not a soft Lagrange free neutrosophic N-groupoid over $N(G)$.
5. Their restricted union $(F,A)\cup_R(K,C)$ is not a soft Lagrange free neutrosophic N-groupoid over $N(G)$.
6. Their OR operation $(F,A)\vee(K,C)$ is not a soft Lagrange free neutrosophic N-groupoid over $N(G)$.

One can easily verify $(1),(2),(3),(4),(5)$ and (6) by the help of examples.

Definition 5.1.20. (F,A) is called soft neutrosophic N-ideal over $N(G)$ if and only if $F(a)$ is a neutrosophic N-ideal of $N(G)$, for all $a\in A$.

Theorem 5.1.21. Every soft neutrosophic N-ideal (F,A) over $N(G)$ is a soft neutrosophic N-groupoid.

Proposition 5.1.22. Let (F,A) and (K,B) be two soft neutrosophic N-ideals over $N(G)$. Then

1. Their extended intersection $(F,A)\cap_E(K,B)$ is soft neutrosophic N-ideal over $N(G)$.
2. Their restricted intersection $(F,A)\cap_R(K,B)$ is soft neutrosophic N-ideal over $N(G)$.
3. Their AND operation $(F,A)\wedge(K,B)$ is soft neutrosophic N-ideal over $N(G)$.

Remark 5.1.23. Let (F,A) and (K,B) be two soft neutrosophic N-ideals over $N(G)$. Then

1. Their extended union $(F,A)\cup_E(K,B)$ is not a soft neutrosophic N-ideal over $N(G)$.
2. Their restricted union $(F,A)\cup_R(K,B)$ is not a soft neutrosophic N-idleal over $N(G)$.
3. Their OR operation $(F,A)\vee(K,B)$ is not a soft neutrosophic N-ideal over $N(G)$.

One can easily proved $(1),(2),$ and (3) by the help of examples

Theorem 5.1.24. Let (F,A) be a soft neutrosophic N-ideal over $N(G)$ and $\{(H_i,B_i):\mathrm{i}\in\mathrm{J}\}$ be a non-empty family of soft neutrosophic N-ideals of (F,A). Then

1. $\underset{i\in J}{\cap}(H_i,B_i)$ is a soft neutrosophic N-ideal of (F,A).
2. $\underset{i\in J}{\wedge}(H_i,B_i)$ is a soft neutrosophic N-ideal of $\underset{i\in J}{\wedge}(F,A)$.

5.2 Soft Neutrosophic Strong N-groupoid

Definition 5.2.1. Let $\mathrm{N(G)}=\{G_1\cup G_2\cup\ldots\cup \mathrm{G}_N,*_1,*_2,\ldots,*_N\}$ be a neutrosophic N-groupoid and (F,A) be a soft set over $\mathrm{N(G)}=\{G_1\cup G_2\cup\ldots\cup \mathrm{G}_N,*_1,*_2,\ldots,*_N\}$. Then (F,A) is called soft neutrosophic strong N-groupoid if and only if $F(a)$ is neutrosophic strong sub N-groupoid of $\mathrm{N(G)}=\{G_1\cup G_2\cup\ldots\cup \mathrm{G}_N,*_1,*_2,\ldots,*_N\}$ for all $a\in A$.

Example 5.2.2. Let $\mathrm{N(G)} = \{G_1 \cup G_2 \cup \mathrm{G}_3, *_1, *_2, *_3\}$ be a neutrosophic 3-groupoid, where

$G_1 = \{\langle Z_{10} \cup I \rangle \mid a * b = 2a + 3b(\mathrm{mod}10); a, b \in \langle Z_{10} \cup I \rangle\}$,

$G_2 = \{\langle Z_4 \cup I \rangle \mid a \circ b = 2a + b(\mathrm{mod}\,4); a, b \in \langle Z_4 \cup I \rangle\}$ and

$G_3 = \{\langle Z_{12} \cup I \rangle \mid a * b = 8a + 4b(\mathrm{mod}12); a, b \in \langle Z_{12} \cup I \rangle\}$.

Let $A = \{a_1, a_2\}$ be a set of parameters. Then (F, A) is a soft neutrosophic N-groupoid over $\mathrm{N(G)} = \{G_1 \cup G_2 \cup \mathrm{G}_3, *_1, *_2, *_3\}$, where

$$F(a_1) = \{0, 5I\} \cup \{0, 2I\} \cup \{0, 2I\},$$

$$F(a_2) = \{0, 5 + 5I\} \cup \{0, 2 + 2\mathrm{I}\} \cup \{0, 2 + 2\mathrm{I}\}.$$

Theorem 5.2.3. Let (F, A) and (H, A) be two soft neutrosophic strong N-groupoids over $N(G)$. Then their intersection $(F, A) \cap (H, A)$ is again a soft neutrosophic strong N-groupoid over $N(G)$.

Theorem 5.2.4. Let (F, A) and (H, B) be two soft neutrosophic strong N-groupoids over $N(G)$. If $A \cap B = \phi$, then $(F, A) \cup (H, B)$ is a soft neutrosophic strong N-groupoid over $N(G)$.

Theorem 5.2.5. If $N(G)$ is a neutrosophic strong N-groupoid, then (F, A) over $N(G)$ is also a soft neutrosophic strong N-groupoid.

Proposition 5.2.6. Let (F, A) and (K, C) be two soft neutrosophic strong N-groupoids over $N(G)$. Then

1. Their extended intersection $(F, A) \cap_E (K, \mathrm{C})$ is a soft neutrosophic strong N-groupoid over $N(G)$.
2. Their restricted intersection $(F, A) \cap_R (K, \mathrm{C})$ is a soft neutrosophic strong N-groupoid over $N(G)$.
3. Their AND operation $(F, A) \wedge (K, \mathrm{C})$ is a soft neutrosophic strong N-groupoid over $N(G)$.

Remark 5.2.7. Let (F, A) and (K, C) be two soft neutrosophic strong N-groupoids over $N(G)$. Then

1. Their extended union $(F, A) \cup_E (K, \mathrm{C})$ is not a soft neutrosophic strong N-groupoid over $N(G)$.
2. Their restricted union $(F, A) \cup_R (K, \mathrm{C})$ is not a soft neutrosophic strong N-groupoid over $N(G)$.
3. Their OR operation $(F, A) \vee (K, \mathrm{C})$ is not a soft neutrosophic strong N-groupoid over $N(G)$.

One can easily verify $(1), (2),$ and (3) by the help of examples.

Definition 5.2.8. Let (F, A) and (H, C) be two soft neutrosophic strong N-groupoids over $N(G)$. Then (H, C) is called soft neutrosophic strong sub N-groupoid of (F, A), if

1. $C \subseteq A$.
2. $H(a)$ is a neutrosophic sub bigroupoid of $F(a)$ for all $a \in A$.

Definition 5.2.9. Let $N(G)$ be a neutrosophic strong N-groupoid and (F, A) be a soft neutrosophic strong N-groupoid over $N(G)$. Then (F, A) is called soft Lagrange neutrosophic strong N-groupoid if and only if $F(a)$ is a Lagrange neutrosophic sub N-groupoid of $N(G)$ for all $a \in A$.

Theorem 5.2.10. Every soft Lagrange neutrosophic strong N-groupoid over $N(G)$ is a soft neutrosophic N-groupoid over $N(G)$ but the converse may not be true.

One can easily see the converse by the help of examples.

Theorem 5.2.11. Every soft Lagrange neutrosophic strong N-groupoid over $N(G)$ is a soft Lagrange neutrosophic N-groupoid over $N(G)$ but the converse may not be true.

Theorem 5.2.12. If $N(G)$ is a Lagrange neutrosophic strong N-groupoid, then (F, A) over $N(G)$ is a soft Lagrange neutrosophic strong N-groupoid but the converse is not true.

Remark 5.2.13. Let (F, A) and (K, C) be two soft Lagrange neutrosophic strong N-groupoids over $N(G)$. Then

1. Their extended intersection $(F, A) \cap_E (K, \mathrm{C})$ may not be a soft Lagrange neutrosophic strong N-groupoid over $N(G)$.
2. Their restricted intersection $(F, A) \cap_R (K, \mathrm{C})$ may not be a soft Lagrange neutrosophic strong N-groupoid over $N(G)$.
3. Their AND operation $(F, A) \wedge (K, \mathrm{C})$ may not be a soft Lagrange neutrosophic strong N-groupoid over $N(G)$.
4. Their extended union $(F, A) \cup_E (K, \mathrm{C})$ may not be a soft Lagrange neutrosophic strong N-

groupoid over $N(G)$.

5. Their restricted union $(F,A)\cup_R (K,C)$ may not be a soft Lagrange neutrosophic strong N-groupoid over $N(G)$.
6. Their OR operation $(F,A)\vee(K,C)$ may not be a soft Lagrange neutrosophic strong N-groupoid over $N(G)$.

One can easily verify $(1),(2),(3),(4),(5)$ and (6) by the help of examples.

Definition 5.2.14. Let $N(G)$ be a neutrosophic strong N-groupoid and (F,A) be a soft neutrosophic strong N-groupoid over $N(G)$. Then (F,A) is called soft weak Lagrange neutrosophic strong N-groupoid if atleast one $F(a)$ is not a Lagrange neutrosophic sub N-groupoid of $N(G)$ for some $a\in A$.

Theorem 5.2.15. Every soft weak Lagrange neutrosophic strong N-groupoid over $N(G)$ is a soft neutrosophic N-groupoid over $N(G)$ but the converse is not true.

Theorem 5.2.16. Every soft weak Lagrange neutrosophic strong N-groupoid over $N(G)$ is a soft weak Lagrange neutrosophic N-groupoid over $N(G)$ but the converse is not true.

Remark 5.2.17. Let (F,A) and (K,C) be two soft weak Lagrange neutrosophic strong N-groupoids over $N(G)$. Then

1. Their extended intersection $(F,A)\cap_E (K,C)$ may not be a soft weak Lagrange neutrosophic strong N-groupoid over $N(G)$.
2. Their restricted intersection $(F,A)\cap_R (K,C)$ may not be a soft weak Lagrange neutrosophic strong N-groupoid over $N(G)$.
3. Their AND operation $(F,A)\wedge(K,C)$ may not be a soft weak Lagrange neutrosophic strong N-groupoid over $N(G)$.
4. Their extended union $(F,A)\cup_E (K,C)$ may not be a soft weak Lagrnage neutrosophic strong N-groupoid over $N(G)$.
5. Their restricted union $(F,A)\cup_R (K,C)$ may not be a soft weak Lagrange neutrosophic strong N-groupoid over $N(G)$.
6. Their OR operation $(F,A)\vee(K,C)$ may not be a soft weak Lagrange neutrosophic strong N-groupoid over $N(G)$.

One can easily verify $(1),(2),(3),(4),(5)$ and (6) by the help of examples.

Definition 5.2.18. Let $N(G)$ be a neutrosophic strong N-groupoid and (F,A) be a soft neutrosophic strong N-groupoid over $N(G)$. Then (F,A) is called soft Lagrange free neutrosophic strong N-groupoid if $F(a)$ is not a Lagrange neutrosophic sub N-groupoid of $N(G)$ for all $a\in A$.

Theorem 5.2.19. Every soft Lagrange free neutrosophic strong N-groupoid over $N(G)$ is a soft neutrosophic N-groupoid over $N(G)$ but the converse is not true.

Theorem 5.2.20. Every soft Lagrange free neutrosophic strong N-groupoid over $N(G)$ is a soft Lagrange neutrosophic N-groupoid over $N(G)$ but the converse is not true.

Theorem 5.2.21. If $N(G)$ is a Lagrange free neutrosophic strong N-groupoid, then (F,A) over $N(G)$ is also a soft Lagrange free neutrosophic strong N-groupoid but the converse is not true.

Remark 5.2.22. Let (F,A) and (K,C) be two soft Lagrange free neutrosophic N-groupoids over $N(G)$. Then

1. Their extended intersection $(F,A)\cap_E (K,C)$ is not a soft Lagrange free neutrosophic strong N-groupoid over $N(G)$.
2. Their restricted intersection $(F,A)\cap_R (K,C)$ is not a soft Lagrange free neutrosophic strong N-groupoid over $N(G)$.
3. Their AND operation $(F,A)\wedge(K,C)$ is not a soft Lagrange free neutrosophic strong N-groupoid over $N(G)$.
4. Their extended union $(F,A)\cup_E (K,C)$ is not a soft Lagrange free neutrosophic strong N-groupoid over $N(G)$.
5. Their restricted union $(F,A)\cup_R (K,C)$ is not a soft Lagrange free neutrosophic strong N-groupoid over $N(G)$.
6. Their OR operation $(F,A)\vee(K,C)$ is not a soft Lagrange free neutrosophic stong N-groupoid over $N(G)$.

One can easily verify $(1),(2),(3),(4),(5)$ and (6) by the help of examples.

Definition 5.2.23. (F,A) is called soft neutrosophic strong N-ideal over $N(G)$ if and only if $F(a)$ is a neutrosophic strong N-ideal of $N(G)$, for all $a\in A$.

Theorem 5.2.24. Every soft neutrosophic strong N-ideal

(F,A) over $N(G)$ is a soft neutrosophic N-groupoid.

Theorem 5.2.25. Every soft neutrosophic strong N-ideal (F,A) over $N(G)$ is a soft neutrosophic N-ideal but the converse is not true.

Proposition 15. Let (F,A) and (K,B) be two soft neutrosophic strong N-ideals over $N(G)$. Then

1. Their extended intersection $(F,A)\cap_E (K,B)$ is soft neutrosophic strong N-ideal over $N(G)$.
2. Their restricted intersection $(F,A)\cap_R (K,B)$ is soft neutrosophic strong N-ideal over $N(G)$.
3. Their AND operation $(F,A)\wedge(K,B)$ is soft neutrosophicstrong N-ideal over $N(G)$.

Remark 5.2.26. Let (F,A) and (K,B) be two soft neutrosophic strong N-ideals over $N(G)$. Then

1. Their extended union $(F,A)\cup_E (K,B)$ is not a soft neutrosophic strong N-ideal over $N(G)$.
2. Their restricted union $(F,A)\cup_R (K,B)$ is not a soft neutrosophic strong N-idleal over $N(G)$.
3. Their OR operation $(F,A)\vee(K,B)$ is not a soft neutrosophic strong N-ideal over $N(G)$.

One can easily proved $(1),(2),$ and (3) by the help of examples

Theorem 5.2.27. Let (F,A) be a soft neutrosophic strong N-ideal over $N(G)$ and $\{(H_i,B_i):\mathrm{i}\in\mathrm{J}\}$ be a non-empty family of soft neutrosophic strong N-ideals of (F,A). Then

1. $\underset{i\in J}{\cap}(H_i,B_i)$ is a soft neutrosophic strong N-ideal of (F,A).
2. $\underset{i\in J}{\wedge}(H_i,B_i)$ is a soft neutrosophic strong N-ideal of $\underset{i\in J}{\wedge}(F,A)$.

Conclusion

This paper is an extension of neutrosphic groupoids to soft neutrosophic groupoids. We also extend neutrosophic bigroupoid, neutrosophic N -groupoid to soft neutrosoph ic bigroupoid, and soft neutrosophic N -groupoid. Their related properties and results are explained with many illustrative examples. The notions related with strong part of neutrosophy also established within soft neutrosophic groupoids.

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Neutrosophic routes in multiverse of communication

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Abstract. Florentin Smarandache and Ştefan Vlăduţescu the authors and coordinators of the book "Communication Neutrosophic Routes", published by Education Publishing, Ohio, USA, on 2014, are two remarcable professors, with many researches in neutrosophical, communication, mathematic, literature domains, social sciences. Logic is a fundamental component of advanced computer classes. Reference is constantly being made to how the rules of logic are incorporated into the fundamental circuits of a computer. The logic used in these classes is known as classical or Boolean logic. Neutrosophic logic is an extension of classical logic, there are two intermediate steps between them. Neutrosophic logic is an idea generated by Florentin Smarandache. Like classical logic, it can be used in many ways, everywhere from statistics to quantum mechanics. Neutrosophy is more than just a form of logic however. Neutrosophic emergences are the unexpected occurrences of some major neutrosophic effects from the interaction of some minor qualitative elements.

Keywords: neutrosophy, multiverse of communication, neutrosophic communication routes

1 New ways of communication

Will really do the Humanity arrived to insensibility limit where it is just reason, where the feeling definitively lost its existential value? If it is true means that Albert Camus was right: only logical solution is suicide. To run from the darkness of the death, of the nightmares that ourselves generate them on its behalf, we have some solutions among which obvious suicide, or why not the optimism of the life spectacle. Suicide is <anti-A>; authentic beside the optimism represented by the neutrosophic <A>. If we accept the suicide or the equivalent or the <anti-A> is such as we should accept to spite ours face. As Brâncuşi said that he is not creating the beauty, he only removes the idle material to be easier for us to identify the beauty beside him. As well in this study is defined (is removed) <anti-A> for the it's beauty and meaning, to be visible the beauty of our existence in front of its non-existence. Of the non-existence fears any existence, even the Universe itself, maybe only nonexistence itself is not afraid of itself, or maybe people who forget in their existence or do not know that they exist there. Likewise, we define (we remove) <anti-A> for the beauty and its meaning… to be visible the beauty of our existence in front of nonexistence (Smarandache F.) .

The study "Communication Neutrosophic Routes" focuses on revealing the predominantly neutrosophic character of any communication and aesthetic interpretation. Neutrosophy, a theory grounded by Florentin Smarandache, is a coherent thinking of neutralities; different from G.W. F. Hegel, neutrality is the rule, the contradiction is the exception; the universe is not a place of contradictions, but one of neutralities; the material and significant-symbolic universe consists predominantly of neutrality relationships. Any communication is accompanied by interpretation; sharply, aesthetic communication, by its strong ambiguous character, forces of the interpretation. Since, due to comprehension, description and explanation, the interpretation manages contradictions, understanding conflicts and roughness of reading, aesthetic interpretation is revealed as a deeply neutrosophic interpretation.

Communication and aesthetic interpretation prevalently manage neutralities but contradictions. As authors asserts**,** any manifestation of life is a component of communication, it is crossed by a communication passage. People irrepressibly generate meanings. As structuring domain of meanings, communication is a place where meanings burst out volcanically. Manifestations of life are surrounded by a halo of communicational meanings. Human material and ideatic existence includes a great potential of communication in continuous extension. The human being crosses the path of or is at the intersection of different communicational thoroughfares. The life of human beings is a place of communication. Consequently, any cognitive or cogitative manifestation presents a route of communication. People consume their lives relating by communicationally. Some communicational relationships are contradictory, others are neutral, since within the manifestations of life there are found conflicting meanings and/or neutral meanings.

Communicational relations always comprise a set of neutral, neutrosophic meanings. Communication in general is a human manifestation of life with recognizable profile. Particularly, we talk about scientific communication, literary communication, pictorial communication, sculptural communication, esthetic communication and so on, as specific manifestations of life. All these include coherent, cohesive and structural series of existential meanings which are contradictory and/or neutral, neutrosophic. It can be asserted that in any communication there are routes of access and neutrosophic routes. Any communication is traversed by neutrosophic routes of communication.

2 Book content

The book is structured in ten chapters, each one presenting and arguing the novelty of neutrosophic concept in different areas. The studies in this book are application of the thesis of neutrosophic routes of communication and highlight neutrosophic paths, trajectories, itineraries, directions and routes in different forms and types of communication.

In Chapter 1, Florentin Smarandache and Ştefan Vlăduţescu develop the thesis of neutrosophic routes in the hermeneutics of text; they emphasize the fact that any text allows an infinity of interpretative routes: some based on linguistic-semiotic landmarks, others sustained by sociologic ideas, others founded by moral reference points, others founded by esthetic aspects and so on. A neutrosophic route can always be found in a text, that is a route of neutral elements, a thoroughfare of neutralities.

Professors Ioan Constantin Dima and Mariana Man reveal, in Chapter 2, that is not insignificant for a system to ensure that the events observed are representative for its universe, that they are observed in a precise, neutrosophic and coherent manner and that there are analysis patterns, deeds scientifically established to enable valid estimations and deductions.

In Chapter 3, Alexandra Iorgulescu (Associate Professor at the University of Craiova, Romania) decodes the neutrosophic inflections of Seneca’s tragedies.
Assistant Professor Alina Ţenescu (University of Craiova) analyzes, in Chapter 4, in the non-space in contemporary French novel. The non-space is identified as a neutrosophic neutrality, which allows an application of the methodology and hermeneutics of neutrosophy. Finally, there is revealed a richness of meaning that provides the neutrosophic approach. In Chapter 5, Mădălina Strechie (Senior Lecturer at the University of Craiova, Romania) illustrates the communication as a key source of neutrality in Ancient Rome communication. In Chapter 6, the contribution of Daniela Gîfu (Senior Lecturer at the University of Iaşi, Romania) gives relevance to the “religious humor” in the reference system created by the two mega-concepts launched and imposed by Florentin Smarandache, neutrosophy and paradoxism. In Chapter 7, prepared by Professor Mihaela Gabriel Păun (a Romanian language and literature teacher), focuses on the neutrosophic determining of Romanian popular incidences in the brilliant sculptural work of Romanian artist Constantin Brâncuşi (an unstoppable spiritual-aesthetic river appeared out of everyday folk tributaries). In Chapter 8, Professors Maria Nowicka-Skowron and Sorin Mihai Radu show that the major moments of reproduction are governed only by generally valid rules, and the main dimension of operating such an economy is the market and mechanisms of the market created in principle from the movement of prices according to the demand and supply ratio on the competitive market.

In Chapter 9, Professors Janusz Grabara and Ion Cosmescu demonstrate that being aware of the role that an information system in the company plays and its impact on individual processes, this article presents an information system used in the selected company. In Chapter 10, Bianca Teodorescu (from University of Craiova) shows that communication represents a category more enlarged than the information and has an ordinate concept; information is a part in the process of global communication.

Conclusion

The current book through its studies, represents a novelty in the field, a proof that neutrosophy is a domain of science that can be applied in any domain as interpretation of neutrality, a point of reference for students, master, doctoral, presenting ideas, principles, connections, relationships, interpretations of various fields as specificity, space and time.

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Abstract

This volume is a collection of ten papers and a review of a book, written by different authors and co-authors (listed in the order of the papers): F. Yuhua, P. K. Maji, A. A. Salama, H. Elghawalby, A. Mukherjee, M. Datta, F. Smarandache, K. Mondal, S. Pramanik, M. Ali, L. Vladareanu, M. Shabir, S. Broumi, S. Ye, J. Ye, S. Sarkar, D. Gifu and M. Teodorescu.

In first paper, the author proposed Pauli Exclusion Principle and the Law of In-cluded Multiple-Middle. Weighted Neutrosophic Soft Sets are proposed in the second paper. Neutrosophic Crisp Sets and Neutrosophic Crisp Relations are studied in third paper. In fourth paper, Interval Valued Neutrosophic Soft Topological Spaces are introduced. Similarly in fifth paper, Multi-criteria Group Decision Making Approach for Teacher Recruitment in Higher Education Under Simplified Neutrosophic Environment is discussed. In paper six, Generalization of Soft Neutrosophic Rings and Soft Neutrosophic Fields are presented by the authors. Neutrosophic Refined Similarity Measure Based on Cosine Function is given in seventh paper. Paper eight is about to study Similarity Measure between Single Valued Neutrosophic Multisets and Its Appication in Medial Diagnosis. In the next paper Several Similarity Measures of Interval Valued Neutrosophic Soft Sets and Their Application in Pattern Recognition Problems are discussed. The authors introduced Soft Neutrosophic Groupoids and Their Generalization in the tenth paper. At the end a book reveiw, Neutosophic routes in multiverse of communication is presented by the authors.

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