The Generators of Quantum Fields
Part 1

Peter Hickman 22 August 2015

ABSTRACT

G matrices are matrices with single entries 1,i. It is shown that the G matrices generate the Cl(p,q), SU(n), SO(n) generators and the matrix representation of CAR and CCR operators. The exponential map of the direct sum of so(n) and su(n) results in an expression for the dimension of a SU(N) group. The spatial dimension n is found to be restricted to 2 or 3 dimensions. It is shown that the unitary representation of SO(3), the spin space of 2x2 SU(2) matrices arises naturally. It is shown that there are 3 generations of chiral electroweak doublets of quarks. The CAR & CCR are invariant under a real scaling of the operators only for a null space-time (n,1) metric condition on the co-ordinates.

Introduction

G matrices are matrices with single entries 1,i. It is shown that the G matrices generate the Cl(p,q), SU(n) generators and the matrix representation of CAR and CCR operators. 2nd quantitisation is avoided by the G matrices forming the quantum algebras - CAR & CCR algebras.

Axiom The infinite set of matrices G_λ called generators with single-entry 1, i over the field $\mathbb{R}$ and are the base space for all physical fields.

1 Fermion Generators

The generators G_λ satisfy the anti-commutation relation

$$\frac{1}{2}\{G_\lambda^\dagger, G_\mu\} = \delta_{\lambda\mu} M_{\lambda\mu} \qquad \textbf{1.1}$$

M are single-entry matrices with entry +1. The generators are also orthonormal. Form the random state S from the generators G_λ

$$S = \beta_\lambda G_\lambda \qquad \textbf{1.2}$$

S will be an element of su(n) or cl(p,q) or so(n) for $\beta = \beta_\lambda$. The exponential map of the direct sums of the so(n) and su(n) generators are elements of the group SU(N) with group dimension d:

$$d = \dim(so(n)) \dim(su(n)) \qquad \textbf{1.3}$$

$$d = \frac{1}{2} n(n-1)(n^2-1) \qquad \textbf{1.4}$$

There are 2 solutions to **1.4**, n=2, n=3. It is conjectured that there are no other solutions to **1.4, h**ence Space is a maximum of 3d.

For n=3, **1.4** results in d=24 and N=5. The su(3) are 3x3 matrices and since su(5) are 5x5 matrices it follows that the so(3) matrices are 2x2. The unitary representation of SO(3) are the 2x2 spin matrices. The su(3) is the algebra of the color space of QCD.

For n=2, so(2) is isomorphic to u(1), so

$$so(2) \oplus su(2) = u(1) \times su(2) \qquad \textbf{1.5}$$

which is the algebra of the electroweak group.

There are $\binom{3}{2} = 3$ 2d sub-spaces of n=3d spaces. Here the spinor of su(2) spin space is 2d hence only 1 chirality. Thus there are 3 chiral electroweak doublets of quarks i.e. 3 generations of chiral quarks.

2 Space-Time

Form matrix representation of fermionic & bosonic operators o_f and the gamma matrices $(x\gamma)_{ip}$. Apply a real scaling transformation to the operators

$$o_f \rightarrow \frac{x_{ip}}{x_0} o_{fp} \qquad \textbf{2.1}$$

where $x_{ip}^{\dagger} = x^{ip}$, $x_0^{\dagger} = x^0$. The CAR & CCR are invariant with respect to the scaled operators **2.1** only if the following condition on the co-ordinates holds:

$$x_{ip}x^{ip} - x_0^2 = 0 \qquad \textbf{2.2}$$

From section 1, space is maximum of 3d, hence the co-ordinates (x_{ip}, x_0) form a Lorentzian space i.e. Space-Time is (3,1).

Conclusion

The G matrices form the generators of the Cl(n), SO(n) and SU(n) groups. The exponential map of the direct sums of the so(n) and su(n) leads to the dimension of space to be 2 or 3. The result is 3 generations of doublets of chiral quarks. Chiral quarks are subject to SU(3) and the electroweak group SU(2)xU(1) groups.