A General Relativity model of subatomic particles

Denny L. Lee.

Email: dlee@directxray.com August 16, 2015

Abstract

The commonly accepted No-Hair Conjecture states that black holes can be completely characterized by three and only three externally observable classical parameters: mass, electrical charge, and angular momentum. The Kerr–Newman metric describes the geometry of space-time in the vicinity of a rotating mass M with charge Q. These three parameters are also the basic parameters of many subatomic particles. In light of the similarities between black holes and subatomic particles, this paper applies the Kerr–Newman solutions to a one half Planck mass spinning with an angular momentum of $\frac{\hbar}{2}$ just like a spin ½ particle. The results exhibit a group of particles with properties similar to all the stable subatomic particles, including the neutrino, electron, position, proton, and anti-proton. The composite state of two or more of these spinning Planck masses exhibits other unstable particles such as the pion, neutron, and kaon, with decay products matching the composite components.

For example, this model leads to a composite of two spin ½ particles with a composite mass of $\left(\frac{2}{\alpha}-1\right)m_e$, where α is the fine structure constant, and m_e is the mass of an electron. The spin of this composite particle is zero. The ultimate decay product can be either, neutrinos and an electron, or positron, similar to a pion (π^- or π^+). The numerical value of $\left(\frac{2}{\alpha}-1\right)m_e$ is 139.54 MeV/c^2 which is very close to the reported mass [9] of a spin zero pion, 139.57 MeV/c^2.

Background and Introduction:

One hundred years ago, in November of 1915, Einstein presented what are now known as the Einstein field equations.

$$G_{\mu\nu} + g_{\mu\nu}\Lambda = \frac{8\pi G}{c^4}T_{\mu\nu} \qquad (1)$$

These equations specify how the geometry of space and time is influenced by whatever matter and radiation are present, and form the core of Einstein's general theory of relativity [1] [2] [3] . The Einstein field equations are nonlinear and very difficult to solve. Einstein used approximation methods in working out initial predictions of the theory. But before the end of 1915, the astrophysicist Karl Schwarzschild found the first non-trivial exact solution to the Einstein field equations, the so-called Schwarzschild metric [4]

$$c^2d\tau^2 = \left(1-\frac{r_s}{r}\right)c^2dt^2 - \left(1-\frac{r_s}{r}\right)^{-1}dr^2 - r^2(d\theta^2 + sin^2\theta d\varphi) \qquad (2)$$

where $r_s = \frac{2MG}{c^2}$.

k

This solution laid the groundwork for the description of the final stages of gravitational collapse, and the objects known today as black holes. In the following years, the first steps towards generalizing Schwarzschild's solution to electrically charged objects were taken. This eventually resulted in the Reissner–Nordström solution [5]

$$ds^2 = \left(1 - \frac{r_s}{r} + \frac{r_Q^2}{r^2}\right) c^2 dt^2 - \left(1 - \frac{r_s}{r} + \frac{r_Q^2}{r^2}\right)^{-1} dr^2 - r^2 d\Omega^2$$

$$\text{where } r_Q^2 = \frac{Q^2 G}{4\pi\varepsilon_0 c^4}, \text{ and } d\Omega^2 = d\theta^2 + sin^2 d\phi^2 \qquad (3)$$

which is now associated with electrically charged black holes. In 1917, Einstein applied his theory to the universe as a whole, initiating the field of relativistic cosmology. In 1965, Ezra "Ted" Newman found the axisymmetric solution of Einstein's field equation for a black hole which is both rotating and electrically charged. This formula for the metric tensor $g_{\mu\nu}$ is called the Kerr–Newman metric. It is a generalization of the Kerr metric for an uncharged spinning point-mass, which had been discovered by Roy Kerr two years earlier. The Kerr–Newman metric [6] describes the geometry of space-time in the vicinity of a rotating mass M with charge Q. The formula for this metric depends upon what coordinates or coordinate conditions are selected. One way to express this metric is by writing down its line element in a particular set of spherical coordinates,[4] also called Boyer–Lindquist coordinates:

$$c^2 d\tau^2 = -\left(\frac{dr^2}{\Delta} + d\vartheta^2\right)\rho^2 + (cdt - asin^2\vartheta d\phi)^2 \frac{\Delta}{\rho^2} - ((r^2 + a^2)d\phi - acdt)^2 \frac{sin^2\vartheta}{\rho^2} \qquad (4)$$

where the coordinates (r, ϑ, ϕ) are standard spherical coordinate system, and the length-scales:

$$a = \frac{J}{Mc}$$
$$\rho^2 = r^2 + a^2 + r_Q^2$$
$$\Delta = r^2 - r_s r + a^2 + r_Q^2$$

have been introduced for brevity. Here r_s is the Schwarzschild radius (in meters) of the massive body, which is related to its mass M by

$$r_s = \frac{2GM}{c^2}$$

where G is the gravitational constant, and r_Q is a length-scale corresponding to the electric charge Q of the mass

$$r_Q^2 = \frac{Q^2}{4\pi\varepsilon_0} \frac{G}{c^4}$$

where $1/4\pi\varepsilon_0$ is Coulomb's force constant.

An alternative metric form of the Kerr Newman Metric can also be written as:

$$c^2 dt^2 = \frac{(\Delta - a^2 sin^2\vartheta)}{\rho^2} c^2 dt^2 - \left(\frac{\rho^2}{\Delta}\right) dr^2 - \rho^2 d\vartheta^2 +$$
$$(a^2 \Delta sin^2\vartheta - r^4 - 2r^2a^2 - a^4)\frac{sin^2\vartheta d\phi^2}{\rho^2} - (\Delta - r^2 - a^2)\frac{2a sin^2\vartheta c dt d\phi}{\rho^2} \quad (5)$$

All these equations and metrics are widely used for describing massive astronomical scale objects from the size of the earth, the sun, neutron stars, quasars, and black holes.

Application of Space Time Metrics to Planck scale particles:

The Planck constant h is one of the fundamental quantities of nature. The energy of electromagnetic wave, light, is $E = hv$, or $\hbar\omega$, where v is the frequency and $\hbar$ is the reduced Planck constant $\hbar = \frac{h}{2\pi}$ and $\omega = 2\pi v$ is the angular frequency. Together with velocity of light c, the gravitational constant G, there are three fundamental units that are naturally composed from these constants: $\sqrt{\frac{\hbar G}{c^3}} = l_p$ is the Planck length ($1.61619926 \times 10^{-35}$ meters); $\sqrt{\frac{\hbar G}{c^5}} = \tau_p$ is the Planck Time ($\sim 5.39106x10^{-44}$ sec), and $\sqrt{\frac{\hbar c}{G}} = m_p$ is the Planck Mass ($2.17651(13)\times10^{-8}$ kg). When these fundamental units are used in the space-time solutions of the Einstein's Equations, some interesting results have followed. An object with the mass of one half Planck Mass, M = ½ $m_p = \frac{1}{2}\sqrt{\frac{\hbar c}{G}}$ has a Schwarzschild radius of one Planck Length $\sqrt{\frac{G\hbar}{c^3}} = l_p$

$$r_s = \frac{2MG}{c^2} = \frac{Gm_p}{c^2} = \frac{G\sqrt{\frac{\hbar c}{G}}}{c^2} = \frac{\sqrt{G\hbar c}}{c^2} = \sqrt{\frac{G\hbar}{c^3}} = l_p.$$

The curvature term of Schwarzschild Equation (2), $\left(1 - \frac{r_s}{r}\right)$ becomes zero and $(1 - \frac{r_s}{r})^{-1}$ term become infinite for a half Planck Mass object at the Schwarzschild radius of $2l_p$. At distances approaching this l_p radius, space-time is highly curved just like an astronomical black hole. It has all the properties just like a "micro-black hole". The local time element, $d\tau$, at a distance r away from the object is slowed down in comparison to the far-away time dt. The local line element in the radial direction is lengthened in comparison to the far away dr according to the following relationship:

$$d\tau(local) = (1 - \frac{2MG}{rc^2})^{\frac{1}{2}} dt, \quad \text{and} \quad \text{dr(local)} = \left(1 - \frac{2MG}{rc^2}\right)^{-\frac{1}{2}} dr. \quad (6)$$

Also, a "probe" particle of mass m, interacting in this field has a constant energy to mass ratio, E/m [(7)] of

$$\frac{E}{mc^2} = \left(1 - \frac{2MG}{rc^2}\right)\frac{dt}{d\tau} = 1 \quad (7)$$

i.e. with a space-time curvature of $\sigma = \left(1 - \frac{2MG}{rc^2}\right)$. For a particle with mass equal to one half Planck mass $\frac{1}{2}\sqrt{\frac{\hbar c}{G}}$, σ becomes zero at the Schwarzschild radius of one Planck length $\sqrt{\frac{\hbar G}{c^3}}$ and the reciprocal of this space-time curvature term is infinite at l_p (a singularity).

Particle with angular moment:

For an object spinning with an angular momentum of J and carrying a charge Q, we can use the Alternative form of Kerr-Newman Metric [6].

$$c^2 \mathrm{d}t^2 = \frac{(\Delta - a^2 sin^2\vartheta)}{\rho^2} c^2 dt^2 - \left(\frac{\rho^2}{\Delta}\right) dr^2 - \rho^2 d\vartheta^2 +$$
$$(a^2 \Delta sin^2\vartheta - r^4 - 2r^2a^2 - a^4)\frac{sin^2\vartheta d\phi^2}{\rho^2} - (\Delta - r^2 - a^2)\frac{2a\vartheta c dt d\phi}{\rho^2}$$

where $a = \frac{J}{Mc}$, $\rho^2 = r^2 + a^2 cos^2\vartheta$, $\Delta = r^2 - r_s r + a^2 + r_Q^2$, and $r_Q^2 = \frac{Q^2}{4\pi\varepsilon_0}\frac{G}{c^4}$.

By re-grouping the time dependent terms, [8] we have

$$c^2 d\tau^2 = \frac{(\Delta - a^2 sin^2\vartheta)}{\rho^2} c^2 dt^2 - (\Delta - r^2 - a^2)\frac{2a sin^2\vartheta c dt d\phi}{\rho^2} + (\text{terms without t}) \quad (8)$$

By replacing $d\phi$ with ωdt where $\omega \equiv \frac{d\phi}{dt}$

$$c^2 d\tau^2 = \frac{(\Delta - a^2 sin^2\vartheta)}{\rho^2} c^2 dt^2 - (\Delta - r^2 - a^2)\frac{2ac\omega sin^2\vartheta dt^2}{\rho^2} + (\text{terms without t})$$

$$c^2 \mathrm{d}\tau^2 = \left[\frac{(\Delta - a^2 sin^2\vartheta)}{\rho^2} c^2 - (\Delta - r^2 - a^2)\frac{2ac\omega sin^2\vartheta}{\rho^2}\right] dt^2 + (terms\ without\ t)$$

A small segment of τ can then be written as

$$\tau = \{\left[\frac{(\Delta - a^2 sin^2\vartheta)}{\rho^2} - (\Delta - r^2 - a^2)\frac{2a\omega sin^2\vartheta}{\rho^2 c}\right] t^2 + (terms\ without\ t)\}^{\frac{1}{2}}$$

If a τ is divided into two sub-segments $\tau = \tau_A + \tau_B$ and the respective r's from M is written as $r = r_A + r_B$,

Then, $\frac{d\tau_A}{dt} = \frac{\left[\frac{(\Delta - a^2 sin^2\vartheta)}{\rho^2} - (\Delta - r^2 - a^2)\frac{2a\omega sin^2\vartheta}{\rho^2 c}\right] t}{\tau_A}$ and $\frac{d\tau_B}{dt} = \frac{\left[\frac{(\Delta - a^2 sin^2\vartheta)}{\rho^2} - (\Delta - r^2 - a^2)\frac{2a\omega sin^2\vartheta}{\rho^2 c}\right] t}{\tau_B}$

Using the Principle of Extremal Aging and setting $\frac{d\tau}{dt} = \frac{d\tau_A}{dt} + \frac{d\tau_B}{dt} = 0$

The constant of motion as the energy for an interacting particle of mass m can be written as

$$\frac{E}{mc^2} = \left[\frac{(\Delta - a^2 sin^2\vartheta)}{\rho^2} - (\Delta - r^2 - a^2)\frac{2a\omega sin^2\vartheta}{\rho^2 c}\right]\frac{dt}{d\tau} \quad (9)$$

$$= \left[1 - \frac{r_s r - r_Q^2}{r^2 + a^2 cos^2\vartheta}\left(1 - \frac{2a\omega sin^2\vartheta}{c}\right)\right]\frac{dt}{d\tau} \quad (10$$

Case I: For Q=0, $a = 0$, a non-rotating electrically neutral object:

$$\frac{E}{mc^2} = \left[1 - \frac{r_s}{r}\right]\frac{dt}{d\tau} = [1 - \frac{2MG}{rc^2}]\frac{dt}{d\tau}$$

The space-time curvature term $[1 - \frac{2MG}{rc^2}]$ becomes zero at $r = \frac{2MG}{c^2} \quad \forall M$

For M $= \frac{1}{2}m_p = \frac{1}{2}\sqrt{\frac{\hbar c}{G}}$, the space-time curvature term $[1 - \frac{2MG}{rc^2}]$ becomes zero at Planck length, l_p and the reciprocal of this term becomes infinite and it is similar to a "micro black hole". This result is the same as using Schwarzschild Metric of Equation (2) above.

Case II: For Q = 0, $(r_Q^2 = 0)$, $a \neq 0$,

An electrically neutral object with an angular momentum,

From Equation (10): $\frac{E}{mc^2} = \left[1 - \frac{r_s r}{r^2 + a^2 cos^2\vartheta}(1 - \frac{2a\omega sin^2\vartheta}{c})\right]\frac{dt}{d\tau}$

(IIA) On the equatorial plane, $\vartheta = \frac{\pi}{2}$ i.e. $cos\vartheta = 0, \quad sin\vartheta = 1$

$$\frac{E}{mc^2} = \left[1 - \frac{r_s r}{r^2}(1 - \frac{2a\omega}{c})\right]\frac{dt}{d\tau} \tag{11}$$

when $2a\omega = c, \quad \frac{E}{mc^2} = [1]\frac{dt}{d\tau} \qquad \forall M \ and \forall r$ (12)

This is to say: This spinning object, independent of its mass, is not causing any space-time curvature on the equatorial plane, just the same as an object of "zero gravitational mass", i.e. equivalent to M = 0. Nevertheless, this object carries a non-zero angular moment of J $(a = \frac{J}{Mc})$.

Furthermore, the condition of $2a\omega = c$ can be written as $2\frac{J}{Mc}\omega = c$.

If $J = \frac{\hbar}{2}$, and $\omega = \frac{1}{2\tau_p}$

then M $= \frac{2J\omega}{c^2} = \frac{\frac{\hbar}{2}\sqrt{\frac{c^5}{\hbar G}}}{c^2} = \frac{1}{2}\sqrt{\frac{\hbar c}{G}} = \frac{1}{2}m_p$

This angular frequency ω_p will be called the Planck Frequency $\left(\omega_p \equiv \frac{1}{\tau_p}\right)$ in this article. A particle with mass equal to one Planck Mass, m_p, spinning at the Planck Frequency, and carrying an angular momentum of $\frac{\hbar}{2}$ satisfies the condition of $\left(1 - \frac{2a\omega}{c}\right) = 0$. Also, particle with mass equal to one half Planck Mass m_p, spinning at one half the Planck Frequency, and carrying an angular momentum of $\frac{\hbar}{2}$ satisfies the condition of $\left(1 - \frac{2a\omega}{c}\right) = 0$. The space time curvature term of such a particle, even though its mass is equal to one half the Planck Mass will be "seen" as a zero mass M = 0 particle on its equatorial plane. Any force acting on this particle can cause it to travel with the velocity of light along its equatorial plane. Unlike the particle of equation (6) and (7) above, this particle having the mass of one Planck Mass, and spinning with an angular momentum of $\frac{\hbar}{2}$, does not contain any singularity of curvature in space-time, and it behaves just like an electrically neutral particle of zero rest mass with spin $\frac{\hbar}{2}$. With the equivalent of zero mass, this particle nevertheless can carry energy and/or transfer an angular momentum of $\frac{\hbar}{2}$ with other interacting particles. All the properties of this particle are very much like that of a neutrino. Could this be a ***neutrino?***

(IIB) Along the polar axis: $\vartheta = 0, \quad cos\vartheta = 1, \quad sin\vartheta = 0$

Equation (10) $\frac{E}{mc^2} = \left[1 - \frac{r_S r}{r^2 + a^2 cos^2\vartheta}\left(1 - \frac{2a\omega sin^2\vartheta}{c}\right)\right]\frac{dt}{d\tau}$ can be written as

$\frac{E}{mc^2} = \left[1 - \frac{r_S r}{r^2+a^2}\right]\frac{dt}{d\tau}$ with $a^2 \neq 0$, $\left[1 - \frac{r_S r}{r^2+a^2}\right] \neq 0$

Since $a = \frac{J}{Mc}$, for a particle of $J = \frac{\hbar}{2}$ and $M = \frac{1}{2}m_p$,

$$a = \frac{\frac{\hbar}{2}}{\frac{1}{2}c\sqrt{\frac{\hbar c}{G}}} = l_p\,, \qquad \text{and} \qquad r_s = \frac{G\sqrt{\frac{\hbar c}{G}}}{c^2} = l_p$$

$$\sigma = \left[1 - \frac{r_S r}{r^2+a^2}\right] = \left[1 - \frac{l_p}{\left(r + \frac{l_p{}^2}{r}\right)}\right] = 1 - \frac{n}{n^2+1} \quad \text{for } r = nl_p. \text{ (see footnote)} \qquad (14)$$

The space-time curvature term $\left[1 - \frac{r_S r}{r^2+a^2}\right]$ is equal to $\frac{1}{2}$ for n=1 in the polar directions. The space-time curvature term does not cause any singularity for all $n \geq 1$ ($\forall\, n \geq 1$) both in the differential space and time coefficients of the Kerr Newman Equation. The space-time curvature is flat, or equal to $1, \forall\, r \gg l_p$ in the polar direction. Since the space-time curvature is not equal to 1 in the polar direction, this is indeed an object with mass and not a "non-object". With the property of M = 0 in the equatorial direction, this particle can move along the equatorial plane with the velocity of light just like as a massless particle.

Case III: $Q \neq 0$, $(r_Q^2 \neq 0)$, $a\omega > 0$

Charge Particle with an Angular Momentum:

From Equation (10), $\frac{E}{mc^2} = \left[1 - \frac{r_S r - r_Q^2}{r^2 + a^2 cos^2\vartheta}(1 - \frac{2a\omega sin^2\vartheta}{c})\right]\frac{dt}{d\tau}$

(IIIA) On the equatorial plane, $\vartheta = \frac{\pi}{2}$ i.e. $cos\vartheta = 0;\ sin\vartheta = 1$

$$\frac{E}{mc^2} = \left[1 - \frac{r_S r - r_Q^2}{r^2}(1 - \frac{2a\omega}{c})\right]\frac{dt}{d\tau} \tag{14a}$$

Again, when $2a\omega = c,\ \frac{E}{mc^2} = [1]\frac{dt}{d\phi}$ $\quad\quad \forall M\ and\ \forall r$

If this particle has an angular momentum J of $\frac{\hbar}{2}$, and if the ω is equal to $\frac{1}{2}\omega_p$, then $2a\omega = 2\frac{J}{Mc}\frac{1}{2\tau_p} = \frac{\hbar}{2Mc}\sqrt{\frac{c^5}{\hbar G}} = c$, therefore, $M = \frac{1}{2}\sqrt{\frac{G\hbar}{c}} = \frac{1}{2}m_p$ The particle mass is equal to one half Planck Mass. Nevertheless, the space-time curvature in the equatorial plane remains flat because of the angular momentum of $\frac{\hbar}{2}$ just like the Case II(A) above.

($IIIA_n$) Negative modulation frequency

Now, if this $M = \frac{1}{2}m_p$ particle is spinning with an angular momentum J $= \frac{\hbar}{2}$ but with a frequency of $\omega = \frac{1}{2}\omega_p - \frac{1}{2}\omega_e$, where $\omega_e = m_e\frac{c^2}{\hbar}$, m_e being the rest mass of an electron, and if with the charge Q is equal to e of an electron,

$$\text{then } \left(1 - \frac{2a\omega}{c}\right) = \left[1 - \frac{\frac{1}{2}\hbar(\omega_p - \omega_e)}{\frac{1}{2}m_p c^2}\right] = \frac{\hbar\omega_e}{m_p c^2} = \frac{m_e}{m_p} \tag{15}$$

and equation (14) will become

$$\frac{E}{mc^2} = \left[1 - \frac{r_S r - r_Q^2}{r^2}(1 - \frac{2a\omega}{c})\right]\frac{dt}{d\tau} = [1 - \frac{r_S r - r_Q^2}{r^2}(\frac{m_e}{m_p})]\frac{dt}{d\tau} \tag{15a}$$

since $r_Q^2 = \frac{Q^2}{4\pi\varepsilon_0}\frac{G}{c^4} = \frac{e^2}{4\pi\varepsilon_0}\frac{G}{c^4}\frac{r}{r} = 2\frac{(\frac{1}{2}\frac{e^2}{4\pi\varepsilon_0 rc^2})Gr}{c^2} = 2\frac{m_e{}'}{c^2}Gr$

where $m_e{}' \equiv (\frac{1}{2}\frac{e^2}{4\pi\varepsilon_0 rc^2})$.

Equation (15a) can be written as

$$\frac{E}{mc^2} = \left[1 - \frac{r_s r - r_Q^2}{r^2}\left(\frac{m_e}{m_p}\right)\right]\frac{dt}{d\tau} = \left[1 - \frac{\frac{m_p G}{c^2}r - 2\frac{m_e'}{c^2}Gr}{r^2}\left(\frac{m_e}{m_p}\right)\right]\frac{dt}{d\tau}$$

$$=[1 - \frac{2\left(\frac{1}{2}m_p - m_e'\right)G}{rc^2}\left(\frac{m_e}{m_p}\right)]\frac{dt}{d\tau} \qquad (16)$$

the term $\left(\frac{1}{2}m_p - m_e'\right)$ is equal to a gravitating mass of one half Planck Mass minus the equivalent mass of the "self energy" divided by c^2, with a spherical radius r. Since $m_e' \ll m_p, \forall r \geq l_p$, Equation (16) can be written as

$$\frac{E}{mc^2} = [1 - \frac{2\left(\frac{1}{2}m_p - m_e'\right)G}{rc^2}\left(\frac{m_e}{m_p}\right)]\frac{dt}{d\tau} = \left[1 - \frac{m_e G}{rc^2}\right]\frac{dt}{d\tau} \qquad (16a)$$

The space-time curvature from the gravitating mass as seen by a "probe" mass m (or test mass) m_e is like the rest mass of an electron, with an equivalent charge radius of the deBroglie wavelength of an electron. The interaction between two of such particles is like two electrons with charge e in each.

(IIIA$_p$) Positive modulation frequency

Now, if this $M = \frac{1}{2}m_p$ particle is spinning with an angular momentum J $= \frac{\hbar}{2}$ but with a frequency of $\omega = \frac{1}{2}\omega_p + \frac{1}{2}\omega_e$, where $\omega_e = m_e \frac{c^2}{\hbar}$,

then $\left(1 - \frac{2a\omega}{c}\right) = \left[1 - \frac{\frac{1}{2}\hbar(\omega_p + \omega_e)}{\frac{1}{2}m_p c^2}\right] = -\frac{\hbar\omega_e}{m_p c^2} = -\frac{m_e}{m_p}$ (17)

This is the same as Equation (15) above with m_e replaced by $-m_e$.

Equation (16) can also be written as

$$\frac{E}{mc^2} = [1 - \frac{2\left(\frac{1}{2}m_p - m_e'\right)G}{rc^2}\left(\frac{-m_e}{m_p}\right)]\frac{dt}{d\tau} = \left[1 - \frac{(-m_e)G}{rc^2}\right]\frac{dt}{d\tau} \qquad (18)$$

The space-time curvature from a mass of $-m_e$.

The interaction between a particle in Case III(A$_n$) with a particle in Case III(A$_p$) will be a repulsive force of $F_q = K\frac{e^2}{r}$. However, since the mass of the particle in Case III(A$_p$) is negative, the acceleration from this "repulsive force" is in the reversed direction, i.e. the interaction between these two particle is "attractive". This is also equivalent to treating the particle in Case III(A$_p$) as a charge of +e with a positive mass of m_e, just like a ***positron***.

(IIIB) Along the polar axis: $\vartheta = 0, \quad cos\vartheta = 1, \quad sin\vartheta = 0$

$$\frac{E}{mc^2} = \left[1 - \frac{r_s r - r_Q^2}{r^2 + a^2}\right]\frac{dt}{d\tau} \tag{19}$$

For Q = e, $\quad M = \frac{1}{2}m_p$, and $J = \frac{\hbar}{2}$,

$$r_s = \frac{Gm_p}{c^2} = \frac{G\sqrt{\frac{\hbar c}{G}}}{c^2} = \sqrt{\frac{\hbar G}{c^3}} = l_p,$$

$$a = \frac{J}{Mc} = \frac{\frac{\hbar}{2}}{\frac{1}{2}m_p c} = l_p \qquad r_Q^2 = \frac{e^2 G}{4\pi\epsilon_0 c^4} = \frac{e^2}{4\pi\epsilon_0 \hbar c}\frac{\hbar G}{c^3} = \alpha\frac{\hbar G}{c^3} = \alpha\, l_p{}^2 \tag{19a}$$

where $\alpha = \frac{e^2}{4\pi\epsilon_0\hbar c} \sim \frac{1}{137}$ *is the fine structure constant*

Let r be equal to an integer n times l_p, i.e. r = n l_p

Then Equation (19) can be written as

$$\frac{E}{mc^2} = \left[1 - \frac{r_s r - r_Q^2}{r^2 + a^2}\right]\frac{dt}{d\tau} = [1 - \frac{n l_p{}^2 - \alpha\, l_p{}^2}{(n^2+1) l_p{}^2}]\frac{dt}{d\tau} = [1 - \frac{(n - \alpha)}{(n^2+1)}]\frac{dt}{d\tau} \tag{20}$$

where n is the number of Planck lengths away from the pole of the spinning object. For n=1, the space-time curvature term σ = [1- ½ + $\frac{\alpha}{2}$] = $\frac{1+\alpha}{2}$. This is very similar to that of the Case II B except with the addition of $\alpha/2$ from the electrical charge. For n ≫ 1, $\sigma \Longrightarrow 1$ of a flat space-time.

Case IV: Q ≠ 0, $(r_Q^2 \neq 0)$, aω < 0, Charge Particle with Angular Momentum and negative angular velocity:

From Equation (10), $\quad \frac{E}{mc^2} = \left[1 - \frac{r_s r - r_Q^2}{r^2 + a^2 cos^2\vartheta}(1 - \frac{2a\omega sin^2\vartheta}{c})\right]\frac{dt}{d\tau}$

(IVA) On the equatorial plane, $\vartheta = \frac{\pi}{2}$ i.e. $cos\vartheta = 0, \quad sin\vartheta = 1$

$$\frac{E}{mc^2} = \left[1 - \frac{r_s r - r_Q^2}{r^2}(1 - \frac{2a\omega}{c})\right]\frac{dt}{d\tau} \tag{21}$$

If $a\omega < 0,$ *and if* $-2a\omega = c$

Then $\frac{E}{mc^2} = \left[1 - \frac{r_s r - r_Q^2}{r^2}(2)\right]\frac{dt}{d\tau} \qquad \forall M \text{ and } \forall r$ (22)

$$\frac{E}{mc^2} = \left[1 - 2\frac{r_s}{r} + 2\frac{r_Q^2}{r^2}\right]\frac{dt}{d\tau}$$

For a particle of M $=\frac{1}{2}m_p$, the Schwarzschild radius $r_s = l_p$.

At two times the Schwarzschild radius $r = 2r_s = 2l_p$,

$$\frac{E}{mc^2} = \left[2\frac{r_Q^2}{4{l_p}^2}\right]\frac{dt}{d\tau} = \frac{1}{2}\frac{\alpha\, {l_p}^2}{{l_p}^2} = \frac{1}{2}\alpha \qquad (22a)$$

Could a composite of this with a neutrino be a π^o?
Could π^+ *be* $\left(\omega_{m_e} - \frac{1}{2}\omega_p\right)$ *and* π^- *be* $(-\omega_{m_e} - \frac{1}{2}\omega_p)$?

There is no space-time singularity for all $r \geq 2l_p$,

(IVA$_p$) Positive modulation frequency

If the particle is spinning at an angular frequency *of*

$$\omega = \omega_{m_0} - \frac{1}{2}\omega_p \qquad (23)$$

where $\omega_{m_0} = m_o\frac{c^2}{\hbar}$, *and* $\frac{\lambda_0}{2\pi} = \frac{c}{\omega_{m_0}} = \frac{\hbar}{m_0 c}$ λo is the deBroglie wavelength of the particle m_o,

and if Q = e, M $=\frac{1}{2}m_p$ and $J = \frac{\hbar}{2}$,

then $\left(1 - \frac{2a\omega}{c}\right) = 2 - \frac{2m_0}{m_p}$

and

$$\frac{E}{mc^2} = \left[1 - \frac{r_s r - r_Q^2}{r^2}\left(2 - \frac{2m_0}{m_p}\right)\right]\frac{dt}{d\tau} \qquad (24)$$

$$\cong [1 - \frac{2m_pG}{rc^2} + \frac{2m_0G}{rc^2}]\frac{dt}{d\tau} = [1 - \frac{2(m_p - m_0)G}{rc^2}]\frac{dt}{d\tau} \qquad (24a)$$

using $r_Q^2 \ll r_s r$.

For $r = nr_p$, Equation 24 becomes $\frac{E}{mc^2} = [1 - \frac{2(m_p - m_0)G}{nr_pc^2}]\frac{dt}{d\tau}$

At $r = 2l_p$, $\frac{E}{mc^2} = \left[\frac{m_0}{m_p}\right]\frac{dt}{d\tau}$,

i.e. the space time curvature at $2l_p$ is $\sigma = \left[\frac{m_0}{m_p}\right]$ (25)

The mass/energy of this object as seen (or measured) from a far away distance $r \gg 2l_p$ will be $E = (m_p - m_0)\left[\frac{m_0}{m_p}\right]c^2 = m_0c^2$, just like a particle of mass m_0, spinning with an angular moment of $J = \frac{\hbar}{2}$, carrying a charge of e. At short distances, the curvature term $[1 - \frac{2(m_p - m_o)G}{rc^2}]$ is the same as the space-time curvature term $\left[1 - \frac{2MG}{rc^2}\right]$ in the Schwarzschild Metric from an object with mass $M = m_p - m_0$. The "gravitational interaction" between two of these masses will be $F_g = \frac{G(m_p - m_0)^2}{r^2}$.

The "electrical interaction" from the charge e will be $F_q = \frac{Ke^2}{r^2}$, where $K = \frac{1}{4\pi\varepsilon_o}$.
The ratio between these two interactions will be

$$\frac{F_g}{F_q} = \frac{G(m_p - m_0)^2}{Ke^2} \cong \frac{Gm_p{}^2}{Ke^2} = \frac{G\frac{\hbar c}{G}}{Ke^2} = \frac{\hbar c}{Ke^2} = \frac{1}{\alpha} \cong 137 \qquad (26)$$

using $m_0 \ll m_p$. Recalling that this particle is spinning at an angular frequency $of\ \omega = \omega_{m_0} - \frac{1}{2}\omega_p$, (Equation 23), where $\omega_o = m_o \frac{c^2}{\hbar}$, If $\frac{\lambda_0}{2\pi} = \frac{c}{\omega_{m_0}} = \frac{\hbar}{m_0 c}$ is the deBroglie wavelength of a hadron, such as a proton, then the F_g (at short range) that is 137 times stronger than the electrical force F_q, is very much like the short range "nuclear strong interaction" of a hadron with a deBroglie wavelength

$\lambda\!\!\!^{-}{}_o = \frac{\lambda_0}{2\pi} = \frac{c}{\omega_{m_0}} = \frac{\hbar}{m_0 c}$ $\lambda\!\!\!^{-}$ of m_o.

At short range, when r = $2l_p$, $\sigma = \left[\frac{m_0}{m_p}\right]$. At this space-time curvature, relativistic distance is lengthened by $\sigma^{-1} = \left[\frac{m_p}{m_0}\right]$ (for extreme-spin Kerr black hole). The energy from the electrical force between two particles at this distance can be written as

$$\mathcal{E}_q = K\frac{e^2}{r_p\sigma^{-1}} = K\frac{e^2}{r_p\frac{m_p}{m_0}} = K\frac{e^2 m_o}{r_p m_p} = K\frac{e^2 m_o}{\sqrt{\frac{G\hbar}{c^3}}\sqrt{\frac{\hbar c}{G}}} = K\frac{e^2 m_o c}{\hbar}$$

$$= \frac{Ke^2}{\hbar c} m_o c^2 = \alpha m_o c^2$$

where α is the fine structure constant ($\sim 1/137$).
If m_o is the mass of a proton, ($\sim 938\ Mev/c^2$), $\mathcal{E}_q \cong 6.85\ Mev$ is approximately equal to the (per nucleon) Binding Energy of nucleus.
For $r > 2l_p$, the σ changes from an extremely small number of $\frac{m_0}{m_p}$ to $\sim(1 - \frac{2}{n})$ for $r = nl_p$ and eventually become 1 (flat space-time) for n≫ 2.

(IVA_n) Negative modulation frequency

If the particle is spinning at an angular frequency of

$$\omega = -\omega_{m_0} - \frac{1}{2}\omega_p \qquad (25)$$

where $\omega_{m_0} = m_o \frac{c^2}{\hbar}$, and $\frac{\lambda_0}{2\pi} = \frac{c}{\omega_{m_0}} = \frac{\hbar}{m_0 c}$, $\lambda\!\!\!^{-}$ is the deBroglie wavelength of the particle m_o,

and if Q = e, M = $\frac{1}{2}m_p$ and $J = \frac{\hbar}{2}$,

then $\left(1 - \frac{2a\omega}{c}\right) = 2 + \frac{2m_0}{m_p}$

and $\frac{E}{mc^2} = \left[1 - \frac{r_s r - r_Q^2}{r^2}\left(2 + \frac{2m_0}{m_p}\right)\right]\frac{dt}{d\tau}$

$$\cong [1 - \frac{2m_p G}{rc^2} - \frac{2m_0 G}{rc^2}]\frac{dt}{d\tau} = [1 - \frac{2(m_p + m_0)G}{rc^2}]\frac{dt}{d\tau} \qquad (26)$$

using $r_Q^2 \ll r_s r$.

Together with the angular momentum of $J = \frac{\hbar}{2}$ the space-time curvature of this M = $\frac{1}{2} m_p$ object is like an object of mass $M = m_p + m_0$ for $r \geq 2l_p$. At $r = 2l_p$, $\frac{E}{mc^2} = \left[\frac{-m_0}{m_p}\right]\frac{dt}{d\tau} = \left[\frac{m_0}{m_p}\right]\frac{dt}{-d\tau}$,

i.e. the space time curvature at $2l_p$ is $\sigma = \left[\frac{m_0}{m_p}\right]$ $with\ -d\tau$,

i.e., local time of the particle is in reversed direction: ***Anti-particle***.

The mass/energy of this object as seen (or measured) from a far away distance $r \gg 2l_p$ will be $E = (m_p + m_0)\left[\frac{-m_0}{m_p}\right]c^2 \cong -m_0c^2$, just like an anti-particle of mass m_0, spinning with an angular moment of $J = \frac{\hbar}{2}$, carrying a charge of e. At short distances, the curvature term $[1 - \frac{2(m_p+m_o)G}{rc^2}]$ is the same as the space-time curvature term $\left[1 - \frac{2MG}{rc^2}\right]$ in the Schwarzschild Metric from an object with mass $M = m_p + m_0$. The "gravitational interaction" between two of these masses will be

$F_g = \frac{G(m_p+m_0)^2}{r^2}$, and the "electrical interaction" from the charge e will be $F_q = \frac{Ke^2}{r^2}$, where $K = \frac{1}{4\pi\varepsilon_0}$. The ratio between these two interactions will be $\frac{F_g}{F_q} = \frac{G(m_p+m_0)^2}{Ke^2} \cong \frac{Gm_p{}^2}{Ke^2} = \frac{G\frac{\hbar c}{G}}{Ke^2} = \frac{\hbar c}{Ke^2} = \frac{1}{\alpha} \cong 137$

using $m_o \ll m_p$.

(IVB) Along the polar axis: $\vartheta = 0,\quad cos\vartheta = 1,\quad sin\vartheta = 0$

$$\frac{E}{mc^2} = \left[1 - \frac{r_s r - r_Q^2}{r^2+a^2}\right]\frac{dt}{d\tau}$$

For Q = e, $\quad$ M = $\frac{1}{2} m_p$, $\quad J = \frac{\hbar}{2}$,

$$r_s = \frac{Gm_p}{c^2} = \frac{G\sqrt{\frac{\hbar c}{G}}}{c^2} = \sqrt{\frac{\hbar G}{c^3}} = l_p,$$

$$a = \frac{J}{Mc} = \frac{\frac{\hbar}{2}}{\frac{1}{2}m_p c} = l_p,\quad r_Q^2 = \frac{e^2 G}{4\pi\epsilon_0 c^4} = \alpha\hbar\frac{G}{c^3} = \alpha\, l_p{}^2$$

where $\alpha = \frac{e^2}{4\pi\epsilon_0\hbar c} \sim \frac{1}{137}$ $is\ the\ fine\ structure\ constant.$

If $r = n\, l_p,\ then$

$$\frac{E}{mc^2} = \left[1 - \frac{r_s r - r_Q^2}{r^2+a^2}\right]\frac{dt}{d\tau} = \left[1 - \frac{nl_p{}^2 - \alpha l_p{}^2}{n^2 l_p{}^2 + l_p{}^2}\right]\frac{dt}{d\tau} = \left[1 - \frac{(n-\alpha)}{(n^2+1)}\right]\frac{dt}{d\tau} \qquad (27)$$

Summery and extension:

Based on the similarity of the basic mass, charge, and angular momentum properties of black holes and fundamental particles, the Kerr-Newman solution to the Einstein Field equation that used to characterize a black hole can be applied to some spin ½ particles. Interesting properties are seen when one half Planck mass is used for the mass, one half of Planck constant is used for the angular momentum and electronic charge e is used as the charge. Table I summarizes the various conditions and the resulting properties that parallel some of the stable particles in nature.

$M=\frac{1}{2}m_p$	Q	J	$\omega \equiv \frac{d\phi}{dt}$	ϑ	$\sigma\ at\ r$ (σ =1 = flat)	$\sigma\ at\ nl_p$	ƛ (size) DeBroglie Wavelength	Mass observed from infinity	object
Case I	0	0	0	$\forall\vartheta$	$[1-\frac{2MG}{rc^2}]$	$(1-\frac{1}{n})$ $\sigma=0$ @ $n=1$	l_p	$\frac{m_p}{2}$	"Micro Schwarzschild Black hole"
Stable particles									
Case IIA	0	$\frac{\hbar}{2}$	$\frac{\omega_p}{2}$	$\frac{\pi}{2}$	1 ∀ r	1	l_p	0	Neutrino
Case IIB				0		$1-\frac{n}{n^2+1}$			
Case IIIA$_n$	-e	$\frac{\hbar}{2}$	$\frac{\omega_p}{2}-\frac{\omega_e}{2}$	$\frac{\pi}{2}$	1 ∀ r	1	$\frac{\hbar}{m_e c}$	m_e	Electron m_e = $electron\ mass$
Case IIIB				0		$1-\frac{(n-\alpha)}{(n^2+1)}$			
Case IIIA$_p$	+e	$\frac{\hbar}{2}$	$\frac{\omega_p}{2}+\frac{\omega_e}{2}$	$\frac{\pi}{2}$	1 ∀ r	1	$\frac{\hbar}{m_e c}$	m_e $d\tau<0$	Positron
Case IVA$_p$	+e	$\frac{\hbar}{2}$	$\frac{\omega_{m_0}}{2}-\frac{\omega_p}{2}$	$\frac{\pi}{2}$	$[1-\frac{2(m_p-m_o)G}{rc^2}]$	$\left[\frac{m_0}{m_p}\right]$ @ n=2	$\frac{\hbar}{m_o c}$	m_o	Proton m_o = $proton\ mass$
Case IVB				0	$\left[1-\frac{r_s r-r_Q^2}{r^2+a^2}\right]$	$1-\frac{(n-\alpha)}{(n^2+1)}$			
Case IVA$_n$	-e	$\frac{\hbar}{2}$	$-(\frac{\omega_{m_0}}{2}+\frac{\omega_p}{2})$	$\frac{\pi}{2}$	$[1-\frac{2(m_p+m_o)G}{rc^2}]$	$\left[\frac{-m_0}{m_p}\right]$ @ n=2	$\frac{\hbar}{m_o c}$	m_o $d\tau<0$	Anti-Proton
Unstable particles			Composite ⊕						
Case IV A$_o$	0	0	$-\frac{\omega_p}{2}\oplus(\frac{\omega_p}{2})$			$\alpha/2$			π^o
Case IV A$_{pe}$	+e	0	$-\frac{\omega_p}{2}$ $\oplus(\frac{\omega_p}{2}+\frac{\omega_e}{2})$			$\alpha/2$	$\frac{\alpha\hbar}{2m_e c}$	$\sim(\frac{2}{\alpha}-1)m_e$	π^+
Case IV A$_{ne}$	-e	0	$-\frac{\omega_p}{2}$ $\oplus(\frac{\omega_p}{2}-\frac{\omega_e}{2})$			$\alpha/2$	$\frac{\alpha\hbar}{2m_e c}$	$\sim\left(\frac{2}{\alpha}-1\right)m_e$ $d\tau<0$	π^-

CaseIIIA$_{un}$	-e	$\frac{\hbar}{2}$	$\frac{\omega_p}{2} - \frac{\omega_\mu}{2}$		1 $\forall$ r	1	$\frac{\hbar}{m_\mu c}$	m_μ	μ^-
CaseIIIA$_{up}$	+e	$\frac{\hbar}{2}$	$\frac{\omega_p}{2} + \frac{\omega_\mu}{2}$		1 $\forall$ r	1	$\frac{\hbar}{m_\mu c}$	m_μ $d\tau < 0$	μ^+
CaseIV A$_{pN}$	0	$\frac{\hbar}{2}$	$(\frac{\omega_{m_o}}{2} - \frac{\omega_p}{2})$ $\oplus(\frac{\omega_p}{2} - \frac{\omega_e}{2})$ $\oplus(\frac{\omega_p}{2})$		$[1 - \frac{2(m_p - m_o)G}{rc^2}]$	$\left[\frac{m_0}{m_p}\right]$ @ n=2	$\sim\frac{\hbar}{m_o c}$	$\sim m_o$	μ^+
Unstable particles of m = m_p			Composite $\oplus$						
M = m_p	Q	J	$\omega \equiv \frac{d\phi}{dt}$	ϑ	σ at r (σ =1 = flat)	σ at nl_p	ƛ (size) DeBroglie Wavelength	Mass observed from infinity	object
Case IV A$_{pe}$	+e	0	$-\omega_p + \frac{\omega_e}{2}$ $\oplus(\frac{\omega_p}{2} + \frac{\omega_e}{2})$			$\left[\frac{m_0}{2m_p}\right]$ @ n=2	$\frac{2\hbar}{m_o c}$	$m_o/2$ $+\delta$	K^+
Case IV A$_{ne}$	-e	0	$-\omega_p$ $\oplus(\frac{\omega_p}{2} - \frac{\omega_e}{2})$			$\left[\frac{m_0}{2m_p}\right]$ @ n=2	$\frac{2\hbar}{m_o c}$	$m_o/2$ $+\delta$	K^-
Case IV A$_o$	0	0	$-\omega_p \oplus (\frac{\omega_p}{2})$			$\left[2\frac{m_0}{m_p}\right]$ @ n=2	$\frac{2\hbar}{m_o c}$	$m_o/2$ $+\delta$	K^0

Other than the non-spinning Planck mass of case I, the space-time curvature of all the spinning particles in the equatorial plane is different from the curvature in the polar directions. For Case II and Case III, the space-time curvature on the equatorial plane is equal to one or just slightly different from one because of the mass equivalent from the energy of the electrical charge of the particle. The properties of the particles in case II and III are very much like those of leptons. However, equatorial plane curvature (σ) for particles in Case IV is very small but not zero. At Planck length, the gravitational interaction of two such particles is very much like the "strong interaction" (Gm_pm_p). When the particles are separated by a large distance, (n >>1), the mass is σ times the Planck Mass, that is the observed mass m_0 of particle as measured in the lab. The gravitational interaction will simply be proportional to (Gm_0m_0). Along the polar direction, all particles from Case II, III, and IV have similar curvature terms of

$(1-\frac{n-\alpha}{n^2+1})$ or $(1-\frac{n}{n^2+1})$ when there is no electrical charge. At the distance of one Planck length (n=1), the curvature is practically equal to ½. A mass of m_0 at infinity will have a relativistic mass of $2m_0$ at Plank length (n=1) from the interacting mass of ½m_p along the polar direction. The gravitational interaction between these two masses will simply be proportional to (Gm_0m_p). The interaction of two particles from the polar to polar direction will be equivalent to the gravitational interaction between a Planck mass and the rest mass of the particle at infinity (Gm_0m_p). The magnitude is similar to that of "weak interaction". Furthermore, the r_Q^2 term in Equation 10, mass arises from energy of the charge of the particle with a magnitude of α times the angular energy of the particle. Since the polarity of the charge is related to the sign of the modulation frequency, electrical charge carried by the particle could be equivalent to the direction of the spin modulation. So, electrical charge could be explained by the spin-spin interaction of the space-time curvature vortex. Should this be the case, then, all four interactions in nature could just be the interactions of space-time geometries.

The properties of the spinning ½m_p entities resemble many of the basic and stable subatomic particles:

(1) Neutrino (Case II above): This particle carries an angular moment of $\frac{\hbar}{2}$. Spinning at one half Planck frequency $\frac{\omega_p}{2}$. It is electrically neutral; it may carry energy and has a zero rest mass. It travels with the speed of light along the equatorial plane. It can interact with other particle with a "weak force" along the polar direction. Since it can only travel along the equatorial plane, only 1/3 of them can be detected from any isotropic emitter. This may account for the "missing neutrinos" from the sun or from any neutrino source on Earth.

(2) Electron (Case IIIA$_n$ above): This particle carries an angular moment of $\frac{\hbar}{2}$. It is spinning with a frequency $-\frac{\omega_e}{2}$ less than one half of Planck frequency where $\omega_e = \frac{m_e c^2}{\hbar}$ is the deBroglie frequency of an electron. The size of this particle is in the order of the deBroglie wavelength of an electron. It carries a unit charge of -e and interact with other charge particles with the coupling constant of k where $\frac{Ke^2}{\hbar c}$ is the fine structure constant. In the polar direction, it also interacts with other particles with "weak interaction" in additional to the interaction from electrical charge.

(3) Positron (Case IIIA$_p$ above): With spinning frequency$+\frac{\omega_e}{2}$ more than $\frac{\omega_p}{2}$, this particle carries a positive charge of +e or –e with -m_e just like an anti-particle of electron.

(4) Proton (Case IVA$_p$ above): With spinning frequency $\frac{\omega_{m_0}}{2}-\frac{\omega_p}{2}$ where ω_{m_0} is the deBroglie frequency $\omega_o = \frac{m_o c^2}{\hbar}$ of a proton, this spin ½ particle carries a positive charge of +e. At 2 Planck length $(2l_p)$, the gravitational force (Gm_pm_p) between

two of these particles is 137 times stronger , ($\frac{\hbar c}{Ke^2}$ $tims$), than the electrical force (Ke^2) just like the "nuclear strong force". The space-time curvature σ at $2l_p$ is $\frac{m_0}{m_p}$, and therefore, when the second particle is moved from $2l_p$ to infinity, (σ=1), the relativistic become m_o, a proton mass.

(5) Anti-proton (Case IVA$_n$ above): With spinning frequency $-\frac{\omega_{m_0}}{2}-\frac{\omega_p}{2}$, this spin ½ particle carries a negative charge of –e, (or +e with a negative mass), and just like the anti-particle of a proton.

The properties of one spinning ½m_p with one or more other ½m_p entity also resembles many of the unstable subatomic particles:

(6) Neutron (composite particle of a proton, an electron, and a neutrino): The space-time curvature σ of a proton polar direction is equal to $1-\frac{(n-\alpha)}{(n^2+1)}$ or (½ $+\frac{\alpha}{2}$) for n = 1 (one Planck length). At this distance, gravitational force between spin one half, ½m_p particles can be held by the "weak force" from the polar to polar direction space time curvature of ½ on both sides. A positive charged proton, a negative charge electron and a neutrino can than be held by both the electrical force and the "weak force" from both sides and exhibited as a spin ½ particle with neutral electrical charge. The time period of the electron at the space-time curvature of one half Planck mass ½m_p will be dilated by $\frac{2\omega_e}{\omega_p}$, i.e. $\tau \cong \frac{2\omega_e}{\omega_p}\tau_e$. Numerically, $\tau \cong 607$ seconds, matching the half-life of a neutron. This composite particle is unstable by itself and decays into an electron, a proton and a neutrino $n \Rightarrow p+e+\bar{\upsilon}$ with a half-life of $\frac{2\omega_e}{\omega_p}\tau_e$, where τ_e the period of deBroglie wave length of an electron.

(7) Pion (composite particle of Case IVA, and Case II, or Case IIIAn or Case IIIAp): The space-time curvature of Case IVA in the equatorial plane at one Planck length is $\frac{\alpha}{2}$. A composite of this with an electron or positron will have a space-time curvature of $(\frac{2}{\alpha}-1)$, and have a mass of $(\frac{2}{\alpha}-1)m_e$. This particle is also belong to the group of "strong interaction" particle as well as "weak interaction" particle.

(π^o) A composite particle with a Case II (neutrino) held together in the polar direction will be a spin zero neutral particle that interacts with both "weak interaction" and "strong interaction" like a Pion Zero π^o.

(π^+) A composite particle with a Case IIIAp (positron) held together in the polar direction will be a spin zero positively charged particle like a pion plus. The mass of the particle will be $\sim(\frac{2}{\alpha}-1)m_e$. The numerical value is 139.54 MeV/c^2 very close to the measured value of 139.57018(35) MeV/c^2 MeV/c^2.

(π^-) A composite particle with a Case IIIAn (electron) held together in the polar direction will be a spin zero positively charged particle like a pion minus. The mass of the particle will be $\sim(\frac{2}{\alpha}-1)m_e$. The numerical value is 139.54 MeV/c^2 very close to the measured value of 139.57018(35) MeV/c^2 [9]

(8) Kaon: Similar to Case IV, M= m_p instead of M=$\frac{1}{2}m_p$ and if the angular frequency is $\omega = \omega_{m_0} - \omega_p$, at r = 2 l_p the space-time curvature will be $\frac{m_0}{2m_p}$. Together with a neutrino, this will be a spin zero particle with a mass about one half of a proton mass like a K^+ . For $\omega = -\omega_{m_0} - \omega_p$, the composite particle is like a K^- . K^O is like a particle of $\omega = -\omega_p$.

(9) Higgs boson: From Equation 24, $\frac{E}{mc^2} = \left[1 - \frac{r_s r - r_Q^2}{r^2}\left(2 - \frac{2m_0}{m_p}\right)\right]\frac{dt}{d\tau}$ and for Q = e, $r_Q^2 = \frac{e^2 G}{4\pi\epsilon_0 c^4} = \alpha\hbar\frac{G}{c^3} = \alpha\, l_p{}^2,\ r_s = l_p,$

at r = $2l_p$, $\frac{E}{mc^2} = \left[1 - \frac{2\, l_p{}^2 - \alpha\, l_p{}^2}{4\, l_p{}^2}\left(2 - \frac{2m_0}{m_p}\right)\right]\frac{dt}{d\tau} = \left[\frac{m_0}{m_p}\right] - \left[\frac{m_0}{m_p}\right]\frac{\alpha}{2} + \frac{\alpha}{2}$ (28)

The third term of the curvature $\frac{\alpha}{2}$ is due to the self energy of the charge with a charge radius of $2l_p$. The charge from the angular frequency of $\omega_{m_0} - \frac{1}{2}\omega_p$ has a charge radius in the order of the deBroglie wave length of the particle m_0 and is many orders of magnitude larger then $2l_p$. The $\frac{\alpha}{2}$ term in (Equation 28) should therefore be neglected. The first term in (Equation 28) is then the mass m_0 of the particle as observed in the lab of flat space-time. In high energy P-P scattering, this term can be cancelled by an appropriate angular momentum between the P-P system. The second term represents a "resonant" in the order of $\frac{\alpha}{2}m_0$. For m_0 of a K particle that has an numerical value of about half a proton, this resonant is in the range of 128 GeV like the "Higgs" peak announced by CERN in July of 2013. For m_0 of a Proton, another resonant should occur in the range of 256 GeV and should be a stronger resonant in the high energy P-P scattering.

Footnote: This model assumes that space is quantized with a minimum length of one Planck length l_p.

References:

(1) Misner, Charles W.; Thorne, Kip S.; Wheeler, John Archibald (1973). *Gravitation*. San Francisco: W. H. Freeman. ISBN 978-0-7167-0344-0.

(2) Edwin F. Taylor, John Archibald Wheeler "Exploring Black Holes, Introduction to General Relativity" Addison Wesley Longman, ISBN 0-201-38423-X

(3) Richard A. Mould "Basic Relativity" Springer, ISBN 0-387-95210-1

(4) https://en.wikipedia.org/wiki/Schwarzschild_metric

(5) https://en.wikipedia.org/wiki/Reissner%E2%80%93Nordstr%C3%B6m_metric

(6) https://en.wikipedia.org/wiki/Kerr%E2%80%93Newman_metric

(7) Equation [3] in page 4-4 of Reference (2) above.

(8) Equation [5 to 11] in page 4-4 of Reference (2) above.

(9) https://en.wikipedia.org/wiki/Pion