Sense and soundness of thought as a biochemical process

Mahmoud A. Mansour

August 17,2015

Abstract

A biochemical model is suggested for how the mind/brain might be modelling objects of thought in analogy with the biochemistry of enzymes, which explains various characters about human thought such as having a top-bottom architecture. The composition of complex ideas out of simple ones is realized as a semigroup $\mathcal{C}$. Then we explain the composition of objects in the act of thought, and prove that every object of thought has a match – another object which makes perfect sense with it – whether meaningful or not. Lastly, we emphasize the role of experience in the evolution of the Algebras of sense.

Making sense as a biochemical process

Suppose we are thinking about a problem that is to reconcile two experiments together, and suppose we are considering one side of the problem. We suggest that the mind, let it be the brain or anything else depending on your philosophical dogma, expresses every idea as a molecule that encodes all information about such concept or idea.

This molecule would have certain active sites like those present in enzymes bearing particular functional groups, which are specific to some chemical agents. These sites define the essence of an idea, which determines that this molecule fits with another one representing another idea, if they really make sense together.

Now, these two ideas, which are symbolically represented by two molecules, are to engage in the act of thought, which now doesn't depend on the semantics of the problem – i.e. the meaning of every symbol –; since meaning has been cooked up in the formulation of these symbolic elements, namely the chemical representation of the meaning, which the two ideas bear with their corresponding active sites.

Now, possibly the brain sets them in close proximity to see what could possibly result from their interaction, and we know that chemical and physical interactions exhibit a broad domain of bonding from repulsion to strong affinity, which might explain why we tend to feel severe degrees of soundness from decoherence to perfect soundness.

So boldly, analyzing an idea is simply adding more functional groups to the molecule representing our idea, which ensures that two ideas fit if they are perfect match.

And we set the logic of making sense by the chemical configuration we compose functional groups with, which describes how ideas should make sense together. All of this constructs what we might call a "Chemical Algebra".

So, now we have two sets of objects, namely, the set of symbols and the set of functional groups, and a dictionary which translates every symbol signifying a certain meaning to a particular functional group.

Now the question is: what is really happening?

If we look closely, we want to solve the problem such that you must start from functional groups and then advance them into composition to see if they really solve the problem, and that is conspicuously impossible exploiting the construction given above.

But to solve the problem we turn it on its head. We say that what's really going on is that the mind fabricates these functional groups according to some Algebra, and then see how things fit together, and if they don't, the mind re-analyzes our problem until everything just works out. And that really offers an explanation why our mind really thinks in top-down architecture unlike computers, which are bottom-top in architecture.
Now the question is this: how the mind really translates meaning into groups? On what measure the mind incorporates its Algebra in the synthesis of functional groups?
Because on the one hand either you know how things are going to fit! Or it's totally random, which renders the system with no reliable Algebra! And if it's the former case, then it seems that the whole process is devoid of meaning, so there must be some experimentation going on.
The answer to this problem lies in Hume's conception of complex and simple ideas. Hume's philosophy was based on the idea that every idea thought of is always traced back to perception, but he had the problem of imagination, which exhibits some ideas that seem to be never been perceived. But his solution to the problem was to divide ideas into simple and complex ideas. Simple ideas are the ones that are directly intelligible through perception, whereas complex ideas are mere compounds of simple ones.
So we establish that simple ideas, or better yet axioms of Algebra, are the things that establish the progressive synthesis of the mind. These rules define the synthesis of which perfect and poor fitting is based on. In other words, they formalize our Algebra used by the mind. So, a complex problem would have a multiple components of these yet simple ideas, and the whole process of analyzing an idea is to realize as many immediate ideas (simple ideas) as possible, and then translate it back to the mind as a modification of the genuine idea, but more pleasant and elegant.
An idea by itself with no framework to conform it to is nil; since in order to judge a certain idea you can't really judge it absolutely; because that would be merely absurd. But on the contrary you judge something relative to something else, and evidently this framework of judgment is just the formal rules of composition of simple ideas, i.e. the algebra incorporated by human reason.
So it really seems that you have to know how to make sense of a certain problem before really engaging in any kind of thought.
Now, it seems hitherto ambiguous how really the mind sets up an algebra and a dictionary to translate meaning or ideas into symbols and consequently into functional groups? But we ask the reader kindly to bear until chapter three where everything becomes clear.

The algebra of sense

We have that for every molecule representing an object of thought – idea or problem – consists of functional groups embedded in a peculiar spatial configuration – which yet to be discussed in chapter 3. So now we have a set F of primary functional groups seated at a particular spatial configuration, and we are going to denote them as $a(r)$ s.t. a is the functional group and $r \in \mathbb{R}^3$ is the location of such group in $\mathbb{R}^3$-space relative to some choice of origin.
Now, the mind(brain) in expressing complex ideas does compose these elements of F together to formulate the molecule endowed with the active sites representing the object of thought.

The composition of these objects is merely the juxtaposition of these $a(r) \in F$ together. So we construct the free semigroup F^{+}, which represents our field of work. But we have to realize two important properties:

- Suppose we have $w = a_1(r_1) \dots a_n(r_n) \in F^{+}$, the order of the letters $a_i(r_i)$ doesn't really matter for the representation of the molecule of such object.
- Suppose we have $w = a_1(r_1) \dots a_n(r_n) \in F^{+}$, if we apply any rotational transformation or translation of the origin the object obtained is the same; because it changes nothing of the physical, chemical and relative configuration of the molecule.

Remark 2.1. *we have to realize that reflection could result in stereoisomers of a certain molecule that the brain could attach a different meaning to, so all transformations are to belong to* $\boldsymbol{T}.\boldsymbol{SO}(\boldsymbol{3})$, *where* $\boldsymbol{T}$ *is the group of translations.*

Now, we define the binary relation $\boldsymbol{R} \subseteq F^{+} \times F^{+}$ s.t. $\forall x, y \in F^{+}$:

- $x\boldsymbol{R}y$ if x is a reordering of the letters of y.

Proposition 2.2. $\boldsymbol{R}$ *is a congruence relation on* F^{+}.
Proof. We have that $\forall x, y, z \in F^{+}$:

- $(x, x) \in \boldsymbol{R}$.
- If we have $(x, y) \in \boldsymbol{R}$, then x is a reordering of the letter of y, which implies that y is also a reordering of the letters of x. Thus $(y, x) \in \boldsymbol{R}$.
- If $(x, y) \in \boldsymbol{R}$ and $(y, z) \in \boldsymbol{R}$, then $(x, z) \in \boldsymbol{R}$.
- If $(x, y) \in \boldsymbol{R}$, then $(zx, zy) \in \boldsymbol{R}$ and $(xz, yz) \in \boldsymbol{R}$.

We have to notice that although the symmetry $\boldsymbol{T}.\mathbf{SO}(\mathbf{3})$ defines an equivalence relation on F^{+}, it doesn't define a congruence.
Now we construct the quotient semigroup $\mathcal{C} = {F^{+}}/{\boldsymbol{R}}$, which we call the composition semigroup. Moreover, the quotient set ${\mathcal{C}}/{\boldsymbol{T}.\mathbf{SO}(\mathbf{3})}$, which is the set of the equivalence classes of the symmetry $\boldsymbol{T}.\mathbf{SO}(\mathbf{3})$, which we call the representation set $\mathcal{R}$. This set consists of the molecules representing the objects of thought regardless of their embedding in space.
Before moving on we have to notice that $\forall a_i(\bar{r}_i) \in \mathcal{C}$ we have

$$a_i(r_i)\, a_1(r_1) \dots a_i(r_i) \dots\ a_n(r_n) = a_1(r_1) \dots a_i(r_i) \dots\ a_n(r_n)$$

, i.e. the semigroup $\mathcal{C}$ is a commutative semigroup consisting entirely of idempotent.

Remark 2.3. *Now, since* $\mathcal{C}$ *is a semigroup,* $\exists \varphi\colon \mathcal{C} \to \mathcal{T}_X$ *a monomorphism, where* $\mathcal{T}_X$ *is the semigroup of maps from the set* X *to itself. And this result says that every chemical representation and consequently every object of thought corresponds to a certain reordering of the elements of some set* X*, which is represented by a map* $f \in \mathcal{T}_X$*. But if we identify the elements in* $\mathcal{C}$ *under the equivalence relation* $\boldsymbol{T}.\boldsymbol{SO}(\boldsymbol{3})$*, and identify the image of such classes with a single representation in* $\mathcal{T}_X$ *then we obtain a set* $Y \subset \mathcal{T}_X$ *and a bijection* $\pi\colon \mathcal{R} \to Y$.

Now, when we talk of two objects of thought making sense together, we imply that their chemical representation, i.e. molecular representation, fit together, like substrates

fit into their active sites at enzymes, which entails that there is a correspondence between their functional groups, which in the act of thought become contingent.
But some objects of thought might correspond to a variety of other objects, for example a theory that might be solving multiple problems, and in that case the problem, which is of smaller size – the one with lesser number of functional groups – evidently would mate at some locus of functional groups.

Definition 2.4. *the size of an element* $w = a_1(r_1) \dots a_n(r_n) \in \mathcal{C}$ *is a map* $size: \mathcal{C} \to \mathbb{Z}^+$ *s.t.*

$$size(w) = size\big(a_1(r_1) \dots a_n(r_n)\big) = n$$

So, the process of assigning soundness to the composition of two objects of thought entails sorting over all possible spatial configurations of the two molecules representing these objects fitting together.
And in order to establish such calculation we define the action of the group $\boldsymbol{T.SO(3)}$ on the composition semigroup $\mathcal{C}$ by:

$$(a, R) \in \boldsymbol{T.SO(3)}$$

$$(a, R)a_1(r_1) \dots a_n(r_n) = a_1((a, R)r_1) \dots a_n((a, R)r_n)$$

Now, we define all possible mating configurations by:
Let $w_1, w_2 \in \mathcal{C}$ s.t. $size(w_2) \leq size(w_1)$, we form the orbit of the two elements and denote it by $(\boldsymbol{T.SO(3)}w_1, \boldsymbol{T.SO(3)}w_2)$.
But to sort out the mating configurations we sort the configurations satisfying that $\exists\{a_i(r_i)\}$ in w_1 s.t. $|\{a_i(r_i)\}| = size(w_2)$, we have $\forall a_k(r_k) \in \{a_i(r_i)\}\ \ \exists!\, a_m(r_m)$ in $w_2$with $r_k = r_m$. And we call these configurations the mating configurations.
Now, to be very specific about the possible mating configurations regardless of their embedding in space we construct the set of equivalence classes of the mating configurations subject to the symmetry $\boldsymbol{T.SO(3)}$, and we call this the orbit of w_1 and w_2, and denote it as $\mathcal{O}(w_1, w_2)$, which consists of the possible mating configurations regardless of embedding in $\mathbb{R}^3$.
Notice that if $w_1 \equiv w$ and $w_2 \equiv \mu$ under the symmetry $\boldsymbol{T.SO(3)}$, we have $\mathcal{O}(w_1, w_2) = \mathcal{O}(w, \mu)$.
Now, we assign to every configuration $c \in \mathcal{O}(w_1, w_2)$ a measure of soundness based on interaction between the representation molecules by the function

$$s: \mathcal{O}(w_1, w_2) \to [0,1]$$

, where $0 \mapsto$ total decoherence and $1 \mapsto$ perfect fit.

Remark 2.5. *Note that these are relative measures of the degree of soundness of two ideas based on chemical and physical interaction, which is assigned through experience – the context of the problem –, which could be understood in the sense of the probability assigned to a certain decision based on the evidence collected through perception in (Joshua I. Gold and Michael N. Shadlen, 2007).*

Now, we can extend this map to a general map

$$S: \mathcal{R} \times \mathcal{R} \to [0,1]$$

, s.t. given $w_1, w_2 \in \mathcal{C}$ we have

$$S(\overline{w_1}, \overline{w_2}) = \begin{cases} \max\limits_{c \in \mathcal{O}(w_1, w_2)} s(c) & \mathcal{O}(w_1, w_2) \neq \emptyset \\ 0 & \mathcal{O}(w_1, w_2) = \emptyset \end{cases}$$

Definition 2.6. *The dictionary map T is the map that translates every object of thought $s \in O$ into a chemical representation*

$$T: O \to \mathcal{R}$$

Definition 2.7. *The collection (F, T, S) is called an AlgebraS, i.e. it identifies the algebra of what makes sense in a certain context.*

Theorem 2.8. *given $w \in \mathcal{C}$, then $\exists \mu \in \mathcal{C}$ s.t. $S(\bar{w}, \bar{\mu}) = 1$.*
Proof. Suppose we have $w = a_1(r_1) \dots a_n(r_n) \in \mathcal{C}$, we compose the following element $\mu = b_1(r_1) \dots b_n(r_n) \in \mathcal{C}$ s.t. $\forall i \leq n$ we have that a_i and b_i perfectly fit.
Then, $\exists c \in \mathcal{O}(w, \mu)$ with $s(c) = 1$. Thus we have $S(\bar{w}, \bar{\mu}) = 1$.

The previous theorem indicates that for every problem of thought there is an answer contingent to the context, but whether this solution is meaningful or absurd, this is up to what does the solution really mean.

Experience and the algebra of sense

A very important point to note is how the mind should further extend a concept i.e. set further analysis by adding extra functional groups to the molecule representing such concept or idea.
Because think of it like this, suppose we have a functional group $a(r)$, and its perfect match $b(s)$, we have that there is a transformation in $\boldsymbol{T}.\mathbf{SO}(\mathbf{3})$ such that they would be indistinguishable. But the two functional groups might represent different meanings; because there is no general criterion to construct these chemical representations.
How is it decided that two problems should fit together?
We start out by two problems each consisting only of one simple idea that is each chemical representation is only composed of one functional group.
But we can realize that the spatial configuration is irrelevant here; because it is self-evident that the soundness of the two ideas together is only determined through the whether or not the two simple concepts are consistent together i.e. the mere chemical affinity of the two functional groups with no reference to either shape or configuration.
Now, we consider the problem encountering a multiplicity of simple objects. Take w to include the following simple objects $\{a_1, \dots, a_n\}$. In order to assign a word in $\mathcal{C}$ we find ourselves in a loss; because there is no apparent reason or algebra of such construction.
But in order to solve this problem we have to understand the role of context apprehended – conceived – through experience.
Its role is to guide the mind/brain through constructing complex ideas out of simple ones. And this is done through understanding how simple ideas or symbols are manifolded within the complex idea. In other words, to see through the concept.
So, by examples through experience the brain/mind learns how to realize simple ideas and guide the topology of their representation – by showing how they stand in relation to each other.
So for example let's say that we have a certain complex concept w, and we want to analyze it. This concept can never be opposed to any other unless being analyzed in a

certain context, and how do we establish such context is through experience or mere synthesis of the mind.
And upon compiling experience, the brain/mind learns the Algebra of how different concepts in a complex object of thought might relate, and how this relation – which is the core of judgment – is expressed as chemical representation.
So, through experience I learn the Algebra that tells me for two distinct concepts A and B composed of the same simple objects $\{a_1, \ldots, a_n\}$ that:

$$A \equiv a_1(r_1) \ldots\ a_n(r_n)\ , \qquad B \equiv a_1(s_1) \ldots\ a_n(s_n)$$

, however,

$$a_1(r_1) \ldots\ a_n(r_n) \neq a_1(s_1) \ldots\ a_n(s_n)$$

, and this is expressed through the chemical and spatial configuration of the molecule.

References

Gold JI, Shadlen MN (2007) The neural basis of decision making. Annu Rev Neurosci 30:535–574.
Busemeyer JR, Wang Z, Townsend JT (2006) Quantum dynamics of human decisionmaking. J Math Psychol 50(3):220–241.
John M. Howie (1995) Fundamentals of semigroup theory.
Brain C. Hall (2003) Lie groups, lie algebras, and representations: an elementary introduction.
David hume A treatise of human nature.