Two conjectures in number theory

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August 3, 2015

Abstract

In this note, I propose a conjecture of generalization of the Lander, Parkin, and Selfridge conjecture; and a conjecture of generalization of the Beal's conjecture.

Conjecture 1. *Let k, n, m be three positive integers such that $k > m + n$ and $m \neq n$. Let $a_1, a_2, a_3, ..., a_k$ be the integers, with $a_k > 0$ and $M = Max\{|a_1|, |a_2|, ..., |a_k|\}$. Let a polynomial $f(x) = a_k x^k + a_{k-1} x^{k-1} + ... + a_0$, then no (n+m) positive integers x_1, x_2, x_n, y_1, y_2,..., y_m greater than M can satisfy an equation as follows:*

$$f(x_1) + f(x_2) + ... + f(x_n) = f(y_1) + f(y_2) + ... + f(y_m) \quad (1)$$

Conjecture 2. *Let n, m be two positive integers such that $m \neq n$. Let k_1, k_2,..., k_n, h_1, h_2,..., h_m be $(n + m)$ positive integers, such that $k_i > n + m$ for $i = 1, 2, ..., n$ and $h_j > n + m$ for $j = 1, 2, ..., m$. Let x_1, x_2,..., x_n, y_1, y_2,..., y_m be $(n + m)$ positive integers satisfy an equation as follows:*

$$x_1^{k_1} + x_2^{k_2} + ... + x_n^{k_n} = y_1^{h_1} + y_2^{h_2} + ... + y_m^{h_m} \quad (2)$$

Then x_1, x_2,...x_n, y_1, y_2,..., y_m have a common prime factor.

References

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