THE GRAVITATIONAL CONSTANT AND ITS RELATIONSHIP TO THE PROPERTIES OF VIRTUAL PARTICLES

Sundar Narayan
Lambton College
Sarnia, Ontario, Canada

ABSTRACT: This paper derives a formula for the lifetime of an unbound or free neutron and shows that neutron lifetime can be related to Newton's gravitational constant, G, providing a much-needed theoretical formula for G, enabling G to be computed with greater accuracy than today's experiments allow. Another equally accurate formula for G is derived based on the properties of the virtual electrons that very briefly exist in a quantum vacuum. Also, Newton's law of gravity and Coulomb's electrostatic law are derived from the same equation, providing a simple proof of the well-known connection between these two laws.

INTRODUCTION

As is well-known, Newton's gravitational constant G first appeared in Newton's inverse-square law of gravitation propounded in the 17th century. Yet, the value of Newton's gravitational constant G is known with less accuracy [Gillies, 1997] than the values of the other physical constants of classical physics. This is because of experimental difficulties and also because no generally-accepted formulas are available to calculate it theoretically [Gillies, 1997]. The many physics equations in which G appears can indeed be inverted to obtain formulas for G, but no such formula gives a value of G that is more accurate than the experimentally known value of G.

Today, the 'official' or CODATA value of G is 6.67384 (± 0.0000080) x 10^{-11} Nm^2 / kg^2 , but this value is accurate only to 5 decimal digits, whereas the values of most other physical constants are known to 8 decimal digits [Gillies, 1997]. Even the CODATA value of G has been called into question by the recent measurements of Fixler et al. [2007] whose experiments show that the value of G is most likely 6.693 x 10^{-11} Nm^2 / kg^2 with the standard deviation of error being 0.027 x 10^{-11} Nm^2 / kg^2.

Therefore, this work derives another formula for G that is based on the QED vacuum model (also called the quantum vacuum model) that is now widely accepted by physicists [Silk, 2005]. This work also derives a new formula for the neutron decay lifetime, and relates it to the gravitational constant.

BRIEF DESCRIPTION OF THE QED VACUUM MODEL

As is well-known, the Universe is mostly vacuum. Even where there is matter, the atoms of such matter are mostly empty space because the size of protons, neutrons and electrons are negligible compared to the size of an atom. Yet, vacuum has actual measurable properties such as the gravitational constant G, vacuum permittivity ε_0 , the magnetic constant μ_0 (formerly known as magnetic permeability) and the radiation density constant ‘a’ (which is related to the Stefan-Boltzmann constant σ).

If vacuum was truly devoid of matter, it should not have any such properties. The fact that vacuum has physical properties led physicists in the 18^{th} and 19^{th} centuries to propose that vacuum actually contains a physical substance called ‘luminiferous ether’. The failure of the Michelson-Morley experiment and the success of Einstein’s theory of relativity made physicists abandon the idea of a luminiferous ether, by the early 20^{th} century.

Yet, the idea that vacuum contains matter was soon resurrected by the work of Dirac which led to the development of the QED (Quantum Electro-Dynamics) vacuum model.

Today, the QED vacuum model is generally accepted by physicists [Silk, 2005] because it explains experimentally observed phenomena such as the Lamb shift and the Casimir effect. Both of these phenomena are well described in many physics textbooks.

According to the QED vacuum model [Silk, 2005] vacuum is not emptiness, but is full of ‘virtual’ particles such as ‘virtual electrons’ and ‘virtual positrons’ which are constantly appearing and disappearing inside the vacuum. These ‘virtual’ electrons and positrons have lifetimes of under 10^{-21} seconds [Close, 2009], so they cannot be detected by physics instruments; they are also theoretically undetectable because of Heisenberg’s Uncertainty Principle [Barrow, 2009]. In fact, it is Heisenberg’s Uncertainty Principle that is the very cause of the spontaneous generation and disappearance of these particles in a vacuum [Barrow, 2009].

The QED vacuum model allows for the spontaneous appearance and disappearance of other particles besides electrons and positrons [Nagashima, 2013]. In theory, any particle or atom could flash into existence in a vacuum and vanish as quickly. The lifetime of such a particle or anti-particle in a QED vacuum can be computed from Bohr’s modified energy-time version of the Heisenberg Uncertainty Principle [Barrow, 2009], which states that the product of the uncertainty in the particle energy, ΔE, multiplied by the time interval Δt during which it exists in the vacuum is greater than the reduced Planck constant $h / 2\pi$, i.e.,

$$\Delta E \times \Delta t \geq h / 2\pi \qquad \text{[equation 1]}$$

Therefore, if $\Delta t \leq h / [2 \pi \Delta E]$, then the particle will be theoretically undetectable, according to the Uncertainty Principle. So the maximum lifetime of such a particle in a vacuum will be [Barrow, 2009]

$$\Delta t = h / [2 \pi \Delta E] \qquad \text{[equation 2]}$$

A NEW THEORY OF NEUTRON DECAY AND GRAVITY

In this paper, a new formula is derived for the gravitational constant G by assuming that G is related to the lifetime and the volume change associated with the appearance and disappearance of virtual particles in a vacuum.

In theory, any sub-atomic particle can briefly form and vanish in a vacuum. This includes neutrons. Thus, neutron-antineutron pairs can spontaneously form in a vacuum. The time of existence Δt in a vacuum of such a virtual neutron can be computed from

$$\Delta t = h / [2 \pi \Delta E] \qquad \text{[equation 3]}$$

where $h = 6.626\ 069\ 57 \times 10^{-34}$ Joule-seconds = Planck's constant
$\Delta E = m_n c^2$
$m_n = 1.674927351 \times 10^{-27}$ kg = mass of one neutron,
$c = 299\ 792\ 458$ m/s = velocity of light in a vacuum,

We get $\Delta t = 7.0055 \times 10^{-25}$ seconds, as the duration in which a virtual neutron will exist in a vacuum, before disappearing back into the vacuum.

<u>A New Formula to Compute the Lifetime of a Free Neutron</u>

The QED vacuum model posits that electron-positron pairs can spontaneously arise in a vacuum and exist for a period of up to 10^{-21} seconds. This is a much shorter time interval than the lifetime of positronium which is an 'exotic atom' composed of only a positron and an electron orbiting a common center of mass [Karshenboim, 2003]. Usually, positronium is formed whenever an electron comes close to a positron, so that these two particles start orbiting their common center of mass. There are 2 types of positronium, ortho-positronium and para-positronium. Both are unstable entities because the electron and positron attract each other and mutually annihilate to form two gamma rays.

Karshenboim [2003] gives the following formula to calculate the lifetime t_p of para-positronium

$$t_p = 2 (h / 2\pi) / [m_e c^2 \alpha^5] = h / [\pi m_e c^2 \alpha^5] \qquad \text{[equation 4]}$$

The CODATA [2010] values for the constants in equation 4 are:

Planck's constant $h = 6.626\ 069\ 57 \times 10^{-34}$ Joule-seconds
electron mass $m_e = 9.109\ 382\ 91 \times 10^{-31}$ kg
light velocity $c = 299\ 792\ 458$ m/s
fine structure constant $\alpha = 0.007\ 297\ 352\ 57$

Substituting these values into equation 4 gives the lifetime of para-positronium as only 1.244 941 97 x 10^{-10} seconds. Experiments show that the para-positronium lifetime is 1.24 x 10^{-10} seconds [Karshenboim, 2003], so equation 4 may be regarded as accurate.

Though Karshenboim's paper does not say so, the fact that the Bohr's hydrogen atom model says that the Planck's constant h equals $2 \pi m_e v_e a_0$, $a_0 = \alpha^{-2} r_e$ and electron velocity v_e equals αc makes it possible to express t_p in terms of the Bohr atomic radius a_0 , or the classical electron radius r_e.

$t_p = 2 a_0 / [c \alpha^4] = 2 r_e / [c \alpha^6]$ [equation 5]

where
Bohr atomic radius = a_0 = 5.2917721092 x 10^{-11} m
Classical electron radius = r_e = 2.8179403267 x 10^{-15} m = 2.8179403267 femtometers

The classical electron radius in equation 5 is rather loosely defined. It is usually derived from classical electrostatics theory from the relationship

r_e = numerical constant x $e^2 / [4 \pi \varepsilon_0 m_e c^2]$ [equation 6]

The numerical constant = ½ or 3/5 or 5/8 or 1, depending on how the electric charge is assumed to be distributed on or inside the electron. When the numerical constant is assumed to be equal to 1, then the above well-known formula for the classical electron radius r_e can be derived, giving r_e = 2.8179403267 femtometers.

When the numerical constant = 5/8, for instance, then equation 6 suggests that we can also define an electron pseudo-radius R_e as

$R_e = (5/8) r_e$, or $r_e = (8/5) R_e$ [equation 7]

Equation 5 can also be re-written as

$(5/8)t_p = 2 \times (5/8) \times r_e / [c \alpha^6]$ [equation 8]

Positronium is an unstable entity. So is a free neutron, i.e., a neutron outside an atomic nucleus. In fewer than 15 minutes, a free neutron decays into a proton, an electron and an electron-antineutrino.

The lifetime τ_n of a free neutron is known from experiments to be between 878.5 seconds to 889.2 seconds [Nakamura, 2010]. A theoretical formula to calculate the lifetime of a free neutron is given by Byrne [1995] as,

$1.71489\tau_n = \{[2\pi^3 . \ln(2). h^3] / [(2\pi)^3 m_e^5 c^4] \} \{G_V^2 + 3 G_A^2 \}^{-1}$ [equation 9] where G_V and G_A are the weak coupling constants used in the Standard Model.

Equation 9 contains coupling constants whose values are not too accurately known [Byrne, 1995]. In fact, equation 9 is usually inverted to compute the values of these coupling constants from the experimentally measured value of neutron lifetime. Thus, an alternative formula to compute neutron lifetime is highly desirable, and is derived below.

Equation 8 suggests that the lifetime of para-positronium is related to the properties of an electron. The density ρ_e of an electron can be defined as its mass m_e divided by its volume computed using the classical electron radius r_e .

$$\rho_e = m_e / [1.333\ \pi\ r_e^3\ ] \qquad \text{[equation 10]}$$

Davalos [1999] points out that the classical proton radius r_p of a proton is related to the classical electron radius, according to

$$r_p = r_e / \mu \qquad \text{[equation 11]}$$

where μ is the ratio of the proton mass m_p to electron mass m_e ; μ equals 1836.15267245.

Therefore, proton density ρ_p equals

$$\rho_p = \mu\ m_e / [1.333\ \pi\ (\ r_e^3 / \mu^3\ )] \qquad = \mu^4\ \rho_e \qquad \text{[equation 12]}$$

Equations 12 shows that a proton is μ^4 times denser than an electron.

<u>Assumption 1:</u> If the lifetime of para-positronium formed from an electron is assumed to be proportional to the density of an electron, i.e., if, equation 8 is rewritten as

$$(5/8)t_p \qquad \alpha \qquad \rho_e \qquad \text{[equation 13]}$$

then the lifetime τ_n of a free neutron that decays into a proton can be assumed to be proportional to the density of a proton.

$$\tau_n \qquad \alpha \qquad \rho_p \qquad \text{[equation 14]}$$

Dividing equation 14 by equation 13, we get

$$\tau_n / 0.625t_p = \rho_p / \rho_e = \mu^4$$

Using assumption 1, this work p the below formula to compute the free neutron lifetime

$$\tau_n = (5/8)\ \mu^4\ t_p = (5/8)\ \mu^4 \times 2r_e / [c\alpha^6\ ] \qquad \text{[equation 15]}$$

Substituting the previously mentioned values of μ and t_p into equation 15, one computes the lifetime of a free neutron as

τ_n = 884.43166 seconds

According to Nakamura [2010], the 'official' value for the neutron lifetime given in the CODATA is simply the average of various experimental values and Nakamura exhorts the experimentalists to improve their accuracy. The experimental values for neutron lifetime lie between 878.5 seconds to 889.2 seconds [Nakamura, 2010]. The above result, 884.43166 seconds for the lifetime a free or unbound neutron, lies in the middle of the range of experimental values.

Therefore, equation 15 may be regarded as a newly derived formula to compute the lifetime of a free neutron.

A NEW MODEL OF GRAVITY BASED ON THE PROPERTIES OF VIRTUAL PARTICLES

A free neutron vanishes in about 884 seconds while decomposing into 3 other particles that are emitted at high velocities. So it can be said that volume associated with such a neutron vanishes in about 884 seconds.

The volume of a neutron or a proton is rather ill-defined. However, a proton is the same as the nucleus of a hydrogen (protium) atom and the volume of a hydrogen atom is readily computable. Since a free neutron decays into a proton, it seems logical to use the volume of a hydrogen atom as the volume associated with a neutron that decays into a proton.

Defining the specific volume 'v' as the volume of a hydrogen atom divided by the mass of one proton, the following equation is obtained.

$$v = [(4/3)\ \pi\ a_0^3\ ]\ /\ m_p \qquad \text{[equation 16]}$$

where a_0 = Bohr atomic radius of a hydrogen atom = $0.529177211 \times 10^{-10}$ m and the mass of a proton, $m_p = 1.67262178 \times 10^{-27}$ kg.

The specific volume of the hydrogen atom can be calculated to be v = 0.000371102826 m^3 / kg. The inverse of the specific volume is density. Therefore the density of the hydrogen atom, according to the present definition, is 2694.67 kg/m^3. To put that figure in context, it may be mentioned that aluminum metal has about the same density.

To compute Newton's gravitational constant G, we model it as the second derivative with respect to time, of the specific volume of a virtual hydrogen atom. Note that d^2v / dt^2 has the same units as G. Both are expressible in m^3 / kg s^2 .

The quantity v/τ_n^2 can be regarded as a measure of the second derivative, with respect to time, of the specific volume 'v'. The formula for G can be written as

$$G = d^2 v/ dt^2 = (3/8)^2 \times [\, v / \tau_n^2 \,] \qquad \text{[equation 17]}$$

In equation 17, the constant 3/8 = 1 – (5/8), where 5/8 is the constant used in equation 7 or 8 and is based on the assumed charge distribution on or inside an electron.

When the appropriate values are substituted into equation 17, the value of G is obtained as

$$G = (3/8)^2 \times 0.000371102826 / 884.43166^2 = 6.67157289 \times 10^{-11} \text{ m}^3 / \text{kg s}^2$$

The above-computed value of G may be compared with the CODATA value of G, which is

$$G = 6.67384 (\pm 0.0000080) \times 10^{-11} \text{ m}^3 / \text{kg s}^2$$

Therefore, equation 17 gives a theoretical value for G that is very close to the 'official' experimentally obtained value of G.

The value of G, as given by equation 17, diverges from the CODATA value of G after the third decimal digit, but the CODATA value of G has been called into question recently after the measurements done by Fixler et al. [2007].

Fixler et al. [2007], recommend a value of $G = 6.693 \times 10^{-11}$ $\text{Nm}^2 / \text{kg}^2$ with the standard deviation of error being 0.027×10^{-11} $\text{Nm}^2 / \text{kg}^2$. The equation 17 value of G falls within this range. It is only 0.7936 standard deviations away from the Fixler et al. value for G.

Of all the physical constants in the CODATA, G is the most difficult to measure experimentally. Therefore, it is not surprising that out of all the constants used in physics, only the official value of G has been challenged in recent years.

Given the experimental difficulties in measuring G, the theoretical formula for G given in this paper, i.e., equation 17, should be of value to physicists, because it is able to compute G to 8 decimal digits, so that G becomes as accurately known as other physical constants such as Planck's constant.

In view of the uncertainty regarding the CODATA value of G, this paper proposes that the value of G be computed from the theory presented in this paper, i.e.,

$G = 6.671\,572\,89 \times 10^{-11}$ $\text{m}^3 / \text{kg s}^2$ or $\text{Nm}^2 / \text{kg}^2$. Thus, equation 17 relates Newton's gravitational constant to the properties of a virtual neutron that continually appears and disappears in a vacuum.

This does not necessarily mean that virtual neutrons are needed to facilitate the propagation of gravity.

In the next section of this paper, it is shown that even virtual electrons that flash into existence and vanish into the vacuum can be related to the gravitational constant.

GRAVITATIONAL CONSTANT AND ELECTRON PROPERTIES

The Bohr atom model points out that the hydrogen atom radius a_0 can be related to the electron radius r_e by the relationship

$$a_0 = r_e / \alpha^2 = \alpha^{-2} r_e \qquad \text{[equation 18]}$$

Therefore, equation 16 can be re-written as

$$v = [(4/3)\,\pi\, a_0^3\,] / m_p = \quad [(4/3)\,\pi\,\alpha^{-6}\, r_e^3\,] / [\,\mu\, m_e\,] \qquad \text{[equation 19]}$$

Substituting equations 19 and 15 into equation 17, we get

$$G = [3/25]\,[\pi\,\alpha^6\, c^2\, r_e\,] / [\,\mu^9\, m_e\,] \qquad \text{[equation 20]}$$

After multiplying both sides of equation 20 by m_e^2 / r_e , equation 20 can be recast into the following form, which has an interesting physical interpretation.

$$G\, m_e^2 / r_e = [3/25]\,[\pi\,\alpha^6 / \mu^9\,]\,[m_e\, c^2\,] \qquad \text{[equation 21]}$$

The left-hand side of equation 21 is the gravitational potential energy on the surface of an electron in a virtual electron-positron pair and the right hand side contains $m_e c^2$ which is the energy associated with the electron's mass. The gravitational potential energy is seen to be proportional to the rest energy of the electron, yet it is only a small fraction of the rest energy.

According to equations 20 and 21, G is solely dependent on the properties of the virtual electron that appears for a brief instant in a vacuum. The properties of a neutron do not appear in any of these equations.

Both equations 20 and 17 give the same value for G, i.e.,

$$G = 6.671\,572\,89 \times 10^{-11}\ m^3 / kg\, s^2 \text{ or } Nm^2 / kg^2$$

Equation 20 is interesting because both Newton's law of gravity and Coulomb's law of electrostatics can both be derived from it, as is done in the next section.

THE RELATIONSHIP BETWEEN GRAVITY AND ELECTROSTATICS

One of the definitions for the fine structure constant α which appears in equation 20, is

$\alpha = [2 \pi q^2] / [4 \pi \varepsilon_0 h c] = 0.00729735$ [equation 22]

where q is the charge carried by one electron, i.e., $q = 1.6029 \times 10^{-19}$ coulombs.

The term α^6 appears in equation 20, but if we write α^6 as $\alpha^5 \times \alpha$, and substitute equation 22 into equation 20, we obtain

$G = [3/25] [\pi \alpha^5 c^2 r_e] / [\mu^9 m_e] \times [2 \pi q^2] / [4 \pi \varepsilon_0 h c]$ [equation 23]

Multiplying throughout by m_e^2 and combining some terms, we obtain

$Gm_e^2 = \{ [6 \pi^2 \alpha^5 r_e m_e c^2] / [25 \mu^9 h c] \} \times [q^2] / [4 \pi \varepsilon_0]$ [equation 24]

Dividing both sides of equation 24 by R^2 where ‘R’ is any arbitrary distance, we get

$Gm_e^2 / R^2 = \{ [6 \pi^2 \alpha^5 r_e m_e c^2] / [25 \mu^9 h c] \} \times [q^2] / [4 \pi \varepsilon_0 R^2]$ [equation 25]

Thus, the left-hand side of equation 25 is the familiar Newton’s law of gravity as applied to a virtual electron-positron pair separated by an arbitrary distance R. The right-hand side of equation 25 is seen to contain the term $q^2 / [4 \pi \varepsilon_0 R^2]$ which is Coulomb’s law of electrostatics as applied to an electron-positron pair separated by an arbitrary distance ‘R’.

Though scientists have known for nearly 200 years that the law of gravitation and the law of electrostatics are related, a simple derivation of one from the other is not to be found in textbooks. Therefore, this paper has presented the above inter-relationship.

Thus, equation 25 says that the gravitational force is proportional to the electrostatic force, and inter-relates Newton’s gravity law and Coulomb’s electrostatic law. The constant C_{ge} in equation 25 enclosed within braces, is

$C_{ge} = [6 \pi^2 \alpha^5 r_e m_e c^2] / [25 \mu^9 h c]$ [equation 26a]

The fact that the Bohr’s hydrogen atom model says that the Planck’s constant h equals $2 \pi m_e v_e a_0$, $a_0 = \alpha^{-2} r_e$ and electron velocity v_e equals αc enables further simplification of equation 26a. Substituting the Bohr model relationships into equation 26a results in

$C_{ge} = [3 \pi \alpha^6] / [25 \mu^9] = 2.39963 \times 10^{-43}$ [equation 26b]

Note that the right-hand side of equation 26b contains only non-dimensional quantities.

The value of the constant C_{ge} can then be computed to be equal to 2.39963 x 10^{-43} , and illustrates the well-known fact that gravity is 43 orders of magnitude weaker than electrostatic attraction.

As will be shown below, the constant C_{ge} in equation 26 is simply the quotient of two well-known constants.

In physics, the unitless gravitational coupling constant α_G is defined as [Barrow and Tipler, 1988]

$$\alpha_G = 2 \pi G m_e^2 / [h c] = [m_e / m_P]^2 = [t_{pl} \omega_c]^2 \qquad \text{[equation 27]}$$

and has a value $\alpha_G = 1.751073 \times 10^{-45}$, where m_P is the Planck mass. The gravitational coupling constant α_G has a simple interpretation. It is the square of the electron mass when electron mass is measured in units of Planck's mass. In equation 27, t_{pl} is Planck's time and ω_c is the Compton angular frequency of the electron in a hydrogen atom. Both of these quantities are defined in Barrow and Tipler [1988] and in other books too.

The constant C_{ge} in equation 26 can be computed to be equal to the gravitational coupling constant α_G defined in equation 27 divided by the fine structure constant α defined in equation 22, i.e.,

$$C_{ge} = \alpha_G / \alpha = \qquad \text{[equation 28]}$$

i.e., $2.3996 \times 10^{-43} = 1.751073 \times 10^{-45} / 0.00729735$

Substituting equation 26b into equation 28 results in a new formula for the gravitational coupling constant.

$$\alpha_G = [3 \pi \alpha^7] / [25 \mu^9] \qquad \text{[equation 29]}$$

Note that equation 29 relates the gravitational coupling constant to the fine structure constant and the ratio of proton mass to electron mass.

Finally, physicists have been long aware that both α and μ are approximately related to by π, by the formulas $\alpha^{-1} = 2^{0.5} \pi^4$ or $\alpha^{-1} = (7/5) \pi^4$ and $\mu = 6 \pi^5$. The reason for these mysterious relationships is not clear and could be pure coincidence. Substituting these formulas into equation 29 will make the gravitational coupling constant α_G related to only π. However, such a formula would be very slightly inaccurate, and not based on real physics, so it is not presented here.

CONCLUSIONS

The following are the conclusions of this paper.

1] The main conclusion of this paper is that Newton's gravitational constant G can be theoretically computed using the properties of virtual particles that are constantly appearing and disappearing inside a vacuum as per the QED vacuum model. This conclusion is based on the fact that 2 new, accurate equations for G have been derived in this work, namely,

a] An equation relating the gravitational potential energy on the surface of a virtual electron with the rest energy of the electron has been derived in this work and it enables the accurate computation of Newton's gravitational constant G.

b] An equation relating G with the volume change associated with a virtual neutron has been derived and it enables the accurate computation of Newton's gravitational constant G.

Both equations enable the computation of G to 8 decimal digits whereas the present CODATA value for G is known only to 5 digits. Therefore, this paper suggests that these equations be used to improve the accuracy of the value of G, so that G becomes as accurately known as other fundamental constants such as Planck's constant.

The value of G computed in this paper is $G = 6.671\ 572\ 89 \times 10^{-11}\ m^3 / kg\ s^2$ or Nm^2 / kg^2. The CODATA value of G agrees with the computed value up to 2 decimal digits.

2] A simple formula for the lifetime of a free neutron has been derived. This formula agrees with experiments and is simpler than the presently used formula for the lifetime of a free neutron.

3] A new equation for the gravitational coupling constant is proposed in this work. This equation enables the independent computation of the gravitational coupling constant – something that is not possible now without using the experimentally known value of G. This equation proves the relationship of the gravitational coupling constant to the fine structure constant and the ratio of proton mass to electron mass.

4] A simple, new derivation for Newton's law of gravity and Coulomb's law of electrostatics is given in this work. This derivation underscores and proves the well-known connection between these 2 laws.

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