Zero and negative energy dissipation at information-theoretic erasure

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Abstract We introduce information-theoretic erasure based on Shannon's binary channel formula. It is pointed out that this type of erasure is a natural energy-dissipation-free way in which information is lost in double-potential-well memories, and it may be the reason why the brain can forget things effortlessly. We also demonstrate a new non-volatile, charge-based memory scheme wherein the erasure can be associated with even negative energy dissipation; this implies that the memory's environment is cooled during information erasure and contradicts Landauer's principle of erasure dissipation. On the other hand, writing new information into the memory always requires positive energy dissipation.

Keywords

Erasure · Zero Energy Dissipation · Negative Energy Dissipation

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1 Introduction: Classical information erasure

In computer memories, the erasure of a bit means resetting its value to zero. This type of erasure, which we call classical erasure, implies a bit-value change if the bit value before the erasure was 1. In accordance with Brillouin's negentropy equation [1–3], any bit-value change gives a minimum dissipation of energy E_d by

$$E_{\mathrm{d}} \geq kT \ln\left(\frac{1}{p_{\mathrm{e}}}\right) , \qquad (1)$$

where p_{e} is the error probability of the operation $(p_{\mathrm{e}} < 0.5)$, k is Boltzmann's constant and T is absolute temperature. In the case of $p_{\mathrm{e}} = 0.5$, which is the limit for the completely inefficient operation, the relevant $kT\ln(2)$ dissipation is the famous Szilard–Brillouin–Landauer limit [1-3].

In this paper, we introduce "information-theoretic erasure", ITE, for which the elimination of the information is guaranteed by information theory. We show that ITE does not cause energy dissipation, and it can even produce negative energy dissipation by cooling the environment. However, the writing of new information into the memory always requires positive energy dissipation.

2 Information-theoretic erasure

In accordance with Shannon's formula for binary channels, the information content (entropy) I_1 of a single bit with error probability p_e is given by

$$I_1 = 1 + p_{\mathrm{e}} \log_2 p_{\mathrm{e}} + (1 - p_{\mathrm{e}}) \log_2 (1 - p_{\mathrm{e}}) \qquad (2)$$

as illustrated in Fig. 1. The case of $p_{\mathrm{e}} = 0$ yields an $I_1 = 1$ bit , while $p_{\mathrm{e}} = 0.5$ corresponds to a random coin with an $I_1 = 0$ bit .

Motivated by these facts, we consider the memory an information channel between the Writer and Reader of information and introduce ITE as follows: Suppose that the bit-operations are error-free and that the bit-value before the erasure is 1. Then the probability $p(1)$ that the bit has the value 1 is

$$p(1) = 1 \ . \quad (3)$$

Similarly, if the bit value before the erasure is 0, then the probability is

$$p(0) = 1 \ . \quad (4)$$

We define information-theoretic erasure so that, after the erasure, these probabilities become

$$p(1) = 0.5 \quad \text{and} \quad p(0) = 0.5 \ , \quad (5)$$

which guarantees total elimination of information from the memory.

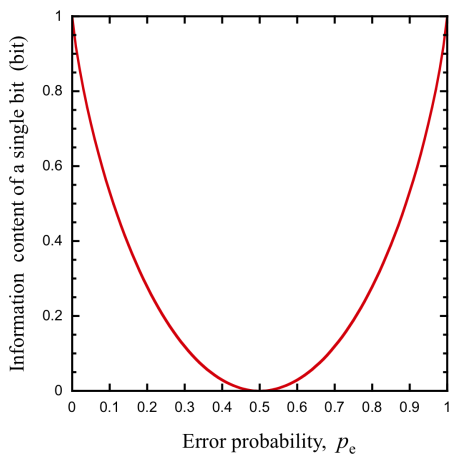

Fig. 1 Information content of a bit versus error according to Eq. (2).

3 Physical realizations

In this section, we show two physical realizations: one is *passive* erasure (thermalization) in memories with double-potential wells and the other is *active* erasure in capacitor-based memories, where even negative energy dissipation is feasible.

3.1 Passive erasure in symmetric potential wells

The most natural process that leads to information-theoretic erasure is thermalization in a symmetric double-well potential system, such as in a magnetic memory; see Fig. 2. When such a system is kept untouched for a number of relaxation events, the exponential nature of relaxation will cause ITE so that

$$p(1) \to 0.5 \quad \text{and} \quad p(0) \to 0.5 \quad (6)$$

occur without energy dissipation because equilibrium thermal fluctuations are utilized for erasure. Of course, such a process may take thousands of years, but the existence of this phenomenon proves that no energy dissipation is required for information erasure. Similar arguments may explain how the brain can easily forget neutral information, while the creation of new information requires efforts.

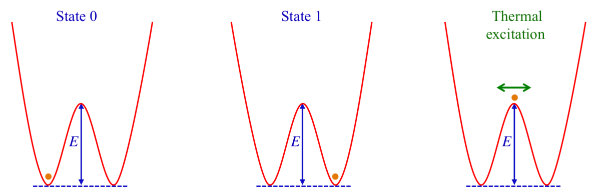

Fig. 2 Passive information-theoretic erasure in a zero-energy-dissipation fashion by waiting for thermalization at ambient temperature, or, in a dissipative way, by heating the memory cell to rapidly thermalize the bit.

Of course, it is possible to heat the memory cell so that kT approaches the barrier height E (see Fig. 2) sufficiently to cause rapid ITE, but this approach involves energy dissipation and is uninteresting from a fundamental scientific point of view.

3.2 Charge-based bit with information-theoretic erasure

We now consider a capacitor-type information cell. Figs. 3–6 show various aspects of its operation. Suppose that positive voltage is interpreted as bit 1 and negative voltage as bit 0.

Fig. 3 shows the writing process. An external resistor and voltmeter are connected to the cell, and thus a parallel RC circuit is present. As a consequence of the measurement and decision process described below, the writing process is strongly dissipative. The resistor will drive a Johnson noise current through the capacitor thereby yielding a noise voltage on the capacitor; see Fig. 4. The voltmeter monitors this voltage, and the resistor is disconnected when the required voltage is reached. The root-mean-square value of the Johnson noise voltage on a parallel *RC* circuit is [3]

$$\sigma = \sqrt{kT/C} \ , \tag{7}$$

and the corresponding mean energy in the capacitor is *kT*/2. Two cases should be considered:

(*i*) If, during the writing process, we choose $+\sigma$ for bit value 1 and $-\sigma$ for bit value 0 then the information-containing capacitor will possess thermal equilibrium energy in accordance with Boltzmann's equipartition theorem for a single thermal degree of freedom.

(*ii*) On the other hand, if we use the voltages $\pm u_0$ for the 1 and 0 bits, respectively, where $u_0 < \sigma$, then the energy in the capacitor will be *less* than the thermal equilibrium level *kT*/2.

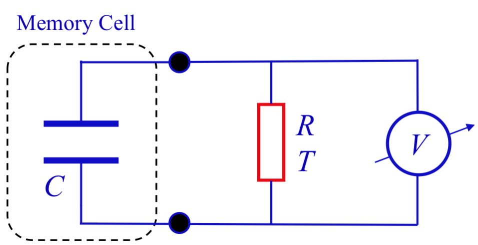

Fig. 3 Writing of information into a capacitor. Johnson noise in the resistor drives the current, and the connection to the memory is terminated when the voltage level corresponding to the information to be stored is reached.

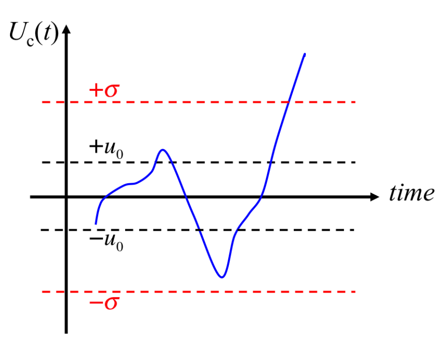

Fig. 4 Johnson noise voltage in a capacitor. The voltage levels $\pm u_0$ pertain to written bit values; see the main text for details.

Fig. 5 shows the corresponding read-out process. It entails measuring the voltage and deciding if it is positive (bit value 1) or negative (bit value 0).

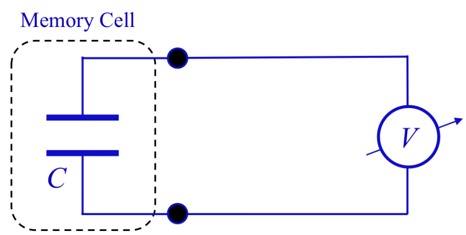

Fig. 5 Reading out information from a capacitor.

Fig. 6 illustrates the erasure process. The capacitor is reconnected to the resistor, but no voltage measurement or decision is necessary. The capacitor will be thermalized within a few events with relaxation time $\tau = RC$, and the conditions of relations (6) are reached. The energy dissipation during erasure is determined by our former choice:

(*iii*) Using writing condition (*i*), the mean energy of the capacitor will not change during erasure, and hence the energy dissipation is zero.

(*iv*) However using writing condition (*ii*), the mean energy of the capacitor is increased during information erasure so that energy dissipation is negative, and consequently the resistor and the environment of the memory cell are cooled in accordance with

$$E_{cool} = \frac{1}{2}\left(Cu_0^2 - kT\right) < 0 \quad . \tag{8}$$

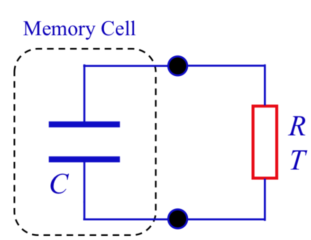

Fig. 6 Information erasure by thermalization of a capacitor. For small values of u_0, energy is extracted from the resistor so that its environment is cooled, thus indicating negative energy dissipation.

4 Conclusion and remarks

Our present study showed schemes and realizations for information erasure, for which energy dissipation can be zero or negative. We trust that these results lend further credence to objections [4–10] against the Landauer theorem [11,12], which claims that information erasure is a dissipative process whereas information writing is not.

Following general practices [11,12] for analysis of information writing and erasure, we neglected the energy dissipation of the external control step for connecting the resistor to the capacitor in the case of erasure. Including the energy for this control [2,3] would imply positive net dissipation in the environment. However, the same happens also during information writing, and consequently information writing still comes out as much more dissipative than erasure.

However, it is still possible to avoid dissipation due to connecting issues, and thus maintain zero and negative energy dissipation, by keeping the resistor permanently connected to the capacitor. Then the passive erasure with zero energy dissipation will take place as in the double-well potential case (see Sec. 3.1) with the following modifications: at condition (*i*) the energy dissipation will be exactly zero, see (*iii*); and at condition (*ii*) the energy dissipation due to erasure will be strictly negative, see (*iv*).

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