Three conjectures in Euclidean geometry

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Abstract

In this note, I introduce three conjectures of generalization of the Lester circle theorem, the Parry circle theorem, the Zeeman-Gossard perspector theorem respectively

1 A conjectures of generalization of the Lester circle theorem

Theorem 1 (Lester)**.** *Let ABC be a triangle, then the two Fermat points, the nine-point center, and the circumcenter lie on the same circle .*

Conjecture 2 ([1], [2], [3])**.** *Let P be a point on the Neuberg cubic. Let P_A be the reflection of P in line BC, and define P_B and P_C cyclically. It is known that the lines AP_A, BP_B, CP_C concur. Let $Q(P)$ be the point of concurrence. Then two Fermat points, P, $Q(P)$ lie on a circle.*

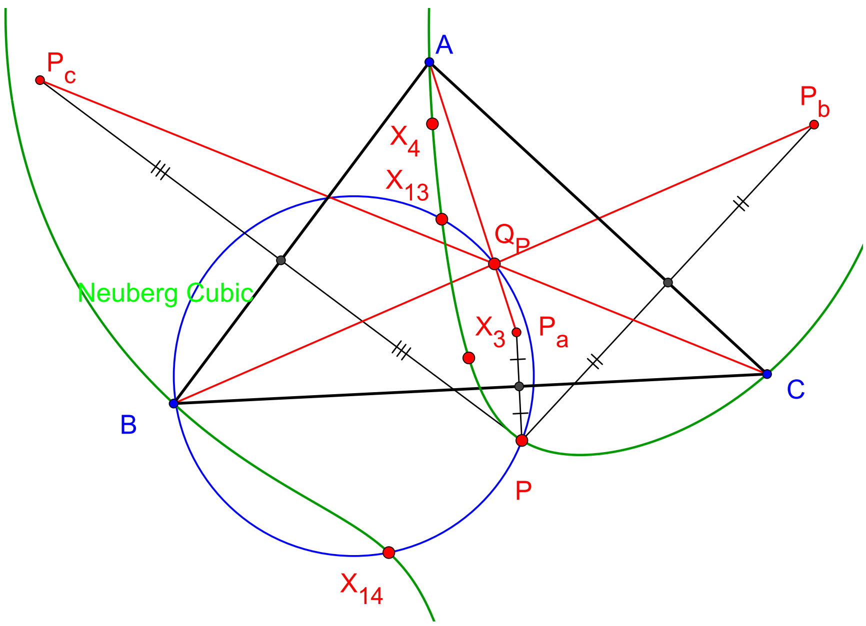

Figure 1: Conjecture 2

When $P = X(3)$, it is well-know that $Q(P) = Q(X(3)) = X(5)$, the conjucture becomes Lester theorem.

2 A conjecture of generalization of the Parry circle theorem

Theorem 3 (Parry). *Let ABC be a triangle, then the triangle centroid, the first and the second isodynamic points, the far-out point, the focus of the Kiepert parabola, the Parry point and two points in Kimberling centers $X(352)$ and $X(353)$ lie on a circle.*

Conjecture 4 ([4], [5]). *Let a rectangular circumhyperbola of ABC, let L be the isogonal conjugate line of the hyperbola. The tangent line to the hyperbola at $X(4)$ meets L at point K. The line through K and center of the hyperbola meets the hyperbola at F_+, F_-. Let I_+, I_-, G be the isogonal conjugate of F_+, F_- and K respectively. Let F be the inverse point of G with respect to the circumcircle of ABC. Then five points I_+, I_-, G, $X(110)$, F lie on a circle. Furthermore K lie on the Jerabek hyperbola.*

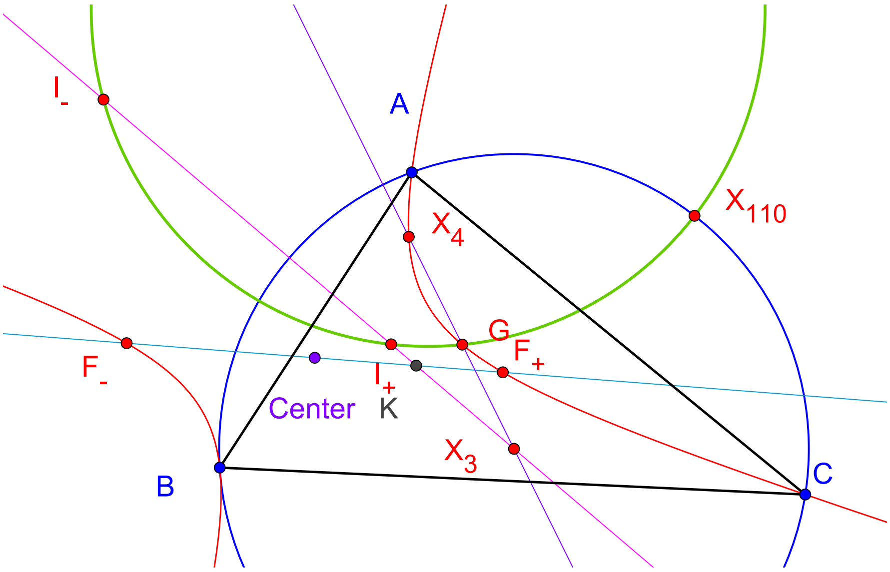

Figure 2: Conjecture 4

When the hyperbolar is the Kiepert hyperbolar the conjecture be comes Parry circle theorem.

3 A conjecture of generalization of the Zeeman-Gossard perspector theorem and related

Theorem 5 ([6]). *Let ABC be a triangle, the three Euler lines of the triangles formed by the Euler line and the sides, taken by twos, of a given triangle, form a triangle perspective with the given triangle and having the same Euler line.*

Conjecture 6 ([7], [8]). *Let ABC be a triangle, Let P_1, P_2 be two points on the plane, the line P_1P_2 meets BC, CA, AB at A_0, B_0, C_0 respectively. Let A_1 be a point on the plane such that B_0A_1 parallel to CP_1, C_0A_1 parallel to BP_1. Define B_1, C_1 cyclically. Let A_2 be a point on the plane such that B_0A_2 parallel to CP_2, C_0A_2 parallel to BP_2. Define B_2, C_2 cyclically. The triangle formed by three lines A_1A_2, B_1B_2, C_1C_2 homothety and congruent to ABC, the homothetic center lie on P_1P_2.*

Conjecture 7 ([7], [8]). *Notation in conjecture 6, then the Newton lines of four quadrilaterals bounded by four lines AB, AC, A_1A_2, L; four lines BC, BA, B_1B_2, L; four lines CA, CB, C_1C_2, L; and four lines AB, BC, CA, L pass through the homothetic center.*

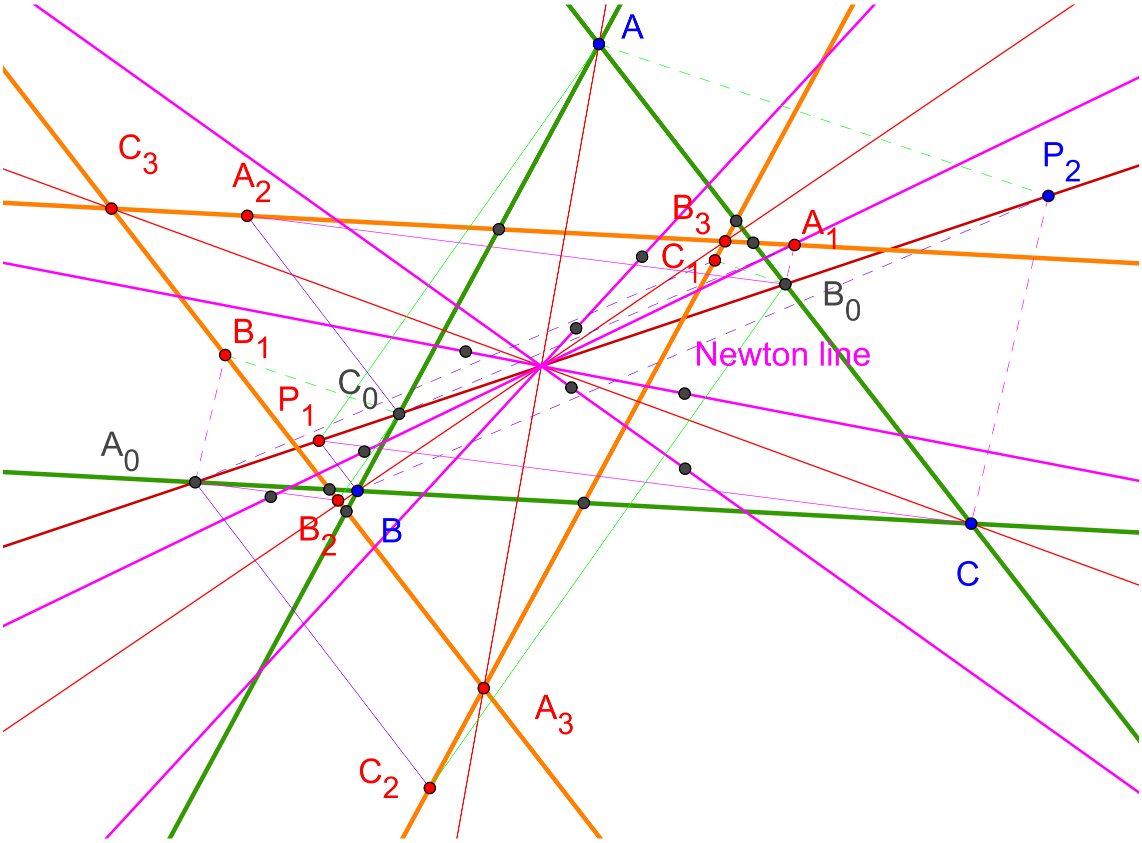

Figure 3: Conjectures 6 and 7

References

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