On the origin of electric charge

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Last modified: 24 juli 2015

Abstract

By starting from a quaternionic separable Hilbert space as a base model the paper uses the capabilities and the restrictions of this model in order to investigate the origins of the electric charge and the electric fields. Also other discrete properties such as color charge and spin are considered.

The paper exploits all known aspects of the quaternionic number system and it uses quaternionic differential calculus rather than Maxwell based differential calculus.

The paper presents fields as mostly continuous quaternionic functions. The electric field is compared with another basic field that acts as a background embedding continuum.

If the paper introduces new science, then it has fulfilled its purpose.

Contents

1 Introduction

Indications suggest that electrical charges are properties of space. The major indication is the fact that quaternionic number systems exist in several versions that differ in their symmetry properties. These symmetry properties are related to the way that these versions are ordered.

As a consequence, it makes sense to introduce the notion of types of spaces where each type has its own symmetry flavor. An important category of these spaces are symmetry centers. Symmetry centers float on a covering background space that has its own symmetry flavor.

Within a separable Hilbert space such types of spaces can coexist as eigenspaces of corresponding types of quaternionic operators. That is why we will use an infinite dimensional separable quaternionic Hilbert space $\mathfrak{H}$ as our base model.

Each infinite dimensional separable quaternionic Hilbert space owns a companion Gelfand triple $\mathcal{H}$, which is a non-separable Hilbert space. In the separable Hilbert space $\mathfrak{H}$ the eigenspaces of operators are countable. In the Gelfand triple $\mathcal{H}$ the eigenspaces of operators can be continuums.

In the separable Hilbert space we introduce the concept of ***well-ordered normal operators***. We will define a well-ordered reference operator $\mathcal{R}$ whose eigenspace acts as a model-wide parameter space. The well-ordered reference operator that provides the countable parameter space in the separable Hilbert space $\mathfrak{H}$ owns a companion reference operator $\mathfrak{R}$ in the Gelfand triple $\mathcal{H}$ that provides a continuum eigenspace.

Fields will appear as continuum eigenspaces of normal operators that reside in the Gelfand triple. We will show that fields can be defined as quaternionic functions that use the eigenspace of the reference operator $\mathfrak{R}$ as their parameter space.

Symmetry centers reside in the separable Hilbert space and are maintained in finite dimensional subspaces. Symmetry centers exist in a small number of types that differ in the corresponding symmetry flavor. Corresponding normal operators $\mathfrak{S}^x$ map these subspaces onto themselves. Superscript ^{x} refers to the symmetry flavor of the symmetry center. The center location of the symmetry center corresponds to the value of a quaternionic mapping function of its quaternionic location in the parameter space that is defined via the well-ordered reference operator $\mathcal{R}$and its companion $\mathfrak{R}$. That value is a location in a background continuum $\mathfrak{C}$.

2 Quaternions

Quaternions can be interpreted as combinations of a real scalar and a three dimensional real vector. The combination supports numeric arithmetic. The vector part introduces a non-commutative multiplication.

We will indicate the real part of quaternion a by subscript a_0 and the vector part will be put in bold font face $\boldsymbol{a}$.

$$a = a_0 + \boldsymbol{a} \tag{1}$$

$$a^* = a_0 - \boldsymbol{a} \tag{2}$$

a^* is the quaternionic conjugate of a.

The sum is defined by:

$$c = c_0 + \boldsymbol{c} = a + b \tag{3}$$

$$c_0 = a_0 + b_0 \tag{4}$$

$$\boldsymbol{c} = \boldsymbol{a} + \boldsymbol{b} \tag{5}$$

The product rule is defined by:

$$c = a\, b = (a_0 + \boldsymbol{a})(b_0 + \boldsymbol{b}) = a_0\, b_0 - \langle \boldsymbol{a}, \boldsymbol{b} \rangle + a_0 \boldsymbol{b} + b_0 \boldsymbol{a} \pm \boldsymbol{a} \times \boldsymbol{b} \tag{6}$$

$$c_0 = a_0\, b_0 - \langle \boldsymbol{a}, \boldsymbol{b} \rangle \tag{7}$$

$$\boldsymbol{c} = a_0 \boldsymbol{b} + b_0 \boldsymbol{a} \pm \boldsymbol{a} \times \boldsymbol{b} \tag{8}$$

The $\pm$ sign signalizes the choice between right handed and left handed external vector product. This choice indicates that quaternionic number systems exist in multiple versions. Due to the four dimensions of quaternions will quaternionic number system exist in sixteen different symmetry flavors.

The norm of a quaternion is defined by:

$$|a| = \sqrt{a a^*} = \sqrt{a_0 a_0 + \langle \boldsymbol{a}, \boldsymbol{a} \rangle} \tag{9}$$

3 Quaternionic Hilbert spaces

Separable Hilbert spaces are linear vector spaces in which an inner product is defined. This inner product relates each pair of Hilbert vectors. The value of that inner product must be a member of a division ring. Suitable division rings are real numbers, complex numbers and quaternions. This paper uses quaternionic Hilbert spaces.

$$\langle x|y\rangle = \langle y|x\rangle^* \tag{1}$$

$$\langle x+y|z\rangle = \langle x|z\rangle + \langle y|z\rangle \tag{2}$$

$$\langle \alpha x|y\rangle = \alpha\langle x|y\rangle \tag{3}$$

$\langle x|$ is a bra vector. $|y\rangle$ is a ket vector. α is a quaternion.

This paper considers Hilbert spaces as no more and no less than structured storage media for dynamic geometrical data that have an Euclidean signature. Quaternions are ideally suited for the storage of such data. Quaternionic Hilbert spaces are described in “Quaternions and quaternionic Hilbert spaces” [8].

The operators of separable Hilbert spaces have countable eigenspaces. Each infinite dimensional separable Hilbert space owns a Gelfand triple. The Gelfand triple embeds this separable Hilbert space and offers as an extra service operators that feature continuums as eigenspaces. In the corresponding subspaces the definition of dimension loses its sense.

3.1 Representing continuums and continuous functions

Operators map Hilbert vectors onto other Hilbert vectors. Paul Dirac introduced the bra-ket notation that eases the formulation of Hilbert space habits [5]. Via the inner product the operator T may be linked to an adjoint operator $T^\dagger$.

$$\langle Tx|y\rangle \equiv \langle x|T^\dagger y\rangle \tag{1}$$

$$\langle Tx|y\rangle = \langle y|Tx\rangle^* = \langle T^\dagger y|x\rangle^* \tag{2}$$

A linear quaternionic operator T, which owns an adjoint operator $T^\dagger$ is normal when

$$T^\dagger T = T T^\dagger \tag{3}$$

$T_0 = \left(T + T^\dagger\right)/2$ is a self adjoint operator and $\boldsymbol{T} = \left(T - T^\dagger\right)/2$ is an imaginary normal operator. Self adjoint operators are also called Hermitian operators. Imaginary normal operators are also called anti-Hermitian operators.

By using bra-ket notation, operators that reside in the Hilbert space and correspond to continuous functions, can easily be defined by starting from an orthogonal base of vectors. This works both in separable Hilbert spaces as well as in non-separable Hilbert spaces.

Let $\{q_i\}$ be the set of rational quaternions in a selected quaternionic number system and let $\{|q_i\rangle\}$ be the set of corresponding base vectors. They are eigenvectors of a normal operator $\mathcal{R} = |q_i\rangle q_i\langle q_i|$. Here we enumerate the base vectors with index i.

$$\mathcal{R} = |q_i\rangle q_i\langle q_i| \tag{4}$$

$\mathcal{R}$ is the configuration parameter space operator.

$\mathcal{R}_0 = \left(\mathcal{R} + \mathcal{R}^\dagger\right)/2$ is a self-adjoint operator. Its eigenvalues can be used to order the eigenvectors. The ordered eigenvalues can be interpreted as ***progression values***.

$\boldsymbol{\mathcal{R}} = (\mathcal{R} - \mathcal{R}^\dagger)/2$ is an imaginary operator. Its eigenvalues can be used to order the eigenvectors. The eigenvalues can be interpreted as ***spatial values*** and can be ordered in several ways.

Let $f(q)$ be a quaternionic function.

$$f = |q_i\rangle f(q_i)\langle q_i| \tag{5}$$

f defines a new operator that is based on function $f(q)$.

Operator f has discrete quaternionic eigenvalues.

In a non-separable Hilbert space, such as the Gelfand triple, the continuous function $\mathcal{F}(q)$ can be used to define an operator, which features a continuum eigenspace.

$$\mathcal{F} = |q\rangle \mathcal{F}(q)\langle q| \tag{6}$$

Via the continuous quaternionic function $\mathcal{F}(q)$, the operator $\mathcal{F}$ defines a curved continuum $\mathcal{F}$. This operator and the continuum reside in the Gelfand triple, which is a non-separable Hilbert space.

$$\mathfrak{R} = |q\rangle q\langle q| \tag{7}$$

The function $\mathcal{F}(q)$ uses the eigenspace of the reference operator $\mathfrak{R}$ as a flat parameter space that is spanned by a quaternionic number system $\{q\}$. The continuum $\mathcal{F}$ represents the target space of function $\mathcal{F}(q)$.

Here we no longer enumerate the base vectors with index i. We just use the name of the parameter. If no conflict arises, then we will use the same symbol for the defining function, the defined operator and the continuum that is represented by the eigenspace.

In general the dimension of a subspace loses its significance in the non-separable Hilbert space.

The continuums that appear as eigenspaces in the non-separable Hilbert space $\mathcal{H}$ can be considered as quaternionic functions that also have a representation in the corresponding infinite dimensional separable Hilbert space $\mathfrak{H}$. Both representations use a flat parameter space $\mathfrak{R}$ or $\mathcal{R}$ that is spanned by quaternions.

The parameter space operators will be treated as reference operators. The rational quaternionic eigenvalues $\{q_i\}$ that occur as eigenvalues of the reference operator $\mathcal{R}$ in the separable Hilbert space map onto the rational quaternionic eigenvalues $\{q_i\}$ that occur as subset of the quaternionic eigenvalues $\{q\}$ of the reference operator $\mathfrak{R}$ in the Gelfand triple. In this way the reference operator $\mathcal{R}$ in the infinite dimensional separable Hilbert space $\mathfrak{H}$ relates directly to the reference operator $\mathfrak{R}$, which resides in the Gelfand triple $\mathcal{H}$.

Embedding occurs in a continuum that is defined by a quaternionic function $\mathfrak{C}(q)$.

4 Well-ordered reference operators

The eigenvalues of a normal operator T that resides in a separable Hilbert space can be ordered with respect to the real part of the eigenvalues. Operator $T_0 = (T + T^\dagger)/2$ is the corresponding self-adjoint operator. If each real value occurs only once, then the operator T and its adjoint $T^\dagger$ can be well-ordered. The imaginary part of the eigenvalues can then still be ordered in different ways. Operator $\boldsymbol{T} = (T - T^\dagger)/2$ is the corresponding anti-Hermitian operator. For example it can be ordered according to Cartesian coordinates or according to spherical coordinates. Also each of these orderings can be done in different ways.

The property of being well-ordered is restricted to operators with countable eigenspaces.

4.1.1 Progression ordering

A single self-adjoint reference operator that offers an infinite set of rational eigenvalues can synchronize a ***category of well-ordered normal operators***. The ordered eigenvalues of the self-adjoint operator act as progression values. In this way the infinite dimensional separable Hilbert space owns a model wide clock. With this choice the separable Hilbert space steps with model-wide progression steps.

A selected well-ordered normal reference operator that resides in an infinite dimensional separable quaternionic Hilbert space is used in the specification of the companion quaternionic Gelfand triple. In that way progression steps in the separable Hilbert space and flows in the companion Gelfand triple. Both reference operators will be used to provide parameter spaces.

The countable set of progression values of the Hermitian part $\mathcal{R}_0 = (\mathcal{R} + \mathcal{R}^\dagger)/2$ of the well-ordered reference operator $\mathcal{R}$ can be used to enumerate other ordered sequences.

4.1.2 Cartesian ordering

The whole separable Hilbert space can at the same time be spanned by the eigenvectors of a reference operator whose eigenvalues are well-ordered with respect to the real parts of the eigenvalues, while the imaginary parts are ordered with respect to a Cartesian coordinate system.

For Cartesian ordering, having an origin is not necessary. In affine Cartesian ordering only the direction of the ordering is relevant. Affine Cartesian ordering exists in eight symmetry flavors.

Cartesian ordering supposes a unique orientation of the Cartesian axes.

The well-ordered reference operator $\mathcal{R}$ is supposed to feature affine Cartesian ordering.

4.1.3 Spherical ordering

Spherical ordering starts with a selected Cartesian set of coordinates. In this case the origin is at a unique center location. Spherical ordering can be done by first ordering the azimuth and after that the polar angle is ordered. Finally, the radial distance from the center can be ordered. Another procedure is to start with the polar angle, then the azimuth and finally the radius. Such, spherical orderings may create a ***symmetry center***. Since the ordering starts with a selected Cartesian coordinate system, spherical ordering will go together with affine Cartesian ordering.

Each symmetry center is described by the eigenspaces of an anti-Hermitian operator $\mathfrak{S}^x$ that map a finite dimensional subspace of Hilbert space $\mathfrak{H}$ onto itself. Superscript ^{x} refers to the ordering type of the symmetry center. $\mathfrak{S}^x$ has no Hermitian part. Only through its ordering it can synchronize with progression steps.

5 Symmetry flavor

Quaternions can be mapped to Cartesian coordinates along the orthonormal base vectors 1, $\boldsymbol{i}$, $\boldsymbol{j}$ and $\boldsymbol{k}$; with $\boldsymbol{ij} = \boldsymbol{k}$

Due to the four dimensions of quaternions, quaternionic number systems exist in 16 well-ordered versions $\{q^x\}$ that differ only in their discrete Cartesian symmetry set. The quaternionic number systems $\{q^x\}$ correspond to 16 versions $\{q_i^x\}$ of rational quaternions.

Half of these versions are right handed and the other half are left handed. Thus the handedness is influenced by the symmetry flavor.

The superscript ^{x} can be ⓪, ①, ②, ③, ④, ⑤, ⑥, ⑦, ⑧, ⑨, ⑩, ⑪, ⑫, ⑬, ⑭, or ⑮.

This superscript represents the ***symmetry flavor*** of the indexed subject.

A reference operator $\mathcal{R}^{⓪} = \left|q_i^{⓪}\right\rangle q_i^{⓪} \left\langle q_i^{⓪}\right|$ in separable Hilbert space $\mathfrak{H}$ maps into a reference operator $\mathfrak{R}^{⓪} = \left|q^{⓪}\right\rangle q^{⓪} \left\langle q^{⓪}\right|$ in Gelfand triple $\mathcal{H}$.

The symmetry flavor of the symmetry center $\mathfrak{S}^x$, which is maintained by operator $\mathfrak{S}^x = |\mathfrak{s}_i^x\rangle \mathfrak{s}_i^x \langle \mathfrak{s}_i^x|$ is determined by its Cartesian ordering and then compared with the reference symmetry flavor, which is the symmetry flavor of the reference operator $\mathcal{R}^{⓪}$. ↑↑↑↑

Now the symmetry related charge follows in three steps.

1. Count the difference of the spatial part of the symmetry flavor of $\mathfrak{S}^x$ with the spatial part of the symmetry flavor of reference operator $\mathcal{R}^{⓪}$.
2. If the handedness changes from **R** to **L**, then switch the sign of the count.
3. Switch the sign of the result for anti-particles.

Symmetry flavor					
Ordering x y z τ	Superscript	Handedness Right/Left	Color charge	Charge * 3	Symmetry center type Names are taken from the standard model
↑↑↑↑	⓪	R	N	+0	neutrino
↓↑↑↑	①	L	R	−1	down quark
↑↓↑↑	②	L	G	−1	down quark
↓↓↑↑	③	L	B	−1	down quark
↑↑↓↑	④	R	B	+2	up quark
↓↑↓↑	⑤	R	G	+2	up quark
↑↓↓↑	⑥	R	R	+2	up quark
↓↓↓↑	⑦	L	N	−3	electron
↑↑↑↓	⑧	R	N	+3	positron
↓↑↑↓	⑨	L	R	−2	anti-up quark
↑↓↑↓	⑩	L	G	−2	anti-up quark
↓↓↑↓	⑪	L	B	−2	anti-up quark
↑↑↓↓	⑫	R	B	+1	anti-down quark
↓↑↓↓	⑬	R	R	+1	anti-down quark
↑↓↓↓	⑭	R	G	+1	anti-down quark
↓↓↓↓	⑮	L	N	−0	anti-neutrino

Per definition, members of coherent sets $\{a_i^x\}$ of quaternions all feature the same symmetry flavor that is marked by superscript ^{x}.

Also continuous functions and continuums feature a symmetry flavor. Continuous quaternionic functions $\psi^x(q^x)$ and corresponding continuums do not switch to other symmetry flavors ^{y}.

The reference symmetry flavor $\psi^y(q^y)$ of a continuous function $\psi^x(q^y)$ is the symmetry flavor of the parameter space $\{q^y\}$.

If the continuous quaternionic function describes the density distribution of a set $\{a_i^x\}$ of discrete objects a_i^x, then this set must be attributed with the same symmetry flavor ^{x}. The real part describes the location density distribution and the imaginary part describes the displacement density distribution.

6 Symmetry centers

Each symmetry center corresponds to a dedicated subspace of the infinite dimensional separable Hilbert space. That subspace is spanned by the eigenvectors $\{|\mathfrak{s}_i^x\rangle\}$ of a corresponding symmetry center reference operator $\mathfrak{S}^x$. Here the superscript ^{x} refers to the type of the symmetry center.

Symmetry flavors relate to affine Cartesian ordering. Each symmetry center will own a single symmetry flavor. The symmetry flavor of the symmetry center relates to the Cartesian coordinate system that acts as start for the spherical ordering. The combination of affine Cartesian ordering and spherical ordering puts corresponding axes in parallel. Spherical ordering relates to spherical coordinates. Starting spherical ordering with the azimuth corresponds to ***half integer spin***. The azimuth runs from 0 to π radians. Starting spherical ordering with the polar angle corresponds to ***integer spin***. The polar angle runs from 0 to 2π radians. These selections add to the properties of the symmetry centers.

The model suggests that symmetry centers are maintained by special mechanisms that ensure the spatial and dynamical coherence of the coupling of the symmetry center to the background space. Several types of such mechanisms exist. Each symmetry center type corresponds to a mechanism type. ***These mechanisms are not part of the separable Hilbert space***.

Symmetry centers are ***resources*** where the coherence ensuring mechanisms can take dynamic locations that are stored in quaternionic eigenvalues of dedicated operators, in order to generate coherent location swarms that represent point-like objects. The type of the point-like object corresponds to the type of the controlling mechanism.

The basic symmetry center is independent of progression. Once created, a symmetry center persists until it is annihilated. Any progression dependence that concerns a symmetry center is handled by a type dependent mechanism. The type depends on the symmetry flavor and on the spin. Further, it depends on other characteristics that will not be treated in this paper, but that will appear as properties of the point-like object that will be supported by the controlling mechanism. An example is the generation flavor of the point-like particle. Symmetry flavor and spin can be related to ordering of the symmetry center. Generation flavor is a property of the controlling mechanism.

Symmetry centers have a well-defined spatial origin. That origin floats on the eigenspace of the reference operator $\mathcal{R}^{⓪}$. Symmetry centers are formed by a dedicated category of ***compact anti-Hermitian operators***.

An infinite dimensional separable Hilbert space can house a set of subspaces that each represent such a symmetry center. Each of these subspaces then corresponds to a dedicated spherically ordered reference operator $\mathfrak{S}^x$. The superscript ^{x} distinguishes between symmetry flavors and other properties, such as spin and generation flavor. Symmetry centers correspond to dedicated subspaces that are spanned by the eigenvectors $\{|\mathfrak{s}_i^x\rangle\}$ of the symmetry center reference operator $\mathfrak{S}^x$.

$$\mathfrak{S}^x = |\mathfrak{s}_i^x\rangle \mathfrak{s}_i^x \langle \mathfrak{s}_i^x| \tag{1}$$

$$\mathfrak{S}^{x\dagger} = -\mathfrak{S}^x \tag{2}$$

Only the location of the center inside the eigenspace of reference operator $\mathcal{R}^{⓪}$ is a progression dependent value. This value is not eigenvalue of operator $\mathfrak{S}^x$. The location of the center is eigenvalue of a central governance operator $\mathscr{g}$.

Symmetry centers feature a ***symmetry related charge*** that depends on the difference between the symmetry flavor of the symmetry center and the symmetry flavor of the reference operator $\mathcal{R}^{⓪}$, which equals the symmetry flavor of the embedding continuum $\mathfrak{C}$. The symmetry related charges raise a ***symmetry related field*** φ. The symmetry related field influences the position of the map of the symmetry center into the field that represents the embedding continuum $\mathfrak{C}$. Both fields use the eigenspace of the reference operator $\mathfrak{R}$ as their parameter space.

The closed subspaces that correspond to a symmetry center have a fixed finite dimension. This dimension is the same for all types of symmetry centers. This ensures that symmetry related charges all appear in the same short list.

Symmetry centers cover a subspace that has a fixed finite dimension. This dimension is the same for all types of symmetry centers.

6.1 Synchronization via coupling

The basic symmetry center is independent of progression. Any progression dependence that concerns a symmetry center is handled by a type dependent mechanisms that controls the usage of the symmetry center. The type dependent mechanism acts in a progression dependent fashion. On certain progression steps the mechanism selects a location from the symmetry center that will be used to embed a point-like object in the background space.

The background space, is maintained by reference operator $\mathcal{R}$. Embedding the symmetry center into the eigenspace of this operator ensures the synchronization of the symmetry center with the background space. That is why the embedding occurs at instances that are selected from the progression values, which are offered as eigenvalues by $\mathcal{R}_0 = (\mathcal{R} + \mathcal{R}^\dagger)/2$. However, the controlling mechanism does not embed the center location, but instead the mechanism uses a stochastic process in order to select a location somewhere inside the symmetry center. Further, not all eigenvalues $\{\mathfrak{s}_i^x\}$ of $\mathfrak{S}^x$ will be used in the embedding process. A special operator $\mathscr{o}$ that is dedicated to the type of the embedded point-like object describes the selections in its eigenvalues.

The embedded location represents a point-like object that resides in the symmetry center. That embedding location is mapped onto the embedding continuum, which resides as the eigenspace of operator $\mathfrak{C}$ in the Gelfand triple $\mathcal{H}$. This continuum is defined as a function $\mathfrak{C}(q)$ over parameter space $\mathfrak{R}$.

The locations in the symmetry center that for the purpose of the embedding are selected, form a coherent location swarm and a hopping path that characterize the dynamic behavior of the point-like object. The embedding process deforms continuum $\mathfrak{C}$.This embedding process is treated in more detail in [14].

7 Central governance

The eigenvalues of the central governance operator $\mathcal{g}$ administer the relative locations of the symmetry centers with respect to the reference operator $\mathcal{R}^{ⓞ}$ which resides in the separable Hilbert space $\mathfrak{H}$ and maps to the reference continuum $\mathfrak{R}^{ⓞ}$ in the Gelfand triple $\mathcal{H}$. A further map projects onto the embedding continuum $\mathfrak{C}$.The central governance operator $\mathcal{g}$ resides in the separable Hilbert space $\mathfrak{H}$.

The reference continuum $\mathfrak{R}^{ⓞ}$ acts as a parameter space of the function $\varphi(q)$ that specifies the symmetry related field φ, which is eigenspace of the corresponding operator.

Each symmetry center owns a symmetry related charge, which is located at its geometric center. Each symmetry related charge owns an individual field that contributes to the overall symmetry related field φ.

The reference continuum $\mathfrak{R}^{ⓞ}$ also acts as a parameter space of the function $\mathfrak{C}(q)$ that specifies the embedding continuum $\mathfrak{C}$, which is eigenspace of the corresponding operator $\mathfrak{C}$.

A fundamental difference exists between field φ and field $\mathfrak{C}$. However both fields obey the same quaternionic differential calculus. The difference originates from the artifacts that cause the discontinuities of the fields. In the symmetry related field φ these artifacts are the symmetry related charges. In the embedding continuum $\mathfrak{C}$ these artifacts are the embedding events. What happens in not too violent conditions will be described by the wave equation of the corresponding field and will be affected by the local and current conditions. Since the elementary point-like objects reside inside their individual symmetry center, the embedding continuum will also be affected by what happens to the symmetry centers.

Double differentiation of field φ shows the relation between φ and $\mathcal{g}$.

$$\nabla^* \nabla \varphi = \mathcal{g} \qquad (1)$$

8 Embedding symmetry centers

Together with the locations of the symmetry centers, the well-ordered eigenspace of a quaternionic normal operator $\mathcal{R}^{\circledcirc}$ that resides in an infinite dimensional separable Hilbert space acts as a reference operator from which the parameter space $\mathfrak{R}^{\circledcirc}$ of the embedding continuum $\mathfrak{C}$ will be derived. This parameter space resides as continuum eigenspace of a corresponding operator $\mathfrak{R}^{\circledcirc}$ in the Gelfand triple. This parameter space also acts as parameter space of a symmetry related field φ. It is sparsely covered by locations of symmetry centers. The central governance operator $\mathcal{g}$ administers these locations. The symmetry centers contain symmetry related charges. The locations of these charges are influenced by the symmetry related field φ.

9 Field dynamics

In the model that we selected, the dynamics of the fields are determined by quaternionic differential calculus. Apart from the eigenspaces of reference operators and the symmetry centers we encountered two fields that are defined by quaternionic functions and corresponding operators. One is the symmetry related field φ and the other is the embedding field $\mathfrak{C}$.

φ determines the dynamics of the symmetry centers. $\mathfrak{C}$ gets deformed by the recurrent embedding of point-like elementary particles that each reside on an individual symmetry center.

Apart from the way that they are affected by point-like artifacts that disrupt the continuity of the field, both fields obey, under not too violent conditions, the same differential calculus.

Under rather general conditions the change of a quaternionic function $f(q)$ can be described by:

$$df(q) = c^{\tau}\, dq_{\tau} + c^{x}\, dq_{x} + c^{y}\, dq_{y} + c^{z}\, dq_{z} = df_{\nu}(q)\boldsymbol{e}^{\nu} = \sum_{\mu=0\dots3} \frac{\partial f}{\partial q_{\mu}} dq^{\mu} = c_{\mu}(q)dq^{\mu} \tag{1}$$

Here the coefficients $c^{\mu}(q)$ are full quaternionic functions. dq_{μ} are real numbers.

Under more moderate conditions the function behaves more linearly.

$$df(q) = c_0^{\tau}\, dq_{\tau} + c_0^{x}\, \boldsymbol{i}\, dq_{x} + c_0^{y}\, \boldsymbol{j}\, dq_{y} + c_0^{z}\, \boldsymbol{k}\, dq_{z} = c_0^{\mu}(q)dq_{\mu} \tag{2}$$

Here the coefficients $c_0^{\mu}(q)$ are real functions.

Thus, in a rather flat continuum we can use the quaternionic nabla ∇.

$$\nabla = \left\{\frac{\partial}{\partial \tau}, \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right\} = \frac{\partial}{\partial \tau} + \boldsymbol{i}\frac{\partial}{\partial x} + \boldsymbol{j}\frac{\partial}{\partial y} + \boldsymbol{k}\frac{\partial}{\partial z} = \nabla_0 + \boldsymbol{\nabla} \tag{3}$$

$$\Phi = \Phi_0 + \boldsymbol{\Phi} = \nabla\psi = (\nabla_0 + \boldsymbol{\nabla})(\psi_0 + \boldsymbol{\psi}) \tag{4}$$

$$\Phi_0 = \nabla_0\psi_0 - \langle \boldsymbol{\nabla}, \boldsymbol{\psi} \rangle \tag{5}$$

$$\boldsymbol{\Phi} = \nabla_0\boldsymbol{\psi} + \boldsymbol{\nabla}\psi_0 \pm \boldsymbol{\nabla} \times \boldsymbol{\psi} \tag{6}$$

Double differentiation will then result in the quaternionic wave equation:

$$\rho = \rho_0 + \boldsymbol{\rho} = \nabla^*\nabla\psi = (\nabla_0 - \boldsymbol{\nabla})(\nabla_0 + \boldsymbol{\nabla})(\psi_0 + \boldsymbol{\psi}) = \{\nabla_0\nabla_0 + \langle \boldsymbol{\nabla}, \boldsymbol{\nabla} \rangle\}\psi \tag{7}$$
$$= \frac{\partial^2\psi}{\partial\tau^2} + \frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2}$$

Here ρ is a quaternionic function that describes the density distribution of a set of point-like artifacts that disrupt the continuity of function $\psi(q)$. However, in case of a single artifact, the function ρ will describe the corresponding Green's function.

Function $\psi(q)$ describes the mostly continuous field ψ.

The wave equation can be split into two continuity equations:

$$\Phi = \nabla\psi \tag{8}$$

$$\rho = \nabla^*\Phi \tag{9}$$

If ψ and Φ are normalizable functions and $\|\psi\| = 1$, then with real m and $\|\zeta\| = 1$

$$\nabla\psi = m\,\zeta \tag{10}$$

9.1 Fourier equivalents

In this quaternionic differential calculus, differentiation is implemented as multiplication.

This is revealed by the Fourier equivalents of the equations.

$$\tilde{\Phi} = \tilde{\Phi}_0 + \tilde{\boldsymbol{\Phi}} = p\,\tilde{\psi} = (p_0 + \boldsymbol{p})(\tilde{\psi}_0 + \tilde{\boldsymbol{\psi}}) \tag{1}$$

$$\tilde{\Phi}_0 = p_0\tilde{\psi}_0 - \langle \boldsymbol{p}, \tilde{\boldsymbol{\psi}} \rangle \tag{2}$$

$$\tilde{\boldsymbol{\Phi}} = p_0\tilde{\boldsymbol{\psi}} + \boldsymbol{p}\tilde{\psi}_0 \pm \boldsymbol{p} \times \tilde{\boldsymbol{\psi}} \tag{3}$$

The equivalent of the quaternionic wave equation is:

$$\tilde{\rho} = \tilde{\rho}_0 + \tilde{\boldsymbol{\rho}} = p^* p\ \tilde{\psi} = \{p_0 p_0 + \langle \boldsymbol{p}, \boldsymbol{p} \rangle\}\tilde{\psi} \tag{4}$$

The continuity equations result in:

$$\tilde{\Phi} = p\tilde{\psi} \tag{5}$$

$$\tilde{\rho} = p^* \tilde{\Phi} \tag{6}$$

10 Conclusion

By introducing a background space and a set of symmetry center types, this paper exploits the way in which quaternionic number systems can be ordered. This distinguishes between Cartesian ordering and spherical ordering and it reveals that these ordered versions of the number systems exist in several distinct symmetry flavors. The background space needs no origin and as a consequence it does not feature spin. The coupling of symmetry centers onto the background space offers the possibility to define an algorithm that computes corresponding symmetry related charges that are in agreement with the short list of electric charges and other discrete properties of elementary particles. For example, also the diversity of color charge and spin can be explained in this way. This indicates that elementary particles inherit these properties from the space in which they reside.

An important role is played by controlling mechanisms that are not part of the Hilbert spaces, but that make use of the Hilbert spaces as a structured storage medium. The elementary particles inherit their properties both from the Hilbert space and from these controlling mechanisms.

This paper considers the embedding field because it uses the same parameter space $\mathfrak{R}$ as the symmetry related field does. The embedding field obeys the same quaternionic differential calculus as the symmetry related field, but the triggers that cause discontinuities differ fundamentally between these fields. That is why these fields behave differently. Still both fields determine the kinematics of elementary particles. This is treated in more detail in [14].

11 Related papers

[1] Quantum logic was introduced by Garret Birkhoff and John von Neumann in their 1936 paper. G. Birkhoff and J. von Neumann, *The Logic of Quantum Mechanics*, Annals of Mathematics, Vol. 37, pp. 823–843

[2] The lattices of quantum logic and classical logic are treated in detail in: http://vixra.org/abs/1411.0175 .

[3] The Hilbert space was discovered in the first decades of the 20-th century by David Hilbert and others. http://en.wikipedia.org/wiki/Hilbert_space.

[4] In the second half of the twentieth century Constantin Piron and Maria Pia Solèr proved that the number systems that a separable Hilbert space can use must be division rings. See: “Division algebras and quantum theory” by John Baez. http://arxiv.org/abs/1101.5690 and http://www.ams.org/journals/bull/1995-32-02/S0273-0979-1995-00593-8/

[5] Paul Dirac introduced the bra-ket notation, which popularized the usage of Hilbert spaces. Dirac also introduced its delta function, which is a generalized function. Spaces of generalized functions offered continuums before the Gelfand triple arrived.

[6] In the sixties Israel Gelfand and Georgyi Shilov introduced a way to model continuums via an extension of the separable Hilbert space into a so called Gelfand triple. The Gelfand triple often gets the name rigged Hilbert space. It is a non-separable Hilbert space. http://www.encyclopediaofmath.org/index.php?title=Rigged_Hilbert_space .

[7] Potential of a Gaussian charge density: http://en.wikipedia.org/wiki/Poisson%27s_equation#Potential_of_a_Gaussian_charge_density .

[8] Quaternionic function theory and quaternionic Hilbert spaces are treated in:
http://vixra.org/abs/1411.0178 .

[9] In 1843 quaternions were discovered by Rowan Hamilton.
http://en.wikipedia.org/wiki/History_of_quaternions

Later in the twentieth century quaternions fell in oblivion.

[10] http://en.wikipedia.org/wiki/Wave_equation#Derivation_of_the_wave_equation

[11] http://en.wikipedia.org/wiki/Yukawa_potential

[12] "The Dirac equation in quaternionic format"; http://vixra.org/abs/1505.0149

[13] "Quaternionic versus Maxwell based differential calculus"; http://vixra.org/abs/1506.0111

[14] "Foundation of a mathematical model of physical reality"; http://vixra.org/abs/1502.0186 .