To the quantum theory of gravity

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Abstract

We discuss the gravitational collapse of a photon. It is shown that when the photon gets Planck energy, it turns into a black hole (as a result of interaction with the object to be measured). It is shown that three-dimensional space is a consequence of energy advantage in the formation of the Planck black holes. New uncertainty relations established on the basis of Einstein's equations. It is shown that the curvature of space-time is quantized.

1 The collapse of the photon and the Planck length

The Planck length ℓ_{P} is defined as $\ell_{\mathrm{P}} = \sqrt{\frac{\hbar G}{c^3}} \approx 1.616\ 199(97) \times 10^{-35}$ m, where c is the speed of light in a vacuum, G is the gravitational constant, and $\hbar$ is the reduced Planck constant.

Simple dimensional analysis shows that the measurement of the position of physical objects with precision to the Planck length is problematic. Indeed, we will discuss the following thought experiment. Suppose we want to determine the position of an object using electromagnetic radiation, i.e., photons. The greater the energy of photons, the shorter their wavelength and the more accurate the measurement. If the photon has enough energy to measure objects the size of the Planck length, it would collapse into a black hole and the measurement would be impossible (as a result of interaction with the object to be measured). Thus, the Planck length sets the fundamental limits on the accuracy of length measurement [1].

According to general relativity, any form of energy, including collision energy of a photon with the target, should generate a gravitational field. The higher the energy of the photon, the more powerful gravitational field is generated. We know that the photon has a kinetic energy $E_{kin} = P\,c$, where P is the photon momentum, and c its speed. This energy is positive. But the photon has, according to general relativity, gravitational (potential) energy. This energy is negative. We find its formula from the analogy with the potential energy of massive particles. For a homogeneous sphere of radius r and mass M, its gravitational energy has the form

$$E_{pot} \approx -G\,M^2/r$$

where G is the gravitational constant, M is the mass of the ball, and r its radius. But a photon has no mass M. Therefore M is replaced by the $M \to P/c$, where P is the photon momentum and c is the speed of light in a vacuum. Then the gravitational energy of the photon has the form

$$E_{pot} \approx -G\,P^2/c^2 r$$

where r is necessary to compare with the photon's wavelength λ. The total energy of the interaction of photons with the target is the sum of kinetic and potential energies and has the following form

$$E = E_{kin} + E_{pot} \approx P\,c - \frac{G\,P^2}{c^2\,\lambda} = P\,c\left(1 - \frac{G\,P}{c^3\,\lambda}\right) = P\,c\left(1 - \frac{\lambda_s}{\lambda}\right) \tag{1.1}$$

(here photon spin is not considered, but it is not essential), $\lambda_s = (G/c^3)P$ is an analogue of the gravitational radius for a massive particle $r_s \approx (G/c^3)mc$.

Consider equation (1.1) from the quantum point of view. We assume that $P\,\lambda \approx \hbar$, where $\hbar$ is the Dirac constant. Using this relation (substituting $P \approx \hbar/\lambda$), we find the function $E(\lambda)$ from the equation (1.1)

$$E(\lambda) = \frac{\hbar c}{\lambda}\left(1 - \frac{\ell_P^2}{\lambda^2}\right) \tag{1.2}$$

where $\ell_P = \sqrt{\hbar\,G/c^3}$ is the fundamental Planck length, which appears here automatically.

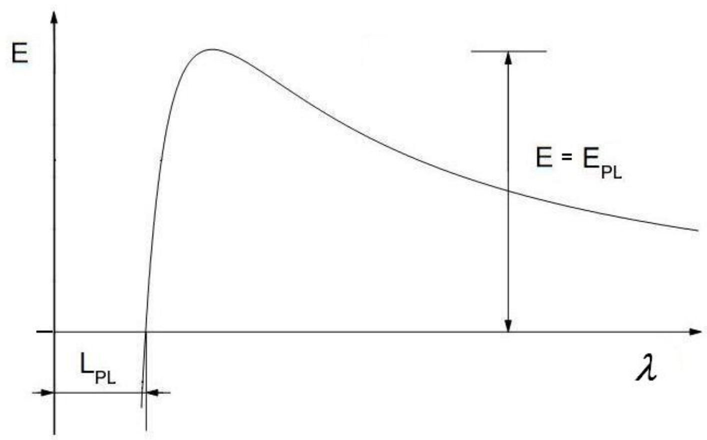

Figure 1: Graphs $E(\lambda)$ of the collapse of the photon

When we construct a graph of the function $E(\lambda)$, we can see that as the photon wavelength decreases, its energy increases, see Fig. 1. The maximum total energy $E(\lambda)$ is approximately equal to the Planck energy, where the photon wavelength is approximately equal to the Planck length. However, if the photon momentum continues to increase, its total energy begins to decrease due to the increase of the gravitational energy of the photon. When the wavelength of the photon is equal to the Planck length, its total energy is zero; The photon collapses and turns into a microscopic black hole, the hypothetical Planck particle (for example, a collision with the target).

To be more accurate, we must proceed from Hamilton-Jacobi equation [2]

$$g^{ik}\partial^2 S/\partial x^i \partial x^k = m^2\,c^2 \tag{1.3}$$

with metric coefficients g^{ik}, taken from Schwarzschild solution, where S is the action and m is the particle mass. It is a generalization of the equation between energy and momentum in special relativity $E^2 - \mathbf{p}^2c^2 = m^2c^4$. Equation (1.3) is generally covariant (physical content of equations does not depend on the choice of coordinate system). This Hamilton-Jacobi equation has the form

$$\left(1-\frac{r_s}{r}\right)^{-1}\left(\frac{\partial S}{c\partial t}\right)^2 - \left(1-\frac{r_s}{r}\right)\left(\frac{\partial S}{\partial r}\right)^2 - \frac{1}{r^2}\left(\frac{\partial S}{\partial \varphi}\right)^2 - m^2c^2 = 0 \tag{1.4}$$

It can be rewritten as follows

$$E^2 = \left(1-\frac{r_s}{r}\right)^2 P^2c^2 + \left(1-\frac{r_s}{r}\right)\frac{N^2c^2}{r^2} + \left(1-\frac{r_s}{r}\right)m^2c^4 \tag{1.5}$$

where N is the angular momentum of a particle and r_s is the gravitational radius of the central attracting body. The following assumptions are necessary for the approach above:

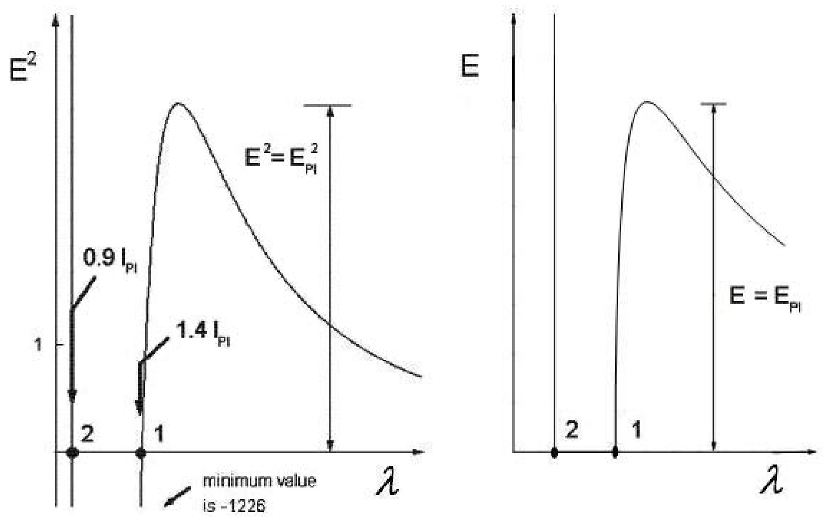

Figure 2: Graphs of the collapse of a photon with angular momentum

1) the mass of the particle m is zero, 2) angular momentum (spin of the photon) N can be neglected, 3) the Heisenberg uncertainty principle is simplified to $P\,r \approx \hbar$. We then obtain an approximate equation for the total energy

$$E \approx \left(1-\frac{r_s}{r}\right)P\,c = \left(1-\frac{2G\,M}{c^2r}\right)P\,c \approx \left(1-\frac{2\ell_P^2}{\lambda^2}\right)\frac{\hbar c}{\lambda} \tag{1.6}$$

where $r = \lambda$ is the wavelength of a photon and $r_s = 2G\,M/c^2$ is the gravitational radius. Mass M should be replaced by P/c; $P = P \approx \hbar/\lambda$ is the momentum of a photon. The resulting equation (1.6) agrees with the equation (1.2) for the total energy to within a factor of 2.

To account for the angular momentum of the photon in the above equation (1.5) it is necessary to substitute N^2 with $\hbar^2 l(l+1)$, where l is the quantum number of the total angular momentum of the photon (see Fig. 2). The angular momentum of a photon leads to the formation of internal event horizon in Planck black hole ($l = 1$, point 2).

Analysis of the Hamilton-Jacobi equation for the photon in spaces of different dimensions n indicates a preference (energy gain) for three-dimensional space for the emergence of the Planck black holes - both real and virtual (quantum foam).

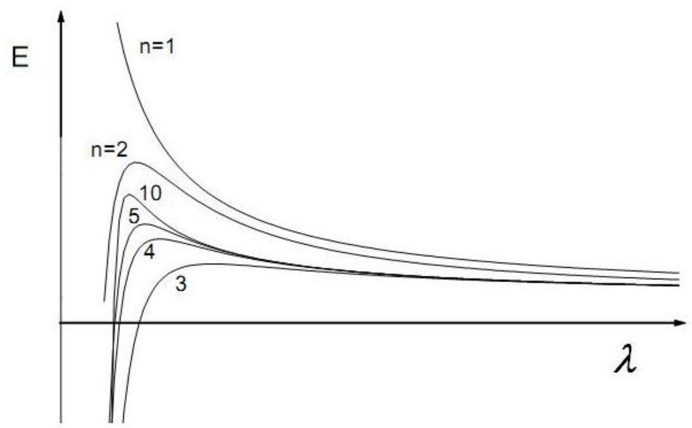

Figure 3: Graphs $E^{(n)}(\lambda)$ of the collapse of the photon in the spaces of different dimensions

Indeed, according to Ehrenfest [3], expressions for the potential energy in spaces of various dimensions are of the form

$$E_{pot}^{(n\geq 3)} \approx -\frac{k\, M^2}{(n-2)r^{n-2}}; \quad n \geq 3 \tag{1.7}$$

$$E_{pot}^{(2)} \approx k\, M^2 \ln r; \quad n = 2 \tag{1.8}$$

$$E_{pot}^{(1)} \approx k\, M^2\, r; \quad n = 1 \tag{1.9}$$

where k - the interaction constant in n-dimensional space. With the usual Newton's constant it is linked through cross-linking potentials for 3-dimensional space and the corresponding n-dimensional space.

For the potential energy of the photon, equations (1.7), (1.8), (1.9) have the form (given that $M \to P/c\,;\ P \approx \hbar/\lambda;\ r = \lambda$)

$$E_{pot}^{(n\geq 3)} \approx -\frac{k\,(P/c)^2}{(n-2)r^{n-2}} = -\frac{k\,(\hbar/\lambda\, c)^2}{(n-2)\lambda^{n-2}}; \quad n \geq 3 \tag{1.10}$$

$$E_{pot}^{(2)} \approx k\,(P/c)^2 \ln r = k\,(\hbar/\lambda\, c)^2 \ln \lambda; \quad n = 2 \tag{1.11}$$

$$E_{pot}^{(1)} \approx k\,(P/c)^2\, r = k\,(\hbar/\lambda\, c)^2\, \lambda; \quad n = 1 \tag{1.12}$$

Then the total energy of the photon is approximately equal to

$$E^{(n)}(\lambda) \approx E_{kin} + E_{pot}^{(n)}$$

where $E_{kin} = P\,c = \hbar\,c/\lambda$ on the space dimension is independent.

Graphics functions $E^{(n)}(\lambda)$ are shown in Fig. 3 (here $k = \hbar = c = 1$). Thus gain in energy, apparently, predetermined three-dimensionality of the observed space, given that the Planck virtual black holes form the so-called quantum foam, which is the foundation of the "fabric" of the Universe.

2 Heisenberg uncertainty principle at the Planck scale.

There is currently no proven physical significance of the Planck length; it is, however, a topic of theoretical research. Physical meaning of the Planck length can be determined as follows:

A particle of mass m has a reduced Compton wavelength

$$\bar{\lambda}_C = \frac{\lambda_C}{2\pi} = \frac{\hbar}{mc}$$

Schwarzschild radius of the particle is

$$r_s = \frac{2G\,m}{c^2} = \frac{2G}{c^3}m\,c$$

The product of these values is always constant and equal to

$$r_s\bar{\lambda}_C = \frac{2G\hbar}{c^3} = 2\ell_P^2$$

Accordingly, the uncertainty relation between the Schwarzschild radius of the particle and Compton wavelength of the particle will have the form

$$\Delta r_s \Delta\bar{\lambda}_C \geq \frac{G\hbar}{c^3} = \ell_P^2$$

which is another form of Heisenberg's uncertainty principle at the Planck scale. Indeed, substituting the expression for the Schwarzschild radius, we obtain

$$\Delta\left(\frac{2Gm}{c^2}\right)\Delta\bar{\lambda}_C \geq \frac{G\hbar}{c^3}$$

Reducing the same characters, we come to the Heisenberg uncertainty relation

$$\Delta\,(mc)\,\Delta\bar{\lambda}_C \geq \frac{\hbar}{2}$$

Uncertainty relation between the gravitational radius and the Compton wavelength of the particle is a special case of the general Heisenberg's uncertainty principle at the Planck scale

$$\Delta R_\mu \Delta x_\mu \geq \ell_P^2 \tag{2.1}$$

where R_μ is the radius of curvature of space-time small domain; x_μ is the coordinate small domain.

Indeed, these uncertainty relations can be obtained on the basis of Einstein's equations

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu} \tag{2.2}$$

where $G_{\mu\nu} = R_{\mu\nu} - (1/2) g_{\mu\nu} R$ is the Einstein tensor, which combines the Ricci tensor, the scalar curvature and the metric tensor, Λ is the cosmological constant, $T_{\mu\nu}$ is energy-momentum tensor of matter, π is the number, c is the speed of light, G is Newton's gravitational constant.

In the derivation of his equations, Einstein suggested that physical spacetime is Riemannian, ie curved. A small domain of it is approximately flat spacetime.

For any tensor field $N_{\mu\nu\ldots}$ value $N_{\mu\nu\ldots}\sqrt{-g}$ we may call a tensor density, where g is the determinant of the metric tensor $g_{\mu\nu}$. The integral $\int N_{\mu\nu\ldots}\sqrt{-g}\, d^4x$ is a tensor if the domain of integration is small. It is not a tensor if the domain of integration is not small, because it then consists of a sum of tensors located at different points and it does not transform in any simple way under a transformation of coordinates [4]. Here we consider only small domains. This is also true for the integration over the three-dimensional hypersurface S^ν.

Thus, Einstein's equations (2.2) for small spacetime domain can be integrated by the three-dimensional hypersurface S^ν. Have

$$\frac{1}{4\pi} \int (G_{\mu\nu} + \Lambda g_{\mu\nu}) \sqrt{-g}\, dS^\nu = \frac{2G}{c^4} \int T_{\mu\nu} \sqrt{-g}\, dS^\nu \tag{2.3}$$

Since integrable spacetime "domain" is small, we obtain the tensor equation

$$R_\mu = \frac{2G}{c^3} P_\mu \tag{2.4}$$

where $P_\mu = \frac{1}{c} \int T_{\mu\nu}\sqrt{-g}\, dS^\nu$ is the 4-momentum of matter, $R_\mu = \frac{1}{4\pi} \int (G_{\mu\nu} + \Lambda g_{\mu\nu}) \sqrt{-g}\, dS^\nu$ is the radius of curvature domain.

The resulting tensor equation can be rewritten in another form. Since $P_\mu = mc\,U_\mu$ then

$$R_\mu = \frac{2G}{c^3} mc\,U_\mu = r_s\,U_\mu \tag{2.5}$$

where r_s is the Schwarzschild radius, U_μ is the 4-speed, m is the gravitational mass. This record reveals the physical meaning of R_μ. There is a similarity between the obtained tensor equation and the expression for the gravitational radius of the body (the Schwarzschild radius). Indeed, for static spherically symmetric field and static distribution of matter have $U_0 = 1, U_i = 0\,(i = 1, 2, 3)$. In this case we obtain

$$R_0 = \frac{2G}{c^3} mc\,U_0 = \frac{2G\,m}{c^2} = r_s \tag{2.6}$$

In a small area of spacetime is almost flat and this equation can be written in the operator form

$$\hat{R}_\mu = \frac{2G}{c^3} \hat{P}_\mu = \frac{2G}{c^3} (-i\hbar) \frac{d}{dx^\mu} = -2i\,\ell_P^2 \frac{d}{dx^\mu} \tag{2.7}$$

where $\hbar$ is the Dirac constant. Then commutator operators $\hat{R}_\mu$ and $\hat{x}_\mu$ is

$$[\hat{R}_\mu, \hat{x}_\mu] = -2i\ell_P^2 \tag{2.8}$$

From here follow the specified uncertainty relations (2.1)

$$\Delta R_\mu \Delta x_\mu \geq \ell_P^2$$

Substituting the values of $R_\mu = \frac{2G}{c^3} m\, c\, U_\mu$ and $\ell_P^2 = \frac{\hbar G}{c^3}$ and cutting right and left of the same symbols, we obtain the Heisenberg uncertainty principle

$$\Delta P_\mu \Delta x_\mu = \Delta(mc\,U_\mu)\Delta x_\mu \geq \frac{\hbar}{2} \tag{2.9}$$

Note that now, according to the equation $R_\mu = (2G/c^3)\, P_\mu$, together with the expressions for the energy-momentum quantum $P_\mu = \hbar\, k_\mu$ valid expressions for the quantum space-time curvature $R_\mu = \ell_P^2\, k_\mu$ (but not quantum space-time), where k_μ - the wave 4-vector. That is, the curvature of space-time is quantized, but the quantization step is extremely small. This can serve as a basis for building a quantum theory of gravity

In the particular case of a static spherically symmetric field and static distribution of matter $U_0 = 1, U_i = 0\,(i = 1, 2, 3)$ and have remained

$$\Delta R_0 \Delta x_0 = \Delta r_s \Delta r \geq \ell_P^2 \tag{2.10}$$

where r_s is the Schwarzschild radius, r is radial coordinate.

Last uncertainty relation (2.10) allows make us some estimates of the equations of general relativity at the Planck scale. For example, the equation for the invariant interval dS in the Schwarzschild solution has the form

$$dS^2 = \left(1 - \frac{r_s}{r}\right) c^2 dt^2 - \frac{dr^2}{1 - r_s/r} - r^2(d\Omega^2 + \sin^2\Omega d\varphi^2) \tag{2.11}$$

Substitute according to the uncertainty relations $r_s \approx \ell_P^2/r$. We obtain

$$dS^2 \approx \left(1 - \frac{\ell_P^2}{r^2}\right) c^2 dt^2 - \frac{dr^2}{1 - \ell_P^2/r^2} - r^2(d\Omega^2 + \sin^2\Omega d\varphi^2) \tag{2.12}$$

It is seen that at the Planck scale $r = \ell_P$ spacetime metric is bounded below by the Planck length, and on this scale, there are real and virtual Planckian black holes [5].

Similar estimates can be made in other equations of general relativity.

It is also seen that the spacetime metric $g_{00} \approx 1 - \ell_P^2/(\Delta r)^2$ is always fluctuates even in the absence of an external gravitational field. This gives rise to the so-called quantum foam, consisting of virtual Planckian black holes. But these fluctuations $\Delta g \approx \ell_P^2/(\Delta r)^2$ in the macrocosm and in the world of atoms are very small compared to 1 and become noticeable only at the Planck scale. Fluctuations need to be considered when using the Minkowski metric of special relativity for very small regions of space and large momenta. For example, fluctuations in the speed of light is equal to the Planck scale $\Delta c = c\Delta g \approx c\,\ell_P^2/(\Delta r)^2$.

This implies that the Planck scale is the limit below which the very notions of space and length cease to exist. Any attempt to investigate the possible existence of shorter distances (less than 10^{-35}m), by performing higher-energy collisions, would inevitably result in black hole production. Higher-energy collisions, rather than splitting matter into finer pieces, would simply produce bigger black holes [6]. Reduction of the Compton wavelength of the particle increases the Schwarzschild radius. The resulting uncertainty relation generates at the Planck scale virtual black holes.

3 Summary

The paper shows that:

1. In the microcosm of the Planck length is the limit of distance.
2. Upon reaching the Planck scale appear Planck black holes.
3. At the Planck level vacuum consists of virtual Planckian black holes.
4. Length measurement is meaningless at the Planck scale
5. Three-dimensional space is a consequence of energy advantage in the formation of the Planck black holes at the Planck scale.
6. The curvature of space-time is quantized. Space-time is not quantized.

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