EXTENDED MATHEMATICS OF UNREDUCED DYNAMIC COMPLEXITY
The Exact Image of Unified Reality, from the Electron to Consciousness

ANDREI P. KIRILYUK

Solid State Theory Department
Institute of Metal Physics, Kyiv, Ukraine
E-mail: *Andrei.Kirilyuk@Gmail.com*

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Abstract

The current crisis in exact description of fundamental and applied systems has the well-defined origin and rigorously substantiated resolution in the form of qualitatively extended, unified mathematical framework of unreduced dynamic complexity. It is based on the unreduced universal solution of arbitrary interaction problem revealing the new, extended qualities with respect to traditional mathematical constructions. We describe the origin of the problem, the proposed causally complete solution and its mathematical novelties confirmed by problem-solving applications in fundamental and applied sciences.

The problem: Increasingly pressing *in*efficiency of traditional mathematics

The popular Wigner's thesis of 1960 about the "unreasonable efficiency of mathematics in the natural sciences" becomes increasingly and even catastrophically compromised in the last decades of first stagnation and then deep crisis of fundamental science, sometimes referred to as the "end of science". While one may evoke various explanations of this obvious "saturation of scientific discoveries", it is definitely related to the mathematical framework of fundamental science (and thus mathematics in the whole), just due to the strong connection between the two, even irrespective of its underlying interpretation.

Contrary to what one might assume based on the huge technological, applied-science progress of the last time, this problem of "suddenly" growing inefficiency of traditional mathematical methods of fundamental science is far from only professional scientific importance. It's rather the opposite relation that becomes the more and more evident: those apparent successes of powerful, but only empirically developing technology meet quickly growing obstacles of both internal and external origin, which can be summarised as "complexity crisis" in either technological drivers or social development around them.

While modern problems become the more and more "critically complex" due to the "global", multicomponent interaction processes involved, the dominating exact-science tools remain basically as "simple" as before, oriented to smooth, effectively one-dimensional, "integrable" behaviour and structures. The *civilisation development* itself asks now for the new power of *rigorously complete*, reliable solution to urgent practical problems of the new level of complexity determining the entire further destiny of humanity.

Even the last-time tendency in *fundamental science* based on the peaking "magic" power of high-tech observation methods shows a strange "inverted-progress effect", where the number of difficult, "unsolvable" problems, or "mysteries", remarkably grows ("dark matter", "crisis in cosmology", etc.), thus compromising the externally bright picture of spectacular science progress of the twentieth century. The accumulated "old" mysteries of science (quantum mechanics, relativity, gravity, unification, high-temperature superconductivity, etc.) also become increasingly disturbing on this background, as all of it seems to push such kind of "objective" knowledge system rather in the opposite direction of supernatural and *therefore* irresolvable enigmas. While basic science sadly declines, superstitions massively flourish, in this strange age of apparently triumphant technology.

The persisting traditional "noncomputability" of the huge spectrum of non-exact, "subjective" knowledge, from "empirical" sciences to the humanities and spiritual matters, also reappears as an integral part of the same critical problem and "supernatural" mystery, especially because the catastrophically peaking interaction power of the now "globalised", practically unified world populations, beliefs and professions urgently needs the respective qualitatively more powerful and unified knowledge basis.

The outlined crisis in today's exact sciences and knowledge system in the whole suggests only one kind of "really good" solution: the new, explicitly extended and now *intrinsically complete content* of rigorous, mathematical description of reality, leaving no place for postulated supernatural mysteries, arbitrary guesses and impasses of development. In the following sections we outline major features of such new mathematical basis for the causally complete and intrinsically sustainable scientific knowledge, in the form of extended unified mathematics of unreduced dynamic complexity (see [3-14] for detailed technical accounts).

The solution:
Extended unified mathematics of the unreduced dynamic complexity

(1) What may be missing. While looking for the causally complete mathematical framework as the solution to the problem of growing inefficiency of traditional mathematics in modern scientific research (previous section), one can start by logically asking *what may be essentially missing* in that traditional mathematics approach and results, especially as compared to real processes and phenomena. It is not really difficult to see the probable answer: usual mathematics does not propose the consistent, unreduced solution to the arbitrary, real interaction problem, while it is the development of such many-component interaction processes that gives rise to all observed phenomena and structures (of increasingly complicated configuration for modern tasks).

As usual approach tries to look for smooth and single-valued, "analytical" or "exact" (in the narrow sense) solutions, it quickly discovers that such solutions for arbitrary interaction can be found only for the strongly limited situation of one-dimensional problem equivalent to interaction of only two material points. Any real interaction in the three-dimensional world, including large numbers of interacting components and their eigenmodes for real cases of interest, falls far outside of those limits of "integrable" problems within which that kind of explicit solution provides the adequate picture of reality.

All that remains then for this conventional approach in real many-body interaction cases is to use an approximation, or "model", where the real interaction problem is replaced with one or many integrable, effectively one-dimensional problems, including the "perturbation theory" description of deviations from real system behaviour (which cannot be correct and gives typical "divergent series" as a sign of unknown unreduced solution).

It is at this point that traditional mathematics makes its (expected) fatal reduction mistake by assuming too easily (without any reasonable substantiation) that the unreduced interaction problem solution and thus the entire picture of any process and structure dynamics would somehow represent at least a *qualitatively similar* version of those reduced, effectively one-dimensional model solutions, where further details can then be adjusted in the direction of observed features by mechanical parameter variation, perturbative modifications or simple heuristic guesses about the "real" solution. This reductive modelling approach, often justified by the subjectively appreciated "beauty" of the proposed abstract structures, leads to the observed growing inefficiency of traditional mathematics, and we shall rigorously specify the origin of these limitations in the next section.

(2) Unreduced solution. If one wants (and needs!) now to return to the natural beauty of the perceived, unreduced reality in its mathematically rigorous picture, then one must restart the entire mathematical enterprise, this time from the *unreduced, non-simplified solution of real interaction problem*, describing the emerging configuration and dynamics of all natural processes and structures, on any scale [1-15].

It is not difficult to obtain a universal enough interaction problem formulation, conveniently provided by a generalised Hamiltonian equation for the system state-function, such as the Hamilton-Jacobi equation from classical mechanics or Schrödinger equation for quantum systems. We call it "existence equation" in the general case, as it does provide a simple description of the arbitrary system configuration, without any further assumptions. A more special case of open system (time-depending interaction) is easily included in the same starting formulation. Many other equations can eventually be reduced to a Hamiltonian formulation, and we self-consistently show in our further analysis of thus introduced interaction process that the results (forming the next-level system components) will always interact and behave according to the generalised, now well-specified Hamilton-Schrödinger formalism [3-14].

The key point of the problem is related to the fact that this unreduced equation for arbitrary real system with interaction (actually in any problem formulation) is "nonintegrable", i. e. it cannot be explicitly solved in that "closed" form of conventional "exact" solution.

One can approach the unreduced interaction problem solution by reformulating it in terms of so-called *optical, or effective, potential method* [1-3] originating in scattering theory and explicitly taking into account the complex, nonlinear interaction links development. One first uses the standard technique of problem expression in terms of eigen-solutions of its free components, transforming it to the equivalent (and equally nonintegrable) system of equations for respective state-function components. One tries then the substitution method for this system of equations using the Green's function technique and obtains a single equation for one state-function component of externally integrable form, but containing, instead of original interaction potential, the *effective potential (EP)* operator with the complicated nonlinear dependence on the problem solutions to be found, rendering the problem again nonintegrable, as should be expected for this equivalent transformation.

It is important, however, that this unreduced EP reveals the *new, universal quality* of the unreduced interaction problem solution designated as *fundamental dynamic multivaluedness, or redundance*, which *can* be properly, rigorously specified and after that helps to express the unreduced solution in a truly exact form accepting suitable, now properly correct approximations where necessary. This new quality of dynamic multivaluedness, constituting the key difference between the new and traditional mathematics frameworks, means that the problem has *many equally real*, locally complete and therefore *mutually incompatible* solutions called system *realisations*, each of them being generally similar to the *unique* solution for the emerging system configuration assumed in the effectively one-dimensional, *dynamically single-valued* approach of usual analysis. The redundant solution multiplication originates in the mentioned nonlinear dependence of the unreduced EP on the problem eigenvalues, which leads to the essential growth of the maximum eigenvalue power in the characteristic equation determining the total solution number [2-14].

Being equally real but mutually incompatible, multiple system realisations thus obtained are forced, by the same driving interaction, to *permanently replace* each other, appearing and disappearing in *causally random* order thus rigorously (and dynamically) defined. We simultaneously obtain therefore the rigorously defined notions of *event*, (real) *change*, *emergence* and the well-specified *origin of physically real, unstoppable* (changing realisations) *and irreversible* (truly random realisation choice) *time flow*. The *dynamically emerging, naturally discrete space* elements are also rigorously obtained as respective eigenvalue separations for the unreduced EP formalism [3-6,12-14].

Moreover, during each of system transitions between its plural "regular" realisations containing its *dynamically entangled* components (or degrees of freedom) the system is forced to transiently disentangle them within a special intermediate realisation, before their new, generally somewhat different dynamic entanglement within the next emerging, randomly chosen realisation. This intermediate, or "main", realisation with transiently disentangled, quasi-free components and chaotically fluctuating structure represents the *universal* extension of thus *causally explained* quantum-mechanical wavefunction and all statistical "distribution functions" at higher interaction levels (including also the wave-like brainfunction at the level of unconscious and conscious brain dynamics [9]). We therefore call the main system realisation thus explicitly obtained in the unreduced interaction analysis the *generalised wavefunction*, or *distribution function*, for any interaction level. It is naturally provided with the rigorously derived and physically transparent *generalised Born rule*, expressing the regular realisation emergence probability as the corresponding value of the generalised wavefunction (or its modulus squared for quantum and other wave-like levels).

While all these results are derived by the rigorous analysis of the unreduced interaction process in the generalised EP method [1-14] (in two independent forms, algebraic and graphical [2,3]), one can provide also a simple transparent explanation of the origin and universality of dynamic multivaluedness by considering a schematic interaction picture between two, for example attracting, objects with N components (or dynamical modes) each. While in the ordinary, dynamically single-valued analysis one would obtain only the same number N of respective binary component or mode combinations, forming elementary solutions (eigen-solutions), in the unreduced interaction analysis one takes into account the complete number N^2 of component or mode combinations, which gives the N-fold redundance with respect to the same number of "places" N for the interaction results. One obtains thus the unstoppable system realisation change in causally random order.

This permanent realisation change implies also the omnipresent *dynamic instability* within any real system due to the unreduced and thus always self-amplifying interaction between multiple components and modes trying to "rearrange" the metastable configuration of any realisation. On the other hand, the fundamental *dynamic discreteness, or quantisation*, of realisations (and thus of any real system dynamics) is also finally due to this self-amplifying, holistic character of the unreduced interaction, which can stop its current tendency only around a metastable system configuration in a regular realisation (which provides another definition of the latter).

The derived causal, dynamic randomness of realisation emergence in any interaction process provides also the *purely dynamic origin* and *a priori* values of *probabilities* of realisation emergence [3-14]. As all elementary realisations are equally real, the probability α_r of the r-th realisation emergence is given by

$$\alpha_r = \frac{1}{N_\Re}, \quad \sum_{r=1}^{N_\Re} \alpha_r = 1 , \tag{1a}$$

where $N_\Re$ is the total realisation number (determined by the number of suitable system component or their eigenmode combinations). Since for many real observations of "self-organised" dynamic regimes with densely packed groups of similar elementary realisations (see below) one deals rather with compound realisations containing many elementary ones, in the general case this dynamic probability definition takes the form

$$\alpha_r = \frac{N_r}{N_\Re} \quad \left(N_r = 1, \ldots, N_\Re ; \ \sum_r N_r = N_\Re \right), \quad \sum_r \alpha_r = 1 , \tag{1b}$$

where N_r is the number of elementary realisations within the r-th actually observed, compound realisation. This original definition of realisation probability correlates, of course, with the above *generalised Born rule* for practical calculation of the same dynamic probabilities from the generalised wavefunction $\Psi(x)$, $\alpha_r = |\Psi(x_r)|^2$.

It is important to emphasize that we obtain in this way the absolutely *universal origin and meaning of randomness* in this world, as well as the universal and dynamic meaning and values of related *probability*, extending essentially the respective formal, non-dynamical concepts of conventional science and mathematics. We see that randomness is omnipresent in real structures (though sometimes in a hidden , "self-organised" form, see below) and can only be of dynamic and genuine, "noncomputable" and "undecidable" character.

Correspondingly, the *general, now truly complete solution* of any real interaction problem, expressed in terms of measured system density, $\rho(x)$, is obtained in the form of *dynamical-*

ly probabilistic sum of individual realisation densities $\{\rho_r(x)\}$ [3-14]:

$$\rho(x) = \sum_{r=1}^{N_{\Re}}{}^{\oplus} \rho_r(x) \ , \tag{2}$$

where the sign $\oplus$ marks the special, dynamically probabilistic meaning of the sum implying that particular summands $\rho_r(x)$ appear there probabilistically, with the corresponding *dynamic* probabilities α_r, so that the result is a probabilistically fluctuating function. In the limit of long enough observations, with large numbers of realisation emergence events, the measured system density tends to the stationary expectation value:

$$\rho(x) = \sum_r \alpha_r \rho_r(x) \ . \tag{3}$$

It is important, however, that contrary to conventional, dynamically single-valued mathematics, our description does not depend on any such "statistical" assumptions, so that the universal dynamic-probability expressions (1)-(2) remain valid and well-defined for any *single* event of realisation emergence and even *before* any event happens at all.

Another qualitatively new feature of the unreduced problem solution (3) is the multilevel, *probabilistically fractal* structure of the complete realisation set, so that summations in (2) and (3) actually include a multilevel hierarchy of sums, especially for higher-complexity systems with many interacting components and eigenmodes. The dynamically multivalued fractal of the complete problem solution is explicitly obtained within the above unreduced EP method [3-5,8,9] and in general (except rare special cases) does not possess the simplified scale symmetry of usual, abstract fractals. It realises instead the absolutely exact and realistic universal symmetry of complexity (see below), including *permanent probabilistic motion* of fractal "branches", which provides the important property of *dynamic adaptability* and related *huge power* of unreduced interaction dynamics, underlying the "magic" features of life and intelligence [4,5,8-12]. If we consider the most fundamental interaction level giving rise to the unified world structure emergence (attraction of two initially homogeneous protofields [3,5,11,13]), then the emerging dynamically multivalued fractal of this process represents the *single, dynamically unified and exact structure of the Universe*.

We can now provide the *universal definition* of the main quantity of *dynamic complexity* C of *any* real structure or process in the form of any growing function of the number of system realisations or rate of their change, equal to zero for the (unrealistic) case of only one realisation (exclusively considered in the usual framework):

$$C = C(N_{\Re}),\ dC/dN_{\Re} > 0,\ C(1) = 0 \ . \tag{4}$$

Examples include $C(N_{\Re}) = C_0 \ln N_{\Re}$, $C(N_{\Re}) = C_0(N_{\Re} - 1)$, generalised energy-mass (temporal rate of realisation change) and momentum (spatial rate of realisation emergence) [3-6,9-14]. It is evident that thus defined complexity automatically includes equally universally defined dynamic randomness, or *chaoticity*, due to the common underlying phenomenon of plural, equally real and mutually incompatible realisations.

We emphasize the universally positive value of this unreduced dynamic complexity of real, dynamically multivalued world structures of any level, starting already from (massive) elementary particles, in strong contrast to vague ideas of complexity in usual science, where it is "intuitively" attributed to externally "sophisticated" structures and phenomena. As to observed externally regular motions and rigid shapes of various objects, they represent but a particular dynamic regime of "self-organised" internal chaoticity of multiple, but similar realisations. In the general case of multi-level dynamics we deal here with the limit of *dynamically multivalued SOC* (self-organised criticality). In the opposite limit of sufficiently different realisations emerging with comparable probabilities we obtain the regime of *global, or uniform, chaos*. The rigorous analysis of the unreduced EP formalism provides the well-defined *criterion of global chaos onset* (the *resonance* between major system motions) and the gradual transition to the opposite limit of multivalued SOC, actually spanning the *entire variety of observed dynamic regimes* and behaviour patterns [2-5,7,9,11,12].

Since the universal complexity thus defined is determined by the system initial configuration (through the total realisation number) it is *conserved* in any system structure emergence and transformation. Something, however, does change during interaction process development, and this change can be rigorously described as unstoppable *transformation* of the initial, potential (hidden) form of dynamic complexity, or *dynamic information*, *I*, to the final, unfolded-structure form of *dynamic entropy*, *S*, which preserves their sum, the total dynamic complexity C: $C = I + S$, $\Delta C = 0, \Delta S = -\Delta I > 0$. Thus obtained *complexity conservation law* includes the generalised and equally universal *entropy growth law* (extended second law of thermodynamics), which now describes also all cases of *externally regular* structure formation, thus solving various respective problems of conventional dynamically single-valued science [3-6,9-14].

As this unified complexity conservation law describes the dynamic symmetry between system realisations practically implemented by system motion in real time, we call it the *universal symmetry of complexity*. It is not difficult to show that the extended, now universal quantity of *action* is a major integral measure of complexity, expressing the above dynamic information, and then one can obtain the differential expression of the universal symmetry of complexity in the form of extended Hamilton-Jacobi equation for action and the related generalised Schrödinger equation for the generalised wavefunction (intermediate realisation) [3-6,9-14]. The obtained *unified Hamilton-Schrödinger formalism* is applicable to any system behaviour and represents the extended generalisation of various "model" dynamic equations. The underlying universal symmetry of complexity also unifies the extended versions of all known, usually postulated laws and principles [3-6,9-14].

Note that the obtained intrinsically complete picture of the dynamically multivalued interaction process development is a qualitative and explicit *extension* of usual, dynamically single-valued description, including all conventional notions of "complexity", "chaoticity", "self-organisation", "attractors", "multistability", etc., which represent however intricate but always dynamically single-valued, point-like, zero-complexity *projection* of the unreduced, dynamically multivalued reality. We also call the entire content of that traditional dynamically single-valued projection *unitary science* (and paradigm), as it neglects the qualitatively inhomogeneous transitions between multiple system realisations and preserves only smooth pseudo-evolution (without real time) within only one realisation (often originating in the described intermediate realisation of the generalised wavefunction). In this way, the *universal science of complexity* presented here clearly specifies the origin and avoids various manifestations of the intrinsic *incompleteness* of usual mathematical framework of science, including the famous Gödel's incompleteness.

(3) Extended mathematics of complexity. We can now provide a concise outline of proper mathematical novelties of the universal science of complexity thus obtained as a result of unreduced solution of arbitrary many-body interaction problem [3,4,8,11,15]:

(i) *Non-uniqueness* of any real problem solution, in the form of *fundamental dynamic multivaluedness (redundance)* of system realisations, as opposed to conventional uniqueness theorems and solution type. Note the essential difference of our *dynamic*, interaction-driven multivaluedness from usual multivalued functions, as well as various unitary imitations of "multistability" within the single solution.

(ii) Omnipresent, universal, dynamic and genuine *randomness* due to equally real and incompatible realisation change in causally random order, providing clear understanding of usual vague notions of nonintegrability, nonseparability, noncomputability, uncertainty (indeterminacy), undecidability, stochasticity, broken symmetry, free will, etc. Truly regular structures, motions and patterns are *absent*, while they exclusively prevail in traditional mathematical framework, including its dynamically single-valued *imitations* of randomness and chaoticity.

(iii) The *absence* of self-identity, $\mathfrak{A} = \mathfrak{A}$, for any structure $\mathfrak{A}$, tacitly assumed in traditional mathematics. In real world and in the new mathematics of complexity we have instead $\mathfrak{A} \neq \mathfrak{A}$ (cf. eq. (2)), giving permanent irreversible change and causal time flow.

(iv) Fractally structured multivalued *dynamic entanglement* of interacting system components in the unreduced problem solution, providing the *rigorous* mathematical definition of the perceived *quality* (or texture) of emerging structures, as opposed to purely abstract, "immaterial" character of usual mathematical structures and models.

(v) *Dynamic discreteness*, or *causal quantisation*, of the unreduced interaction results and dynamics (and thus any real structure and process) eventually due to its *holistic* character, giving rise to qualitatively inhomogeneous, *nonunitary* system evolution, with the opposite quality of the traditional mathematical framework.

One should add to these features the essentially, dynamically *unified* character of the new mathematics of complexity expressed by its *single, unified structure of dynamically probabilistic fractal* and *single, unified law of the universal symmetry of complexity*, which give rise to the entire, now dynamically unified variety of all particular structures, objects, laws and principles. In particular, one obtains the naturally unified picture of reality at the fundamental, lowest complexity levels of elementary particles and fields, where this unification includes the causally specified dynamic origin of particles, their properties, fundamental interaction forces and constants, quantum and relativistic behaviour [3-5,11,13].

Applications and conclusion: The unlimited efficiency of the extended mathematics of complexity

We can only briefly refer to various already realised applications of the universal science of complexity at different complexity levels, from the unified, causally complete fundamental physics to life sciences, intelligence and consciousness, complex computer systems, sustainability and civilisation development [1-15], confirming its problem-solving power, which clearly originates just in the described "dynamically multivalued" extension of traditional dynamically single-valued framework.

The critically acute modern problems of "complexity crisis" in both fundamental science and real world development (see the first section) find thus their universally applicable solution within the framework of extended mathematics of complexity presented above, and we also show why exactly they cannot be solved within the artificially restricted unitary science paradigm. All the real-world structures are obtained as a result of unreduced interaction processes, and it is just the right moment now to extend our scientifically rigorous description of the emerging strong-interaction, "globalised" reality to the causally complete, provably reliable understanding of the underlying complex, multivalued and intrinsically creative interaction dynamics.

References

[1] A. P. Kirilyuk, "Theory of charged particle scattering in crystals by the generalized optical potential method", *Nuclear Instruments and Methods B* **69** (1992) 200-231.

[2] A. P. Kirilyuk, "Quantum chaos and fundamental multivaluedness of dynamical functions", *Annales de la Fondation Louis de Broglie* **21** (1996) 455-480. ArXiv:quant-ph/9511034, ArXiv:quant-ph/9511035.

[3] A. P. Kirilyuk, *Universal Concept of Complexity by the Dynamic Redundance Paradigm: Causal Randomness, Complete Wave Mechanics, and the Ultimate Unification of Knowledge* (Naukova Dumka, Kyiv, 1997). ArXiv:physics/9806002.

[4] A. P. Kirilyuk, "Universal Science of Complexity: Consistent Understanding of Ecological, Living and Intelligent System Dynamics", *ArXiv:0706.3219* (2007), p. 1-13.

[5] A. P. Kirilyuk, *Complex Dynamics of Real Quantum, Classical and Hybrid Micro-Machines* (LAP LAMBERT Academic Publishing, Saarbrücken, 2013). ArXiv:physics/0211071.

[6] A. P. Kirilyuk, "Universal Symmetry of Complexity and Its Manifestations at Different Levels of World Dynamics", *Proceedings of Institute of Mathematics of NAS of Ukraine* **50** (2004) 821-828. ArXiv:physics/0404006.

[7] A. P. Kirilyuk, "Dynamically Multivalued Self-Organisation and Probabilistic Structure Formation Processes", *Solid State Phenomena* **97-98** (2004) 21-26. ArXiv:physics/0405063.

[8] A. P. Kirilyuk, "Complex-Dynamical Extension of the Fractal Paradigm and Its Applications in Life Sciences", In: *Fractals in Biology and Medicine, Vol. IV*, ed. by G. A. Losa et al. (Birkhäuser, Basel, 2005), p. 233-244. ArXiv:physics/0502133. ArXiv:physics/0305119.

[9] A. P. Kirilyuk, *Complex-Dynamic Origin of Consciousness and Sustainability Transition* (LAP LAMBERT Academic Publishing, Saarbrücken, 2014). ArXiv:physics/0409140.

[10] A. P. Kirilyuk, "Towards Sustainable Future by Transition to the Next Level Civilisation", In: *The Future of Life and the Future of Our Civilisation*, ed. by V. Burdyuzha (Springer, Dordrecht, 2006), p. 411-435. ArXiv:physics/0509234.

[11] A. P. Kirilyuk, "Consistent Cosmology, Dynamic Relativity and Causal Quantum Mechanics as Unified Manifestations of the Symmetry of Complexity", *ArXiv:physics/0601140* (2006) 1-46. In Russian: *Nanosystems, Nanomaterials, Nanotechnologies* **11** (2013) 437-517.

[12] A. P. Kirilyuk, "Unreduced Dynamic Complexity: Towards the Unified Science of Intelligent Communication Networks and Software", In: *IFIP, Vol. 229, Network Control and Engineering for QoS, Security, and Mobility, IV*, ed. by D. Gaïti (Springer, Boston, 2007) p. 1-20. ArXiv:physics/0603132. ArXiv:0910.5495.

[13] A. P. Kirilyuk, "Complex-Dynamical Solution to Many-Body Interaction Problem and Its Applications in Fundamental Physics", *Nanosystems, Nanomaterials, Nanotechnologies* **10** (2012) 217-280. ArXiv:1204.3460.

[14] A. P. Kirilyuk, "Dynamic Origin of Evolution and Social Transformation", *Nanosystems, Nanomaterials, Nanotechnologies* **11** (2013) 1-21. hal-00760100, ArXiv:1212.1939.

[15] A. P. Kirilyuk, "The Last Scientific Revolution", In: *Against the Tide: A Critical Review by Scientists of How Physics and Astronomy Get Done*, ed. by M. Lopez Corredoira and C. Castro Perelman (Universal Publishers, Boca Raton, 2008), p. 179-217. ArXiv:0705.4562.