# Unreduced Complex Dynamics of Real Computer and Control Systems

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***Abstract*—The unreduced dynamic complexity of modern computer, production, communication and control systems has become essential and cannot be efficiently simulated any more by traditional, basically regular models. We propose the universal concept of dynamic complexity and chaoticity of any real interaction process based on the unreduced solution of the many-body problem by the generalised effective potential method. We show then how the obtained mathematically exact novelties of system behaviour can be applied to the development of qualitatively new, complex-dynamical kind of computer and control systems.**

## I. Introduction

The construction of intelligent production, control, computer and communication systems (e. g. [1]–[5]) has now entered into a qualitatively new stage, with the effective intensity of interactions involved having crossed the "complexity threshold", after which none of multiple interaction links in a versatile system can be neglected without essential loss to the system dynamics [6], [7]. Under these conditions the rigorously defined *unreduced dynamic complexity* of intelligent system behaviour, including strong *dynamic randomness*, becomes essential and cannot be efficiently simulated by usual, basically regular dynamics of control and computer systems.

In this report we describe the rigorous framework of the unreduced dynamic complexity of any real many-body system with ubiquitous interaction [6]–[12] (section II) and then review its application to modern computer systems of intelligent control and communication, leading to what we call "complexity revolution" in their design and use (section III).

## II. Unreduced Dynamic Complexity, Emergence and Randomness in a Many-Body System with Arbitrary Interaction

### A. Interaction Problem Solution by the Generalised Effective Potential Method

We describe the unreduced dynamics of a real system with arbitrary interacting entities starting from the "existence equation", which is a universal, Hamiltonian (and nonintegrable) expression of a closed system configuration [6]–[12]:

$$\left\{ \sum_{k=0}^{N} \left[ h_k(q_k) + \sum_{l>k}^{N} V_{kl}(q_k, q_l) \right] \right\} \Psi(Q) = E\Psi(Q), \quad (1)$$

where $h_k(q_k)$ is the "generalised Hamiltonian" (later specified as a complexity measure) for the isolated $k$-th system component, $q_k$ stands for the $k$-th component degree(s) of freedom, $V_{kl}(q_k, q_l)$ is the arbitrary interaction potential for the $k$-th and $l$-th components, $Q \equiv \{q_0, q_1, ..., q_N\}$, $\Psi(Q)$ is the system state-function expressing its configuration, $E$ is the generalised Hamiltonian eigenvalue (energy), and summations are performed over all ($N$) system components. This dynamic equation covers also (after the standard transformation) the less fundamental case of time-dependent formalism.

We then separate in (1) some "common" degree(s) of freedom, $q_0 \equiv \xi$, describing e. g. system's spatial configuration:

$$\begin{gathered} \{h_0(\xi) + \sum_{k=1}^{N} [h_k(q_k) + V_{0k}(\xi, q_k) + \\ + \sum_{l>k}^{N} V_{kl}(q_k, q_l)]\} \Psi(\xi, Q) = E\Psi(\xi, Q), \end{gathered} \quad (2)$$

where now $Q \equiv \{q_1, ..., q_N\}$ and $k, l \geq 1$. This form is convenient for the standard problem expression in terms of the known eigen-solutions for the free system components:

$$h_k(q_k)\varphi_{kn_k}(q_k) = \varepsilon_{n_k}\varphi_{kn_k}(q_k), \quad (3)$$

$$\begin{gathered} \Psi(\xi, Q) = \sum_n \psi_n(\xi)\varphi_{1n_1}(q_1)...\varphi_{Nn_N}(q_N) \equiv \\ \equiv \sum_n \psi_n(\xi)\Phi_n(Q), \end{gathered} \quad (4)$$

where $\{\varphi_{kn_k}(q_k), \varepsilon_{n_k}\}$ is the complete set of orthonormal eigenfunctions and eigenvalues of the $k$-th component Hamiltonian $h_k(q_k)$, $n \equiv \{n_1, ..., n_N\}$ runs through all eigenstate combinations, and $\Phi_n(Q) \equiv \varphi_{1n_1}(q_1)...\varphi_{Nn_N}(q_N)$.

Substituting expansion (4) into (2) we obtain the system of equations for $\{\psi_n(\xi)\}$ in a standard way [6], [8], [10], [11]:

$$\begin{gathered} [h_0(\xi) + V_{00}(\xi)]\psi_0(\xi) + \sum_n V_{0n}(\xi)\psi_n(\xi) = \eta\psi_0(\xi) \\ [h_0(\xi) + V_{nn}(\xi)]\psi_n(\xi) + \sum_{n' \neq n} V_{nn'}(\xi)\psi_{n'}(\xi) = \\ = \eta_n\psi_n(\xi) - V_{n0}(\xi)\psi_0(\xi), \end{gathered} \quad (5)$$

where $n, n' \neq 0$, $\eta \equiv \eta_0 = E - \varepsilon_0$, $\eta_n = E - \varepsilon_n$, $\varepsilon_n = \sum_k \varepsilon_{n_k}$,

$$V_{nn'}(\xi) = \sum_k \left[ V_{k0}^{nn'}(\xi) + \sum_{l>k} V_{kl}^{nn'} \right], \quad (6)$$

$$V_{k0}^{nn'}(\xi) = \int_{\Omega_Q} dQ \Phi_n^*(Q) V_{k0}(q_k, \xi) \Phi_{n'}(Q), \quad (7)$$

$$V_{kl}^{nn'}(\xi) = \int_{\Omega_Q} dQ \Phi_n^*(Q) V_{kl}(q_k, q_l) \Phi_{n'}(Q), \quad (8)$$

and we have isolated the equation for the generalised "ground state" $\psi_0(\xi)$ (with minimum complexity defined below). The obtained system of equations (5) is equivalent to the starting existence equation (1)–(2) and as well "nonintegrable".

We now try to solve this nonintegrable system of equations (5) with the help of the generalised effective, or optical, potential method [13], [14], where one expresses $\psi_n(\xi)$ through $\psi_0(\xi)$ from the equations for $\psi_n(\xi)$ in (5) using the standard Green function technique and then inserts the result into the equation for $\psi_0(\xi)$, obtaining thus the externally "integrable" *effective existence equation* [6], [8], [10]–[12]:

$$h_0(\xi)\,\psi_0(\xi) + V_{\text{eff}}(\xi;\eta)\,\psi_0(\xi) = \eta\psi_0(\xi)\,, \qquad (9)$$

with the *effective potential (EP)* operator $V_{\text{eff}}(\xi;\eta)$ given by

$$\begin{gathered} V_{\text{eff}}(\xi;\eta) = V_{00}(\xi) + \hat{V}(\xi;\eta)\,, \\ \hat{V}(\xi;\eta)\,\psi_0(\xi) = \int_{\Omega_\xi} d\xi' V(\xi,\xi';\eta)\psi_0(\xi')\,, \end{gathered} \qquad (10)$$

$$V(\xi,\xi';\eta) = \sum_{n,i} \frac{V_{0n}(\xi)\,\psi^0_{ni}(\xi)\,V_{n0}(\xi')\,\psi^{0*}_{ni}(\xi')}{\eta - \eta^0_{ni} - \varepsilon_{n0}}\,, \qquad (11)$$

where $\varepsilon_{n0} \equiv \varepsilon_n - \varepsilon_0$ and $\left\{\psi^0_{ni}(\xi), \eta^0_{ni}\right\}$ is the complete set of eigen-solutions of a *truncated* system of equations:

$$[h_0(\xi) + V_{nn}(\xi)]\,\psi_n(\xi) + \sum_{n' \neq n} V_{nn'}(\xi)\psi_{n'}(\xi) = \eta_n\psi_n(\xi)\,. \qquad (12)$$

The eigenfunctions, $\{\psi_{0i}(\xi)\}$, and eigenvalues, $\{\eta_i\}$, of the effective equation (9) are used to obtain other state-function components:

$$\psi_{ni}(\xi) = \hat{g}_{ni}(\xi)\,\psi_{0i}(\xi) \equiv \int_{\Omega_\xi} d\xi' g_{ni}(\xi,\xi')\,\psi_{0i}(\xi')\,, \qquad (13)$$

$$g_{ni}(\xi,\xi') = V_{n0}(\xi') \sum_{i'} \frac{\psi^0_{ni'}(\xi)\,\psi^{0*}_{ni'}(\xi')}{\eta_i - \eta^0_{ni'} - \varepsilon_{n0}}\,, \qquad (14)$$

and the total system state-function $\Psi(\xi, Q)$ (see (4)):

$$\Psi(\xi,Q) = \sum_i c_i \left[\Phi_0(Q) + \sum_n \Phi_n(Q)\,\hat{g}_{ni}(\xi)\right] \psi_{0i}(\xi)\,, \qquad (15)$$

where coefficients $c_i$ are determined by the state-function matching conditions at the boundary where interaction vanishes. The measured system density $\rho(\xi, Q)$ is obtained as state-function squared modulus, $\rho(\xi,Q) = |\Psi(\xi,Q)|^2$ (for "wave-like" levels), or as state-function itself, $\rho(\xi,Q) = \Psi(\xi,Q)$ (for "particle-like" structures) [8], [10], [11].

Although the EP problem formulation, (9)–(11), remains as nonintegrable as the initial formulation, (1), (2), (5), the interaction *dynamical links* in the EP version reveal the *qualitatively new properties* of unreduced the problem solution, leading to its full, correctly adjustable form [6]–[12], [14].

The key property of any real interaction result (9)–(15) is its *dynamic multivaluedness, or redundance*, meaning that one has a *redundant* number of individually complete and therefore *mutually incompatible* solutions describing *equally real* system configurations, or *realisations*. This major property of system realisation (solution) plurality, underlying the new mathematics of complexity [8], [10], [11], [15]–[17] and the imminent complexity revolution in modern computer and control systems (section III), is due to the nonlinear and self-consistent dependence of the unreduced EP, (9)–(11), on the solutions to be found, which leads to the growth of the highest power of the characteristic equation determining the eigen-solution number and reflects the physically obvious plurality of interacting eigen-mode combinations [6]–[12], [14]–[16].

If $N_\xi$ and $N_q$ are the numbers of terms in sums over $i$ and $n$ in (11), then the total eigenvalue number of equation (9) is $N_{\max} = N_\xi(N_\xi N_q + 1) = (N_\xi)^2 N_q + N_\xi$, giving the $N_\xi$-fold redundance of the usual "complete" set of $N_\xi N_q$ eigen-solutions of equations (5) plus an additional, "incomplete" set of $N_\xi$ solutions. It means that the number of "regular", locally complete realisations is $N_\Re = N_\xi$, while the additional set of $N_\xi$ solutions forms a special, "intermediate" realisation serving as the transitional state in system jumps between regular realisations and provides the universal, *causally complete* extension of the quantum-mechanical *wavefunction* and classical *(probability) distribution function* [8], [10]–[12], [16].

In a simplified scheme of pair-wise attraction between two objects with $N$ modes/elements each [16], the total number of direct "interaction links", $N^2$, reflects the number of all "eigen-solutions", while we still have only $N$ "accessible places" for any emerging configuration. The system is forced to permanently "switch" between its $N_\Re = N^2/N = N$ incompatible realisations with $N$ elements each. This provides an estimate for the realisation number $N_\Re$ as being equal to the number $N$ of system elements/eigenmodes, while in more complicated cases it is determined by the number of combinations of system interaction links, $N!$ (see section III).

The same property of fundamental dynamic multivaluedness of any real interaction is confirmed by the straightforward graphical analysis of the EP equation [8], [14], [18].

The fundamental dynamic multivaluedness thus rigorously derived for any real interaction process implies the intrinsic property of *causal, or dynamic, randomness* within any real system, in the form of its *permanently changing* realisations, which are forced (by the same driving interaction) to replace each other in *truly random* (unpredictable, undecidable, non-computable) order thus naturally defined. This *omnipresent* randomness in any, even *externally* regular system behaviour provides the universal, consistent version of *(dynamical) chaos*, which is essentially different from any its version in usual, *dynamically single-valued*, exact-solution, or *unitary*, description inevitably reduced to "involved regularity", including *incorrectly* assumed "exponential amplification of deviations" (as a result of invalid extension of a perturbation theory approximation) [8].

It means that the complete *general solution* of arbitrary interaction problem is given by the *dynamically probabilistic* sum of system density values for separate realisations:

$$\rho(\xi,Q) = \sum_{r=1}^{N_\Re}{}^{\oplus}\, \rho_r(\xi,Q), \qquad (16)$$

where the summation is performed over all system realisations, $N_\Re$ is their number, and the $\oplus$ sign designates the special, dynamically probabilistic meaning of the sum. It implies that any measured quantity (16) is *intrinsically unstable* (even for a totally isolated system) and its current value *will* unpredictably change to that of another, *randomly* chosen realisation. Such kind of *permanently unstable* dynamics is readily observed in nature and underlies the phenomenon of life itself [8], [10]–[12], [15], but is avoided in unitary theory and traditional technological/control systems, where it is associated with linear "noncomputability" (e. g. [19]) and technical failure. The omnipresent dynamic multivaluedness from the unreduced interaction problem solution forms thus the unique basis for the truly consistent, *causally complete* understanding of biological and artificial "bio-inspired" and "intelligent" systems, where causal randomness can now be transformed from an obstacle to the key advantage (section III).

Thus obtained causal randomness of the unreduced problem solution (9)–(16) is accompanied by the *dynamic probability definition*. As elementary system realisations are equal in their "right to emerge", the dynamically derived, *a priori probability* of $r$-th realisation emergence, $\alpha_r$, is given by

$$\alpha_r = \frac{1}{N_\Re}\ , \quad \sum_r \alpha_r = 1\,. \tag{17}$$

Actual observations often deal with "self-organised" groups of similar, practically indiscernible elementary realisations (see section II-B). The dynamic probability of such $r$-th *compound realisation* is naturally determined by the number, $N_r$, of elementary realisations it contains:

$$\alpha_r\left(N_r\right) = \frac{N_r}{N_\Re}\ \left(N_r = 1, ..., N_\Re;\ \sum_r N_r = N_\Re\right). \tag{18}$$

The stationary *expectation value*, $\rho_{\exp}\left(\xi, Q\right)$, is directly obtained from (16)–(18) for statistically large event numbers:

$$\rho_{\exp}\left(\xi, Q\right) = \sum_r \alpha_r \rho_r\left(\xi, Q\right). \tag{19}$$

However, contrary to usual theory, our *dynamically derived* randomness and probability (16)–(18) remain valid for any *single* event of realisation emergence and even *before* it happens. The dynamic realisation probability distribution is obtained also from the *Born probability rule* for the *generalised wavefunction* (section II-B) [8], [11], [12], [15], [16].

Another major property of any real interaction process is the *dynamic entanglement* of system components (degrees of freedom) in each realisation, described by the dynamically weighted eigenfunction products with different degrees of freedom $(\xi, Q)$ in the state-function expression (15). It provides the well-specified meaning of "interaction" itself and the *mathematically exact* version of the tangible *quality*, or texture, of the emerging system structure, which is absent in unitary models dealing with abstract, "immaterial" entities.

The obtained *dynamically multivalued entanglement* of the unreduced interaction result describes a *living* structure, permanently changing, developing and *probabilistically adapting* its tangible configuration, thus providing the rigorous definition of (structure) *emergence* and a well-specified basis for biomedical and bio-inspired technology applications (section III). The properties of dynamically multivalued entanglement and adaptability are further amplified due to the complex-dynamical, *probabilistic fractality* of the unreduced general solution [8], [10]–[12], [15] obtained by application of the same EP method to solution of the truncated system of equations (12) from the first-level EP expression (11) (see section II-B).

Now we can *universally* define *dynamic complexity*, $C$, of *any* real system or interaction process as a growing function of the number of its *explicitly obtained* realisations, or rate of their change, equal to zero for the (unrealistic) case of only one realisation [6]–[8], [10]–[12], [15], [16], [18]:

$$C = C(N_\Re)\ ,\ dC/dN_\Re > 0\ ,\ C(1) = 0\,. \tag{20}$$

Dynamic complexity examples are provided by $C\left(N_\Re\right) = C_0 \ln N_\Re$, $C\left(N_\Re\right) = C_0\left(N_\Re - 1\right)$, generalised energy/mass (temporal rate of realisation change) and momentum (spatial rate of realisation emergence) (section II-B). It becomes clear that the entire *dynamically single-valued* paradigm of usual theory (including its versions of "complexity", "chaos" and *imitations* of "multi-stability" in *abstract* "spaces") corresponds to exactly *zero* value of unreduced complexity equivalent to effectively zero-dimensional, *point-like projection* of reality. The proposed universal concept of complexity and its applications appear thus as the explicit and causally complete *extension* of usual theory to the unreduced, dynamically multivalued picture of reality. In particular, the above unified definition of dynamical randomness shows that the unreduced dynamic chaoticity and complexity are closely related and practically synonymous features of any real interaction process.

Thus universally defined complex behaviour involves *essential, or dynamic, nonlinearity*. It is provided by feedback links of developing interaction as they are expressed by the EP dependence on the problem solutions (see (9)–(11)). It is the *dynamically emerging* nonlinearity, since it appears even for a formally "linear" initial problem expression (1)–(2), (5), whereas usual, mechanistic "nonlinearity" is but a perturbative reduction of this essential nonlinearity of the unreduced EP formalism (see also section II-B). Essential nonlinearity leads to irreducible *dynamic instability* of *any* system state: both are determined by the same interaction feedback mechanism.

### *B. Probabilistic dynamic fractality, unified dynamic regimes and the symmetry of complexity*

A more involved feature of *dynamically multivalued, or probabilistic, fractal* (of emerging system structure) appears as a result of partial incompleteness of the first-level solution (9)–(19) relying upon the yet unknown solutions of the truncated system of equations (12). We can now apply the same unreduced EP method to solution of this truncated problem, which gives the second-level effective equation resembling the first-level equation (9):

$$\left[h_0\left(\xi\right) + V_{\text{eff}}^n\left(\xi; \eta_n\right)\right]\psi_n\left(\xi\right) = \eta_n \psi_n\left(\xi\right), \tag{21}$$

where the second-level EP $V_{\text{eff}}^n\left(\xi; \eta_n\right)$ is similar to its first-level version (10)–(11):

$$V_{\text{eff}}^n\left(\xi;\eta_n\right)\psi_n\left(\xi\right) = V_{nn}\left(\xi\right)\psi_n\left(\xi\right) + \\ + \sum_{n' \neq n, i} \frac{V_{nn'}\left(\xi\right)\psi_{n'i}^{0n}\left(\xi\right)\int\limits_{\Omega_\xi} d\xi' \psi_{n'i}^{0n*}\left(\xi'\right)V_{n'n}\left(\xi'\right)\psi_n\left(\xi'\right)}{\eta_n - \eta_{n'i}^{0n} + \varepsilon_{n0} - \varepsilon_{n'0}}\,, \tag{22}$$

and $\left\{\psi_{n'i}^{0n}(\xi), \eta_{n'i}^{0n}\right\}$ is the complete eigen-solution set of the second-level truncated system:

$$h_0(\xi)\psi_{n'}(\xi) + \sum_{n''\neq n'} V_{n'n''}(\xi)\psi_{n''}(\xi) = \eta_{n'}\psi_{n'}(\xi), \quad (23)$$

with $n' \neq n, 0$. Similar to the first-level EP (10)–(12), its second-level version also splits into many incompatible realisations (numbered by index $r'$) due to the self-consistent dependence on the eigen-solutions to be found, leading to respective splitting of the first-level truncated system solutions:

$$\left\{\psi_{ni}^{0}(\xi), \eta_{ni}^{0}\right\} \rightarrow \left\{\psi_{ni}^{0r'}(\xi), \eta_{ni}^{0r'}\right\}. \quad (24)$$

This hierarchical dynamical splitting of emerging system structure progresses with ever more truncated auxiliary systems of equations till the last, exactly solvable system (of two equations). Substituting the dynamically multivalued solutions of each truncated system to the previous-level EP, we get the dynamically probabilistic fractal of the now *truly complete problem solution* in the form of multilevel hierarchy of probabilistically changing realisations:

$$\rho(\xi, Q) = \sum_{r,r',r''...}^{N_{\Re}} {}^{\oplus} \rho_{rr'r''...}(\xi, Q), \quad (25)$$

where indexes $r, r', r'', ...$ enumerate realisations at consecutive levels of dynamically probabilistic fractality. Similar to the dynamic probabilities of realisation emergence events of the first level, (17)–(18), we obtain the hierarchy of *causal realisation probabilities* $\{\alpha_{rr'r''...}\}$ for all levels of dynamically multivalued fractal:

$$\alpha_{rr'r''...} = \frac{N_{rr'r''...}}{N_{\Re}}, \quad \sum_{rr'r''...} N_{rr'r''...} = N_{\Re}. \quad (26)$$

The *expectation value* of the dynamically probabilistic fractal density of the complete problem solution is obtained as:

$$\rho_{\exp}(\xi, Q) = \sum_{r,r',r''...}^{N_{\Re}} \alpha_{rr'r''...}\rho_{rr'r''...}(\xi, Q). \quad (27)$$

The dynamically probabilistic fractal is the *essential extension* of conventional fractals: the latter are not solutions to any real interaction problems and show the simplified "scale symmetry" and basic regularity. By contrast, our dynamically multivalued fractal in general does not possess the scale invariance (with approximate exceptions for limited scale ranges) and realises instead the much deeper law of the universal symmetry of complexity (see below). It is also different from any approximate (and usually diverging) "series expansion": the possibly long, but finite sums of the dynamically probabilistic fractal solution (25), (27) provide the *exact* version of the *real* multilevel system structure.

The universality of our analysis shows that the entire world structure emerges as a single, physically unified dynamical fractal of the underlying simplest interaction between two primal entities ("protofields"), with all the observed properties and laws at all levels of the world structure rigorously derived as emergent features of that unified fractal dynamics [6]–[12], [14]–[18] (see also below). Among those properties one may note especially the *dynamic adaptability* related to the interactive dynamic origin of probabilistic realisation change of the multilevel fractal structure (absent in any unitary description). The high power of the related process of "sensible search" of the optimal structure creation underlie, in particular, the "magic" properties of life and intelligence expressed by the huge exponential growth of the fractal realisation number $N_{\Re}$ and complexity (20), which is of special interest to the complex computer and control systems (section III).

The multivalued fractal dynamics also unifies *all possible dynamic regimes in one classification*, from strong chaoticity to external regularity [8]–[11], [15], [20]. One limiting case called *uniform, or global, chaos* is obtained from the main EP formalism (9)–(15) in the form of essentially different realisations with a quasi-homogeneous probability distribution, i.e. $N_r \approx 1$ and $\alpha_r \approx 1/N_{\Re}$ for all $r$ in (18). It emerges when energy level separations or frequencies of intra-component and inter-component motions are close to each other, which leads to a strong "conflict of interest" and the resulting "huge disorder", without any dominating regular-motion component.

The opposite extreme regime of *multivalued self-organisation or self-organised criticality (SOC)* emerges for sufficiently different interaction frequencies, so that, as follows from (10), (15), one or few rigid, low-frequency components "enslave" a great number of high-frequency and rapidly changing, but configurationally similar realisations (i.e. the realisation probability distribution is highly inhomogeneous, $N_r \sim N_{\Re}$), while the EP (9)–(10) and state-function (15) operators approach local functions [8]–[11], [20]. However, the difference of that extended, multivalued SOC from usual self-organisation and SOC is important: despite the quasi-regular *external* system shape in this regime, it confines an intense "internal life" and *chaos* of changing "enslaved" realisations (which are *not* superposable unitary "modes"). This is the unique key to consistent solution of the well-known entropy-growth problems, in particular for living and bio-inspired systems (see also below). Another important advance is that this real, multivalued SOC unifies the extended versions of a whole variety of separated unitary "models", including usual "self-organisation" (or "synergetics"), SOC, "synchronisation", "control of chaos", "attractors", and "mode locking".

All occurring dynamic regimes fall between these limiting cases of uniform chaos and multivalued SOC (including their multi-level combinations), and they emerge at respective intermediate parameter values. The point of transition to the global chaos regime is given by the *universal criterion of global chaos onset* derived from the unreduced EP formalism (9)–(15):

$$\kappa \equiv \frac{\Delta\eta_i}{\Delta\eta_n} = \frac{\omega_\xi}{\omega_q} \cong 1, \quad (28)$$

where $\kappa$ is the *chaoticity* parameter, $\Delta\eta_i$, $\omega_\xi$ and $\Delta\eta_n \sim \Delta\varepsilon$, $\omega_q$ are energy-level separations and frequencies for the inter-component and intra-component motions, respectively. At $\kappa \ll 1$ one has the externally regular multivalued SOC regime, which degenerates into global chaos as $\kappa$ grows from 0 to 1, and the maximum irregularity at $\kappa \approx 1$ is again transformed to a SOC kind of structure at $\kappa \gg 1$ (with the "inverse" system configuration). One can compare the unified and physically transparent criterion of chaos onset of equation (28) with various nonuniversal and contradictory criteria and definitions of chaoticity from unitary theory, such as "overlapping resonances", "positive Lyapunov exponents", "multistability", "coexisting attractors", or "unstable periodic orbits", all of them referring to the dynamically single-valued and thus basically regular problem description (see [8], [11] for more details). In particular, our criterion (28) remains valid for the case of *quantum chaos*, where it describes the emergence of genuine quantum dynamic randomness, in agreement with the quantum-classical correspondence principle [8], [11], [18], whereas usual theory fails to find any true quantum chaos.

The obtained unified criterion of chaos (28) provides also the extended meaning of the phenomenon of resonance as the condition of strong chaoticity of system dynamics (absent in unitary understanding of resonance). The same analysis of the unreduced EP equations reveals a similar role of higher resonances as "sources of increased chaoticity", so that when chaoticity $\kappa$ grows from 0 (quasi-regularity) to 1 (global chaos), the degree of randomness makes a higher jump each time $\kappa$ passes through a higher resonance, $\kappa = m/n$ , with integer $n > m$ [8], [11], [18], [20]. As those ever higher (and weaker) resonances form a dense network of rational values of $\kappa$, we obtain a well-specified version of the "fractal structure of chaos", here in the system parameter space. This chaos-inducing role of resonance expressed by (28) is important for computer and control system applications (section III).

The dynamically multivalued fractal is thus the *unified structure* of the world or any its part exactly described by the unreduced interaction problem solution (9)–(27) and containing various dynamic regimes between global chaos and multivalued SOC. There is also the *single, unifying law* of dynamic existence and development of this world structure, the universal *complexity conservation law*. It originates in the fact that the system realisation number underlying its dynamic complexity according to (20) is determined by the initial system structure (its number of component eigenmode combinations) and therefore remains unchanged during any further system evolution.

However, while the total dynamic complexity does not change, a related quality should change in the process of structure development. It is easy to see that as branches and levels of the dynamical fractal emerge in this process, the *potential form* of interaction complexity, or *dynamic information* $I$, is transformed to its realised, *unfolded form* of *dynamic entropy* $S$, so that their sum, the *total dynamic complexity* $C = I + S$ remains unchanged, $\Delta C = 0,\ \Delta I = -\Delta S < 0$ [8], [10], [11], [15], [16], [20], [21]. Both complexity forms are measured, of course, in the same way, by suitable functions of realisation number (equation (20) and below). Their change only reflects progressive system structure emergence and development.

Contrary to unitary conservation laws, here the dynamic *symmetry* between changing realisations and their number *conservation* mean *the same*, so that there is no difference any more between a "symmetry" and the respective "conservation law" (cf. "Noether's theorem"), and we obtain the *universal symmetry of complexity* implying *complexity conservation by transformation from dynamic information to dynamic entropy*. Another difference from unitary symmetries is that the latter reflect abstract and "ideal" (regular) structure transformations and therefore often become "broken" in real world, while the universal symmetry of complexity does the opposite by relating quite *irregular* realisation structures within the *absolutely exact* symmetry of complexity, which is thus *never violated* (as it should be for a genuine symmetry law). It also unifies the extended, complex-dynamical versions of *all* (correct) symmetries and laws (see below) separated in usual theory.

In order to obtain a useful dynamic expression of the universal symmetry of complexity, we introduce the unified definition of elementary complexity forms known as *time* and *space*, now *explicitly emerging* in interaction process [8], [10], [11], [15], [16], [20], [21]. The *space element*, or *elementary size*, $\Delta x$, is given by the eigenvalue separation of the unreduced EP formalism (9)–(12), $\Delta x = \Delta \eta_i^r$, where the separation of eigenvalues within the same realisation (numbered by $i$) provides the *space point size*, $r_0 \simeq \Delta x_i = \Delta_i \eta_i^r$, while the separation of eigenvalues from neighbouring realisations (numbered by $r$) gives the elementary length (smallest distance between points), $\lambda \simeq \Delta x_r = \Delta_r \eta_i^r$. The *elementary time interval*, $\Delta t$, is obtained as *intensity*, specified as *frequency*, $\nu$, of universally defined *events* of realisation change, $\Delta t = \tau = 1/\nu$. Because of unstoppable realisation change in dynamically random order, the resulting *time flow* is also *unstoppable* and *irreversible*, $\Delta t > 0$. Whereas the events and time flow result from the dynamic multivaluedness of real interaction, a useful expression for $\Delta t = \tau$ is based on the above elementary length $\lambda = \Delta x_r$ and the (known) velocity $v_0$ of signal propagation in the interaction component material, $\tau = \lambda / v_0$.

As the emergent time and space intervals characterise the realisation change process, while dynamic complexity (20) is a growing function of realisation number, it becomes clear that a *fundamental complexity measure* is provided by the simplest combination of space and time variables, known as *action*, $\mathcal{A}$, which acquires now the extended, universal and complex-dynamical meaning [8], [10], [11], [15], [16], [20], [21]:

$$\Delta \mathcal{A} = p \Delta x - E \Delta t \,, \tag{29}$$

where the coefficients $p$ and $E$ are recognised as (now extended) *momentum* and (total) *energy*:

$$p = \left. \frac{\Delta \mathcal{A}}{\Delta x} \right|_{t=\mathrm{const}} \simeq \frac{\mathcal{A}_0}{\lambda} \,, \tag{30}$$

$$E = - \left. \frac{\Delta \mathcal{A}}{\Delta t} \right|_{x=\mathrm{const}} \simeq \frac{\mathcal{A}_0}{\tau} \,, \tag{31}$$

$\mathcal{A}_0$ being the characteristic action magnitude at the considered complexity level, and the evident vector versions of all relations are implied if necessary. We see that the extended action is a universal *integral complexity measure*, while momentum and energy are unified *differential complexity measures*.

Because of the above irreversible time flow ($\Delta t > 0$) and positive total energy ($E > 0$), action can only decrease with time, $\Delta \mathcal{A} < 0$ (see (31)). Due to the dynamically random realisation choice, it measures a *consumable*, irreversibly decreasing complexity form coinciding thus with the dynamic information $I$ from the above symmetry of complexity, $\mathcal{A} = I$ (we shall also call it *complexity-action*). Conservation of total complexity $C = I + S$ can now be expressed as

$$\Delta C = \Delta \mathcal{A} + \Delta S = 0 \,,\ \Delta S = -\Delta \mathcal{A} > 0 \,, \tag{32}$$

where the dynamic entropy, or *complexity-entropy*, $S$ can only grow, at the expense of complexity-action $\mathcal{A}$, thus specifying the time arrow direction and providing the extended, *universal* versions of the *second law of thermodynamics* (energy degradation principle) and the *least-action principle*, applicable now to *any* system dynamics [8], [10], [11], [15], [16], [20], [21].

We can now obtain the desired dynamic expression of the symmetry of complexity (32) by dividing it by $\Delta t|_{x=\text{const}}$:

$$\frac{\Delta \mathcal{A}}{\Delta t}|_{x=\text{const}} + H\left(x, \frac{\Delta \mathcal{A}}{\Delta x}|_{t=\text{const}}, t\right) = 0, \ H = E > 0, \tag{33}$$

where the *generalised Hamiltonian*, $H = H(x,p,t)$, considered as a function of emerging space coordinates $x$, momentum $p = (\Delta \mathcal{A}/\Delta x)|_{t=\text{const}}$ (see (30)) and time $t$, expresses the unfolded, entropy-like form of differential complexity, $H = (\Delta S/\Delta t)|_{x=\text{const}}$, while the last inequality reflects the generalised second law (or the time arrow direction), in agreement with energy definition (31). We obtain thus the differential dynamic expression of the symmetry of complexity in the form of *generalised*, universally applicable *Hamilton-Jacobi equation* revealing its true, *complex-dynamical* origin. The finite-increment form of (33) reflects the natural *discreteness* of multivalued interaction dynamics and will tend to the continuous limit in suitable cases. The generalised Hamilton-Jacobi equation takes a simpler form for conservative (closed) systems with time-independent Hamiltonians:

$$H\left(x, \frac{\Delta \mathcal{A}}{\Delta x}|_{t=\text{const}}\right) = E\,, \tag{34}$$

with the conserved total energy $E$ defined by equation (31).

The dynamic entropy growth law constituting an integral part of the universal symmetry of complexity (32)–(33) can be amplified with the help of *generalised Lagrangian*, $L$, defined as the total (discrete) time derivative of complexity-action $\mathcal{A}$:

$$L = \frac{\Delta \mathcal{A}}{\Delta t} = \frac{\Delta \mathcal{A}}{\Delta t}|_{x=\text{const}} + \frac{\Delta \mathcal{A}}{\Delta x}|_{t=\text{const}}\frac{\Delta x}{\Delta t} = pv - H, \tag{35}$$

where $v = \Delta x/\Delta t$ is the velocity of global system motion as a whole. Irreducible dynamic randomness of realisation choice at every step of system dynamics implies permanent *decrease* of dynamic information, or complexity-action, equivalent to dynamic entropy growth, (32), meaning that

$$L < 0\,, \quad E, H(x,p,t) > pv \geq 0\,. \tag{36}$$

As noted above, it is important that now this "generalised second law" refers to both externally chaotic and *externally regular* structure emergence thus solving the respective entropy-growth problems of usual unitary theory.

The generalised Hamilton-Jacobi equation (33)–(34) describing the evolution and behaviour of the ensemble of "regular" system realisations has an important complement dealing with the dynamics of special, "intermediate" realisation revealed in the unreduced EP formalism (section II-A) and forming the transitional state of briefly disentangled, quasi-free system components before they take the next regular, properly entangled realisation. This intermediate realisation and state, the *generalised wavefunction* $\Psi(x)$, is the realistic and universal extension of the quantum-mechanical wavefunction and various distribution functions from unitary theory. It has a chaotically fluctuating structure due to the dynamically random emergence of regular realisations whose dynamic probability obeys both the main rule of the unreduced EP formalism (17)–(18) and the *generalised Born rule* causally following from this transitional role of the generalised wavefunction and rigorously obtained from the above matching conditions for the state-function coefficients $c_i$ in (15) [8], [10], [11], [16], [20]:

$$\alpha_r = \alpha(x_r) = |\Psi(x_r)|^2\,, \tag{37}$$

where $x_r$ is the $r$-th realisation configuration and for particle-like complexity levels one should imply the generalised distribution function itself at the right-hand side (instead of its modulus squared for wave-like complexity levels).

Now, in order to find the dynamic equation for $\Psi(x)$ similar to the Hamilton-Jacobi equation (33)–(34) for regular realisations, we can use the *causal quantisation condition* following from the symmetry of complexity applied now to one cycle of transition from the wavefunction to a regular realisation and back [8], [10]–[12], [16], [20]:

$$\Delta(\mathcal{A}\Psi) = 0\,, \quad \Delta \mathcal{A} = -\mathcal{A}_0 \frac{\Delta \Psi}{\Psi}\,, \tag{38}$$

where $\mathcal{A}_0$ is a characteristic complexity-action magnitude that here may contain a numerical constant reflecting specific features of the considered complexity sublevels (thus at quantum sublevels $\mathcal{A}_0 = i\hbar$, where $\hbar = h/2\pi$ is Planck's constant). Using relation (38) in the Hamilton-Jacobi equation (33), we obtain the causally derived *universal Schrödinger equation* for the *realistically interpreted* generalised wavefunction at *any* complexity level (starting from the lowest, quantum levels, now liberated from all postulated "mysteries" [8], [11], [16], [20]):

$$\mathcal{A}_0 \frac{\Delta \Psi}{\Delta t}|_{x=\text{const}} = \hat{H}(x,\hat{p},t)\,\Psi(x,t)\,, \tag{39}$$

$$\hat{p} = -\mathcal{A}_0 \frac{\Delta}{\Delta x}|_{t=\text{const}},$$

where the momentum operator $\hat{p}$ and the Hamiltonian operator, $\hat{H}(x,\hat{p},t)$, are obtained from momentum $p$ and the Hamiltonian function $H = H(x,p,t)$ of (30), (33) by the same causal quantisation (38). For the closed system case we similarly obtain from (34) the stationary Schrödinger equation:

$$\hat{H}(x,\hat{p})\,\Psi(x) = E\Psi(x)\,. \tag{40}$$

This causally derived and now complete dynamic expression of the universal symmetry of complexity, the *universal Hamilton-Schrödinger formalism* (33)–(40) does apply, together with the initial expression (32), to any system dynamics (*thus justifying* the Hamiltonian form of the initial existence equation (1)) and therefore underlies *any* (correct) law, "principle" and equation from unitary theory (usually postulated in a semi-empirical way). For a direct demonstration, we can expand the Hamiltonian $\hat{H}(x,\hat{p},t)$ in (39) in a power series of $\hat{p}$ (and $\Psi$), which gives (for the continuous-derivative notation):

$$\begin{gathered}\frac{\partial \Psi}{\partial t} + \sum_{\substack{m=0\\n=1}}^{\infty} h_{mn}(x,t)\,[\Psi(x,t)]^m \frac{\partial^n \Psi}{\partial x^n} + \\ + \sum_{m=0}^{\infty} h_{m0}(x,t)\,[\Psi(x,t)]^{m+1} = 0\,,\end{gathered} \tag{41}$$

where $h_{mn}(x,t)$ are arbitrary functions and the dependence on $\Psi$ may arise from the EP. We see that various usual model equations are but particular cases of (41) providing thus their true, causally specified origin, including the complex-*dynamic* origin of *any*, usually postulated *nonlinearity* (with similar results for a series expansion in (33), (34) and (40)). Details for quantum, relativistic and other laws can be found elsewhere [8]–[12], [15], [16], [20], [21].

## III. Complexity Transition and the Main Principles of Intelligent Production, Computer and Control Systems Operation and Design

As shown in the previous section, any real system with multiple interacting objects possesses the irreducible dynamic complexity appearing as a fractally structured hierarchy of permanently and chaotically changing realisations. Usual engineering approach to the operation and design of complex production, computer, communication and control systems tries to treat their dynamics semi-empirically and find "order in chaos" by actually using the unreduced complexity of *human* designer and operator brain involved. However, with today's critically growing complexity and intensity of the whole system dynamics (one could call it "generalised globalisation"), one cannot rely any more on human factors and must ask for the adequately complex, or "intelligent", *autonomous* dynamics of controlling systems. We call this important change *complexity transition, or revolution*, and shall specify its content and perspectives in this section based on the above rigorous framework of the universal science of complexity.

Mathematically, the complexity transition is expressed by the main features of the unreduced interaction process from section II, including (1) the fundamental dynamic multivaluedness (i. e. *non*-uniqueness) of *any real* problem solution, (2) the related omnipresent and *genuine* dynamic randomness, even within *externally* regular processes and structures, providing consistent understanding of usually vague notions of *nonintegrability*, *nonseparability*, *noncomputability*, *uncertainty (indeterminacy)*, *undecidability*, stochasticity, broken symmetry, free will, etc. (cf. e. g. [19]), (3) the *absence* of usually assumed self-identity, $\mathfrak{A} = \mathfrak{A}$, for any structure $\mathfrak{A}$, leading to irreversible change (time flow), (4) fractally structured multivalued *dynamic entanglement* of interacting system components determining the perceived *quality* of emerging structures, and (5) *dynamic discreteness*, or *causal quantisation*, of the unreduced interaction results and dynamics (due eventually to its *holistic* character), giving rise to qualitatively inhomogeneous, *nonunitary* system evolution.

The obtained *qualitatively new mathematics of complexity* [8], [10], [15]–[17] is summarised by the *dynamically probabilistic fractal* (25)–(27) as its single, dynamically unified structure and the *universal symmetry of complexity* (32)–(40) as the unique law determining this structure evolution and dynamics (including all the observed local dynamic regimes and laws, now properly classified). In fact, this qualitatively new structure and symmetry constitute the main guiding principle of complex-dynamic system engineering, underlying all its particular principles and laws.

More applied aspects of the necessary intelligent, complex-dynamical operation and design of production, computer, communication and control systems are provided by the following *major principles* specifying the universal symmetry of complexity [6], [7], [10]–[12], [22]:

(I) The *complexity correspondence principle* implies efficient or sensible interaction mainly between systems of comparable dynamic complexity. This direct corollary to the universal symmetry of complexity means, in particular, that a system of certain complexity can be efficiently designed and controlled only by systems and techniques of higher, but not lower, dynamic complexity, with various important applications to intelligent system design and related sustainability problems [6], [7], [10]–[12], [22], [23]. In fact, this principle underlies the general, now *rigorously* substantiated necessity of the above complexity revolution in both technological and social development practices.

(II) The *complex-dynamical control principle* is based on the complexity-transformation aspect of the universal symmetry of complexity (section II-B) and states that any truly efficient and sustainable control implies suitable *complexity development* (of both controlled and controlling systems), with inevitable *partially random change*, in contrast to "limiting" or "fixing" approach of usual, unitary control theory (including its formally complex-dynamical aspects, such as "chaos control"). Proper control leading to genuine system sustainability is reduced thus to design and monitoring of *optimal interaction complexity development* (rather than its maximum restriction in usual approach), emphasizing intrinsic *creativity* aspects of unreduced complex dynamics. It is stability through (suitable) development (including the necessary chaotic deviations), instead of traditional regularity limitation.

(III) The *unreduced (free) interaction principle* refers to the exponentially huge power and efficiency of natural, multicomponent system interaction processes, as opposed to their only power-law efficiency considered within the conventional unitary-model projection [6], [7], [10]–[12], [15], [16], [22]. Referring to the self-developing dynamically multivalued fractal of unreduced interaction process (section II-B), one can easily understand that its maximum operation power $P_{\rm real}$, determined by the total (fractal) realisation number $N_{\Re}$ (proportional to the unreduced complexity $C$), can be estimated as the number of system link combinations:

$$P_{\rm real} \propto N_{\Re} = L! \to \sqrt{2\pi L}\left(\frac{L}{e}\right)^L \sim L^L \ggg L\,, \qquad (42)$$

where the number of system links $L$ can already be a very large number, essentially exceeding the number of interacting system components $N$ (thus for both human brain and genome $N > 10^{10}$, $L > 10^{14} \gg N$). The obtained exponentially huge power of unreduced complex dynamics $P_{\rm real}$, dramatically exceeding its unitary-model estimates, $P_{\rm reg} \propto L^{\beta}$, $\beta \sim 1$, $P_{\rm real}/P_{\rm reg} \sim L^{L-\beta} \to \infty$, provides the origin of the "miraculous" properties of life, intelligence and consciousness, which now should and *can* be reproduced in artificial "bio-inspired" computer, production and control systems. In particular, the *genuine* artificial intelligence and machine consciousness become indispensable to modern technological system control and can only be realised using these principles of unreduced complex dynamics [6], [7], [11], [12].

Technical realisation of major principles I–III of complex artificial system dynamics and other manifestations of the universal symmetry of complexity (section II and items (1)-(5) above) will include such important features as (a) qualitatively new, *teleological, or purposeful, programming*, (b) *creative intermittency of chaoticity and self-organisation*, and (c) genuine, *autonomous sustainability* in system operation and development, which reproduce the respective features of natural system dynamics in their unreduced efficiency:

(a) Teleological (purposeful) programming relies on the complexity transformation from dynamic information to dynamic entropy as the unique realisation of the universal symmetry of complexity and the *intrinsic purpose* of system development (section II-B). Therefore, instead of regular-sequence programming of usual computer and control systems, in teleological programming one will impose a hierarchy of suitable purposes to achieve while letting the system itself to choose optimal ways of their fulfilment with the help of complex-dynamic adaptability of unreduced interaction processes. One can profit here from the exponentially huge power of the probabilistic fractal dynamics (item (III) above).

(b) Creative intermittency of chaoticity and self-organisation is based on the controlled use of the unified criterion (28) of strong chaoticity onset around system resonances, where one can ensure the *efficient chaotic search* for the desired self-organised states (including system purposes from (a)), with the optimal intermittency of such strongly chaotic and quasi-regular stages of structure development.

(c) Autonomous sustainability of system development implies realisation of the huge power of intelligent complex dynamics (42) for maintenance of failure-proof and unlimited, in principle, technological system development taking into account e. g. *adaptable ageing* effects.

Further applications of these features and principles (1)–(5), I–III, (a)–(c) will eventually include the entire, *inevitably unified techno-human process* of truly sustainable, noosphere civilisation *and* intelligence development, where one can emphasize the mentioned cases of complex-dynamic ecology, biomedical and bio-inspired ICT technologies, artificial intelligence and consciousness systems, with the urgent need for the complexity transition based on the suitable fundamental framework [6], [7], [10]–[12], [22], [23].

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