

The Mathematical Formula Of The Causal Relationship k

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Abstract: The deterministic relationship between cause and effect is deeply connected with our understanding of the physical sciences and their explanatory ambitions. Though progress is being made, the lack of theoretical predictions and experiments in quantum gravity makes it difficult to use empirical evidence to justify a theory of causality at quantum level in normal circumstances, i. e. by predicting the value of a well-confirmed experimental result. For a variety of reasons, the problem of the deterministic relationship between cause and effect is related to basic problems of physics as such. Despite the common belief, it is a remarkable fact that a theory of causality should be consistent with a theory of everything and is because of this linked to problems of a theory of everything. Thus far, solving the problem of causality can help to solve the problems of the theory of everything (at quantum level) too.

Key words: Cause, Effect, Cause and effect, causal relationship, causality.

1. Introduction

On the one hand, as already mentioned above, and this may not come as a surprise, it is highly desirable to formulate a quantum mechanical version of the relationship between cause and effect. But at least one of the difficult questions that chaos theory raises for the epistemology of determinism of the relationship between cause and effect, can there exist a deterministic relationship between a cause and an effect at all. In other words, what is necessity, what is randomness? Quantum gravity for instance, can provide us a completely new view concerning the most fundamental of all relationships, the deterministic relationship between the cause and the effect. Although numerous attempts have been made in this topic, there is no commonly accepted solution of quantum gravity up to the present day. Research in quantum gravity, extremely difficult due to the missing close relationship between theory and experiment, is owing both, a technical and a conceptual difficulty too. A non-negligible minority of the physicist focus their attention on what is now called loop quantum gravity while the majority of the physicists is working in the field called string theory. Thus far, there is no single, generally agreed theory in quantum gravity. However, it is still quite unclear, in principle and even in practice, how to make any concrete predictions in these theories. Under these conditions, quantum gravity and the deterministic relationship between a cause and an effect appear to be intimately connected with one another. The solution of the problems of causation can help to solve the problems of quantum gravity too.

2. Definitions

2.1. Definition. The Expectation Value And The Variance Of A Random Variable X

Let ${}_R X$ denote a random variable which can take the value ${}_0 X$ with the probability $p({}_0 X)$, the value ${}_1 X$ with the probability $p({}_1 X)$ and so on up to value ${}_N X$ with the probability $p({}_N X)$. Then the expectation of a single random variable ${}_i X$ is defined as

$$E({}_i X) \equiv p({}_i X) \times {}_i X \quad (1)$$

while the expectation value of the population $E({}_R X)$ is defined as

$$E({}_R X) \equiv E({}_0 X) + E({}_1 X) + \dots + E({}_N X) \quad (2)$$

More important, all probabilities p_i add up to one ($p_0 + p_1 + \dots + p_N = 1$). Quite naturally, the expected value can be viewed something like the weighted average, with p_i 's being the weights.

$$E({}_R X) \equiv \frac{p({}_0 X) \times {}_0 X + p({}_1 X) \times {}_1 X + \dots + p({}_N X) \times {}_N X}{p({}_0 X) + p({}_1 X) + \dots + p({}_N X)} \quad (3)$$

Under conditions where all outcomes ${}_i X$ are equally likely (that is, $p_0 = p_1 = \dots = p_N$), the weighted average turns finally into a simple average. Let ${}_i X^*$ denote the complex conjugate of the random variable ${}_i X$. The complex conjugate of a random variable ${}_i X^*$ is defined as

$${}_i X^* \equiv \frac{E({}_i X)}{{}_i X^2} \equiv \frac{p({}_i X)}{{}_i X} \quad (4)$$

thus that $p({}_i X) \equiv {}_i X \times {}_i X^* \equiv {}_i X \times \frac{E({}_i X)}{{}_i X^2}$. Let $\sigma({}_i X)^2$ denote the variance of the random variable ${}_i X$. The variance of the random variable ${}_i X$ at a single Bernoulli trial t is defined as

$$\sigma({}_i X)^2 \equiv E({}_i X^2) - E({}_i X)^2 \equiv ({}_i X^2) \times p({}_i X) \times (1 - p({}_i X)) \quad (5)$$

or as

$$\sigma({}_i X)^2 \equiv {}_i X \times E({}_i X) - E({}_i X)^2 \equiv E({}_i X) \times ({}_i X - E({}_i X)) = E({}_i X) \times E({}_i \underline{X}) \quad (6)$$

where $E({}_i \underline{X}) = ({}_i X - E({}_i X)) = ({}_i X) \times (1 - p({}_i X))$ denotes something like an expectation value of anti ${}_i X$. Sometimes, this is called the "hidden" variable. Let $\sigma({}_i X)$ denote the standard deviation of the random variable ${}_i X$. The standard deviation of the random variable ${}_i X$ is defined as

$$\sigma({}_i X) \equiv \sqrt[2]{E({}_i X^2) - E({}_i X)^2} \equiv \sqrt[2]{({}_i X^2) \times p({}_i X) \times (1 - p({}_i X))} \quad (7)$$

2.2. Definition. The Logical Contradiction And The Inner Contradiction Of A Random Variable X

Let $\Delta({}_iX)^2$ denote the logical contradiction. We define

$$\Delta({}_iX)^2 \equiv \frac{\sigma({}_iX)^2}{{}_iX^2} \equiv \frac{E({}_iX^2) - E({}_iX)^2}{{}_iX^2} \equiv p({}_iX) \times (1 - p({}_iX)) \quad (8)$$

Let $\Delta({}_iX)$ denote the inner contradiction. We define

$$\Delta({}_iX) \equiv \frac{\sigma({}_iX)}{{}_iX} \equiv \sqrt[2]{\frac{E({}_iX^2) - E({}_iX)^2}{{}_iX^2}} \equiv \sqrt[2]{p({}_iX) \times (1 - p({}_iX))} \quad (9)$$

Scholium

Under conditions of special theory of relativity, ${}_R E$ can denote the expectation value as determined by the stationary observer R while ${}_O X$ can denote the value (i. e. after the collapse of the wave function) as determined by the moving observer O.

The Cause

2.3. Definition. Expectation Value Of The Cause $E({}_R U_t)$ At A Certain Bernoulli Trail t

In general, we define the expectation value of the cause ${}_R U_t$ at one single Bernoulli trial t (i. e. at a certain point in space-time et cetera) as

$$E\left({}_R U_t\right) \equiv p\left({}_R U_t = {}_R W_t\right) \times {}_R U_t \quad (10)$$

where $p\left({}_R U_t = {}_R W_t\right)$ denotes the probability at one single Bernoulli trial t that the random variable ${}_R U_t$ is the cause of the effect, the random variable ${}_R W_t$.

2.4. Definition. Expectation Value Of The Cause squared $E({}_R U_t^2)$ At A Certain Bernoulli Trail t

In general, we define the expectation value of the cause squared at one single Bernoulli trial t (i. e. at a certain point in space-time et cetera) as

$$E\left({}_R U_t^2\right) \equiv p\left({}_R U_t = {}_R W_t\right) \times {}_R U_t^2 \quad (11)$$

where $p\left({}_R U_t = {}_R W_t\right)$ denotes the probability at one single Bernoulli trial t that the random variable ${}_R U_t$ is the cause of the effect, the random variable ${}_R W_t$.

2.5. Definition. The Variance $\sigma({}_R U_t)^2$ Of The Cause ${}_R U_t$ At A Certain Bernoulli Trail t

In general, we define the variance $\sigma({}_R U_t)^2$ of the cause at one single Bernoulli trial t (i. e. at a certain point in space-time et cetera) as

$$\sigma\left({}_R U_t\right)^2 \equiv E\left({}_R U_t^2\right) - \left(E\left({}_R U_t\right)\right)^2 = \left({}_R U_t\right) \times p\left({}_R U_t = {}_R W_t\right) \times \left(1 - p\left({}_R U_t = {}_R W_t\right)\right) \quad (12)$$

The standard deviation $\sigma({}_R U_t)$ Of The Cause ${}_R U_t$ At one single Bernoulli trail t follows as

$$\sigma\left({}_R U_t\right) \equiv \sqrt{\sigma\left({}_R U_t\right)^2} \equiv \sqrt{E\left({}_R U_t^2\right) - \left(E\left({}_R U_t\right)\right)^2} = \left({}_R U_t\right) \times \sqrt{p\left({}_R U_t = {}_R W_t\right) \times \left(1 - p\left({}_R U_t = {}_R W_t\right)\right)} \quad (13)$$

The Effect

2.6. Definition. Expectation Value Of The Effect $E({}_0 W_t)$ At A Certain Bernoulli Trail t

In general, we define the expectation value the effect as $E({}_0 W_t)$ at one single Bernoulli trial t (i. e. at a certain point in space-time et cetera) as

$$E\left({}_0 W_t\right) \equiv p\left({}_0 W_t = {}_R W_t\right) \times {}_0 W_t \quad (14)$$

where $p\left({}_0 W_t = {}_R W_t\right)$ denotes the probability at one single Bernoulli trial t that the random variable ${}_0 W_t$ is the cause of the effect, the random variable ${}_R W_t$.

2.7. Definition. Expectation Value Of The Effect squared $E({}_0 W_t^2)$ At A Certain Bernoulli Trail t

In general, we define the expectation value of the effect squared at one single Bernoulli trial t (i. e. at a certain point in space-time et cetera) as

$$E\left({}_0 W_t^2\right) \equiv p\left({}_0 W_t = {}_R W_t\right) \times {}_0 W_t \times {}_0 W_t \quad (15)$$

where $p\left({}_0W_t={}_R W_t\right)$ denotes the probability at one single Bernoulli trial t that the random variable ${}_0W_t$ is the cause of the effect, the random variable ${}_R W_t$.

2.8. Definition. The Variance $\sigma({}_0W_t)^2$ Of The Effect ${}_0W_t$ At A Certain Bernoulli Trail t

In general, we define the variance $\sigma({}_0W_t)^2$ of the cause at one single Bernoulli trial t (i. e. at a certain point in space-time et cetera) as

$$\sigma\left({}_0W_t\right)^2 \equiv E\left({}_0W_t^2\right) - \left(E\left({}_0W_t\right)\right)^2 = \left({}_0W_t\right) \times p\left({}_0W_t={}_R W_t\right) \times \left(1 - p\left({}_0W_t={}_R W_t\right)\right) \quad (16)$$

The standard deviation of $\sigma({}_0W_t)$ of the effect ${}_0W_t$ at one single Bernoulli trail t follows as

$$\sigma\left({}_0W_t\right) \equiv \sqrt{\sigma\left({}_0W_t\right)^2} \equiv \sqrt{E\left({}_0W_t^2\right) - \left(E\left({}_0W_t\right)\right)^2} = \left({}_0W_t\right) \times \sqrt{p\left({}_0W_t={}_R W_t\right) \times \left(1 - p\left({}_0W_t={}_R W_t\right)\right)} \quad (17)$$

The Cause And The Effect

2.9. Definition. Expectation Value Of The Cause ${}_R U_t$ And Effect ${}_0W_t$ At A Certain Bernoulli Trail t

In general, we define the expectation value of cause ${}_R U_t$ and the effect ${}_0W_t$ at one single Bernoulli trial t (i. e. at a certain point in space-time et cetera) as

$$E\left({}_R U_t, {}_0W_t\right) \equiv p\left({}_R U_t \cap {}_0W_t\right) \times {}_R U_t \times {}_0W_t \quad (18)$$

where $p\left({}_R U_t \cap {}_0W_t\right)$ denotes the joint probability distribution of cause and effect at one single Bernoulli trial t.

2.10. Definition. The Co-Variance Of The Cause ${}_R U_t$ And The Effect ${}_0W_t$ At A Certain Bernoulli Trail t

In general, we define the variance of the cause and effect as $\sigma({}_R U_t, {}_0W_t)$ at one single Bernoulli trial t (i. e. at a certain point in space-time et cetera) as

$$\sigma\left({}_R U_t, {}_0W_t\right) \equiv E\left({}_R U_t, {}_0W_t\right) - \left(E\left({}_R U_t\right) \times E\left({}_0W_t\right)\right) = \left({}_R U_t \times {}_0W_t\right) \times \left(p\left({}_R U_t \cap {}_0W_t\right) - p\left({}_R U_t\right) \times p\left({}_0W_t\right)\right) \quad (19)$$

2.11. Definition. The Mathematical Formula Of The Causal Relationship k

In general, we define the mathematical formula of the causal relationship k (Einstein's Weltformel) from the standpoint of philosophy, classical logic and probability theory as

$$\left| k \left({}_R U_t, {}_0 W_t \right) \right| \equiv \frac{\sigma \left({}_R U_t, {}_0 W_t \right)}{\sigma \left({}_R U_t \right) \times \sigma \left({}_0 W_t \right)} \equiv \frac{E \left({}_R U_t, {}_0 W_t \right) - \left(E \left({}_R U_t \right) \times E \left({}_0 W_t \right) \right)}{\sqrt{E \left({}_R U_t \right) \times \left({}_R U_t - E \left({}_R C_t \right) \right) \times E \left({}_0 W_t \right) \times \left({}_0 W_t - E \left({}_0 W_t \right) \right)}} \quad (20)$$

or something as

$$\left| k \left({}_R U_t, {}_0 W_t \right) \right| \equiv \frac{\sigma \left({}_R U_t, {}_0 W_t \right)}{\sigma \left({}_R U_t \right) \times \sigma \left({}_0 W_t \right)} \equiv \frac{\left({}_R U_t \times {}_0 W_t \right) \times \left(p \left({}_R U_t \cap {}_0 W_t \right) - p \left({}_R U_t \right) \times p \left({}_0 W_t \right) \right)}{\sqrt{\left({}_R U_t \times {}_R U_t \right) \times \left(p \left({}_R U_t \right) - p \left({}_R U_t \right)^2 \right) \times \left({}_0 W_t \times {}_0 W_t \right) \times \left(p \left({}_0 W_t \right) - p \left({}_0 W_t \right)^2 \right)}} \quad (21)$$

or as

$$\left| k \left({}_R U_t, {}_0 W_t \right) \right| \equiv \frac{\sigma \left({}_R U_t, {}_0 W_t \right)}{\sigma \left({}_R U_t \right) \times \sigma \left({}_0 W_t \right)} \equiv \frac{\left(p \left({}_R U_t \cap {}_0 W_t \right) - p \left({}_R U_t \right) \times p \left({}_0 W_t \right) \right)}{\sqrt{p \left({}_R U_t \right) \times \left(1 - p \left({}_R U_t \right) \right) \times p \left({}_0 W_t \right) \times \left(1 - p \left({}_0 W_t \right) \right)}} \quad (22)$$

Scholium.

It is important to note, that the mathematical formula of the causal relationship k is not identical with Pearson's coefficient of correlation.

2.12. Axioms.

The following theory is based on the following axioms.

Axiom I.

$$+1 = +1. \quad (\text{Axiom I})$$

Axiom II.

$$\frac{+0}{+0} \equiv +1 \quad (\text{Axiom II})$$

Axiom III.

$$\frac{+1}{+0} \equiv +\infty \quad (\text{Axiom III})$$

Consequently, it is $+1 \equiv +\infty \times +0$.

3. Theorems

3.1. Theorem. The Cause ${}_R U_t$.

Claim.

In general, under some circumstances, the effect is determined as

$${}_R U_t = \frac{\sigma\left({}_R U_t\right)}{\sqrt[2]{p\left({}_R U_t = {}_R W_t\right) \times \left(1 - p\left({}_R U_t = {}_R W_t\right)\right)}}. \quad (23)$$

Proof.

Starting with Axiom I it is

$$+1 = +1. \quad (24)$$

Multiplying this equation by standard deviation of $\sigma({}_R U_t)$ it is

$$\sigma\left({}_R U_t\right) = \sigma\left({}_R U_t\right). \quad (25)$$

Due to our definition of the standard deviation of $\sigma({}_R U_t)$ of the cause ${}_R U_t$ at a certain Bernoulli trail t as $\sigma\left({}_R U_t\right) = \sqrt[2]{\sigma\left({}_R U_t\right)^2} = \sqrt[2]{E\left({}_R U_t^2\right) - E\left({}_R U_t\right)^2} = \left({}_R U_t\right) \times \sqrt[2]{p\left({}_R U_t = {}_R W_t\right) \times \left(1 - p\left({}_R U_t = {}_R W_t\right)\right)}$ it is

$$\sigma\left({}_R U_t\right) = \left({}_R U_t\right) \times \sqrt[2]{p\left({}_R U_t = {}_R W_t\right) \times \left(1 - p\left({}_R U_t = {}_R W_t\right)\right)}. \quad (26)$$

After division it follows that

$${}_R U_t = \frac{\sigma\left({}_R U_t\right)}{\sqrt[2]{p\left({}_R U_t = {}_R W_t\right) \times \left(1 - p\left({}_R U_t = {}_R W_t\right)\right)}}. \quad (27)$$

Quod erat demonstrandum.

Scholium.

Such a definition of a cause is useful **under conditions** where there is a probability and a standard deviation et cetera.

3.2. Theorem. The Effect ${}_0W_t$.

Claim.

In general, under some circumstances, the effect ${}_0W_t$ is determined as

$${}_0W_t = \frac{\sigma\left({}_0W_t\right)}{\sqrt[2]{P\left({}_0W_t={}_R W_t\right) \times \left(1 - P\left({}_0W_t={}_R W_t\right)\right)}}. \quad (28)$$

Proof.

Starting with Axiom I it is

$$+1 = +1. \quad (29)$$

Multiplying this equation by standard deviation of $\sigma({}_0W_t)$ it is

$$\sigma\left({}_0W_t\right) = \sigma\left({}_0W_t\right). \quad (30)$$

Due to our definition of the standard deviation of $\sigma({}_0W_t)^2$ of the effect ${}_0W_t$ at a certain Bernoulli trail t as $\sigma\left({}_0W_t\right) = \sqrt[2]{\sigma\left({}_0W_t\right)^2} = \sqrt[2]{E\left({}_0W_t^2\right) - \left(E\left({}_0W_t\right)\right)^2} = \left({}_0W_t\right) \times \sqrt[2]{P\left({}_0W_t={}_R W_t\right) \times \left(1 - P\left({}_0W_t={}_R W_t\right)\right)}$ it is

$$\sigma\left({}_0W_t\right) = \left({}_0W_t\right) \times \sqrt[2]{P\left({}_0W_t={}_R W_t\right) \times \left(1 - P\left({}_0W_t={}_R W_t\right)\right)}. \quad (31)$$

After division it follows that

$${}_0W_t = \frac{\sigma\left({}_0W_t\right)}{\sqrt[2]{P\left({}_0W_t={}_R W_t\right) \times \left(1 - P\left({}_0W_t={}_R W_t\right)\right)}}. \quad (32)$$

Quod erat demonstrandum.

Scholium.

Such a definition of the effect is useful **under conditions** where there is a probability and a standard deviation et cetera.

3.3. Theorem. The Cause ${}_R U_t$ And The Effect ${}_0 W_t$.

Claim.

In general, under some circumstances, the cause ${}_R U_t$ and the effect ${}_0 W_t$ are determined as

$${}_R U_t \times {}_0 W_t = \frac{\sigma\left({}_R U_t, {}_0 W_t\right)}{p\left({}_R U_t \cap {}_0 W_t\right) - p\left({}_R U_t\right) \times p\left({}_0 W_t\right)}. \quad (33)$$

Proof.

Starting with Axiom I it is

$$+1 = +1. \quad (34)$$

Multiplying this equation by the co-variance $\sigma({}_R U_t, {}_0 W_t)$ of the cause ${}_R U_t$ and the effect ${}_0 W_t$ it is

$$\sigma\left({}_R U_t, {}_0 W_t\right) = \sigma\left({}_R U_t, {}_0 W_t\right). \quad (35)$$

Due to our definition of the co-variance of the cause ${}_R U_t$ and the effect ${}_0 W_t$ at a certain Bernoulli trail t as

$$\sigma\left({}_R U_t, {}_0 W_t\right) \equiv E\left({}_R U_t, {}_0 W_t\right) - \left(E\left({}_R U_t\right) \times E\left({}_0 W_t\right)\right) = \left({}_R U_t \times {}_0 W_t\right) \times \left(p\left({}_R U_t \cap {}_0 W_t\right) - p\left({}_R U_t\right) \times p\left({}_0 W_t\right)\right)$$

we obtain

$$\sigma\left({}_R U_t, {}_0 W_t\right) = \left({}_R U_t \times {}_0 W_t\right) \times \left(p\left({}_R U_t \cap {}_0 W_t\right) - p\left({}_R U_t\right) \times p\left({}_0 W_t\right)\right). \quad (36)$$

After Division, it follows that

$${}_R U_t \times {}_0 W_t = \frac{\sigma\left({}_R U_t, {}_0 W_t\right)}{p\left({}_R U_t \cap {}_0 W_t\right) - p\left({}_R U_t\right) \times p\left({}_0 W_t\right)}. \quad (37)$$

Quod erat demonstrandum.

Scholium.

It is necessary to make a difference between one single Bernoulli trial t and the whole population (i. e. sample) of the size N.

3.4. Theorem. The Mathematical Formula Of The Causal Relationship k

Claim.

In general, the mathematical formula of the causal relationship k is determined as

$$k\left({}_R U_t, {}_0 W_t\right) = \frac{\sigma\left({}_R U_t, {}_0 W_t\right)}{\sigma\left({}_R U_t\right) \times \sigma\left({}_0 W_t\right)} = \frac{p\left({}_R U_t \cap {}_0 W_t\right) - p\left({}_R U_t\right) \times p\left({}_0 W_t\right)}{\sqrt[2]{p\left({}_R U_t = {}_R W_t\right) \times \left(1 - p\left({}_R U_t = {}_R W_t\right)\right)} \times \sqrt[2]{p\left({}_0 W_t = {}_R W_t\right) \times \left(1 - p\left({}_0 W_t = {}_R W_t\right)\right)}} \quad (38)$$

Proof.

Starting with Axiom I it is

$$+1 = +1. \quad (39)$$

Multiplying this equation by the cause ${}_R U_t$ it is

$${}_R U_t = {}_R U_t. \quad (40)$$

Under the assumption of commutativity, the multiplication by the effect ${}_0 W_t$ yields

$${}_R U_t \times {}_0 W_t = {}_R U_t \times {}_0 W_t. \quad (41)$$

Due to our theorem above, it is

$${}_R U_t = \frac{\sigma\left({}_R U_t\right)}{\sqrt[2]{p\left({}_R U_t = {}_R W_t\right) \times \left(1 - p\left({}_R U_t = {}_R W_t\right)\right)}}.$$

Thus far, it follows that

$${}_R U_t \times {}_0 W_t = \frac{\sigma\left({}_R U_t\right)}{\sqrt[2]{p\left({}_R U_t = {}_R W_t\right) \times \left(1 - p\left({}_R U_t = {}_R W_t\right)\right)}} \times {}_0 W_t. \quad (42)$$

Due to our theorem above, it is

$${}_0 W_t = \frac{\sigma\left({}_0 W_t\right)}{\sqrt[2]{p\left({}_0 W_t = {}_R W_t\right) \times \left(1 - p\left({}_0 W_t = {}_R W_t\right)\right)}}.$$

Consequently, it is

$${}_R U_t \times {}_0 W_t = \frac{\sigma\left({}_R U_t\right)}{\sqrt[2]{p\left({}_R U_t = {}_R W_t\right) \times \left(1 - p\left({}_R U_t = {}_R W_t\right)\right)}} \times \frac{\sigma\left({}_0 W_t\right)}{\sqrt[2]{p\left({}_0 W_t = {}_R W_t\right) \times \left(1 - p\left({}_0 W_t = {}_R W_t\right)\right)}}. \quad (43)$$

Due to our theorem concerning the co-variance of cause and effect, it is

$${}_R U_t \times {}_0 W_t = \frac{\sigma({}_R U_t, {}_0 W_t)}{p({}_R U_t \cap {}_0 W_t) - p({}_R U_t) \times p({}_0 W_t)}.$$

Thus far, we obtain in the following the next relationship as

$$\frac{\sigma({}_R U_t, {}_0 W_t)}{p({}_R U_t \cap {}_0 W_t) - p({}_R U_t) \times p({}_0 W_t)} = \frac{\sigma({}_R U_t)}{\sqrt[2]{p({}_R U_t = {}_R W_t) \times (1 - p({}_R U_t = {}_R W_t))}} \times \frac{\sigma({}_0 W_t)}{\sqrt[2]{p({}_0 W_t = {}_R W_t) \times (1 - p({}_0 W_t = {}_R W_t))}}. \tag{44}$$

Rearranging equation yields

$$\frac{\sigma({}_R U_t, {}_0 W_t)}{\sigma({}_R U_t) \times \sigma({}_0 W_t)} = \frac{p({}_R U_t \cap {}_0 W_t) - p({}_R U_t) \times p({}_0 W_t)}{\sqrt[2]{p({}_R U_t = {}_R W_t) \times (1 - p({}_R U_t = {}_R W_t))} \times p({}_0 W_t = {}_R W_t) \times (1 - p({}_0 W_t = {}_R W_t))}} \tag{45}$$

which is equivalent to our formula of the causal relationship k at each Bernoulli trial t as

$$k({}_R U_t, {}_0 W_t) = \frac{\sigma({}_R U_t, {}_0 W_t)}{\sigma({}_R U_t) \times \sigma({}_0 W_t)} = \frac{p({}_R U_t \cap {}_0 W_t) - p({}_R U_t) \times p({}_0 W_t)}{\sqrt[2]{(p({}_R U_t = {}_R W_t) - p({}_R U_t = {}_R W_t))^2} \times (p({}_0 W_t = {}_R W_t) - p({}_0 W_t = {}_R W_t))^2}} \tag{46}$$

Quod erat demonstrandum.

Scholium.

The following illustration may be of help somehow.

Fig.		Effect		
		yes	no	
Cause	yes	${}_0 U$	${}_{\Delta} U$	${}_R U$
	no	${}_0 \underline{U}$	${}_{\Delta} \underline{U}$	${}_R \underline{U}$
		${}_0 W$	${}_{\Delta} W$	${}_R W$

The above formula of the causal relationship is ensuring the deterministic relationship at **every single Bernoulli trial t**. Under the assumption that the probabilities from trial to trial are constant and not changing (i. e. conditions of special theory of relativity, v=constant). We

obtain the following picture. $\sum_{t=1}^N k({}_R U_t, {}_0 W_t) = N \times k({}_R U_t, {}_0 W_t)$, while N is the population size or the number of trials.

3.5. Theorem. The Formula Of The Causal Relationship k Of A Binomial Random Variable.

Claim.

Under conditions where **the causal relationship k is constant from trial to trial**, the mathematical formula of the causal relationship k can be simplified as

$$k\left({}_R U_t, {}_0 W_t\right) \equiv \frac{N \times \sum\left({}_R U \cap {}_0 W\right) - \left(\sum {}_R U \times \sum {}_0 W\right)}{\sqrt{\left(N \times \sum {}_R U - \sum {}_R U \times \sum {}_R U\right) \times \left(N \times \sum {}_0 W - \sum {}_0 W \times \sum {}_0 W\right)}} . \quad (47)$$

Proof.

Starting with Axiom I it is

$$+1 = +1 . \quad (48)$$

Multiplying this equation by the causal relationship $k\left({}_R U_t, {}_0 W_t\right)$ it is

$$k\left({}_R U_t, {}_0 W_t\right) = k\left({}_R U_t, {}_0 W_t\right) \quad (49)$$

which is equivalent to

$$k\left({}_R U_t, {}_0 W_t\right) = 1 \times k\left({}_R U_t, {}_0 W_t\right) \quad (50)$$

or to

$$k\left({}_R U_t, {}_0 W_t\right) = \frac{N \times N}{N \times N} \times k\left({}_R U_t, {}_0 W_t\right) \quad (51)$$

where N denotes the total number of Bernoulli trials t, the sample size et cetera. Due to our theorem above, this equation is equivalent with

$$k\left({}_R U_t, {}_0 W_t\right) \equiv \frac{N \times N \times \left(p\left({}_R U_t \cap {}_0 W_t\right) - p\left({}_R U_t\right) \times p\left({}_0 W_t\right)\right)}{N \times N \times \sqrt{\left(p\left({}_R U_t = {}_R W_t\right) \times \left(1 - p\left({}_R U_t = {}_R W_t\right)\right) \times p\left({}_0 W_t = {}_R W_t\right) \times \left(1 - p\left({}_0 W_t = {}_R W_t\right)\right)\right)}} \quad (52)$$

or with

$$k\left({}_R U_t, {}_0 W_t\right) \equiv \frac{N \times N \times p\left({}_R U_t \cap {}_0 W_t\right) - \left(N \times N \times p\left({}_R U_t\right) \times p\left({}_0 W_t\right)\right)}{\sqrt{N \times N \times \left(p\left({}_R U_t = {}_R W_t\right) - p\left({}_R U_t = {}_R W_t\right)^2\right) \times N \times N \times \left(p\left({}_0 W_t = {}_R W_t\right) - p\left({}_0 W_t = {}_R W_t\right)^2\right)}} . \quad (53)$$

We define $\sum({}_R U \cap {}_0 W) \equiv E({}_R U \cap {}_0 W) \equiv \sum_{t=1}^N {}_R U_t \times {}_0 W_t \times p({}_R U_t \cap {}_0 W_t) = N \times p({}_R U_t \cap {}_0 W_t)$,
 $\sum({}_R U) \equiv E({}_R U) \equiv N \times p({}_R U_t)$ and $\sum({}_0 W) \equiv E({}_0 W) \equiv N \times p({}_0 W_t)$. The formula above can be simplified as

$$k({}_R U_t, {}_0 W_t) \equiv \frac{N \times \sum({}_R U \cap {}_0 W) - (\sum {}_R U \times \sum {}_0 W)}{\sqrt{\left(N \times \sum {}_R U - \sum {}_R U \times \sum {}_R U \right) \times \left(N \times \sum {}_0 W - \sum {}_0 W \times \sum {}_0 W \right)}} . \quad (54)$$

Quod erat demonstrandum.

Scholium.

Under the conditions above, the significance of the causal relationship k can be tested using the Chi-Square distribution with one degree of freedom. The following 2x2 table may provide an overview.

Fig.		Effect		
		yes	no	
Cause	yes	${}_R U \cap {}_0 W$	${}_{\Delta} U$	${}_R U$
	no	${}_0 \underline{U}$	${}_{\Delta} \underline{U}$	${}_R \underline{U} = N - {}_R U$
		${}_0 W$	${}_{\Delta} W$	${}_R W = N$

In statistics, the phi coefficient, introduced by Karl Pearson, is one of the known measures of association for two binomial random variables. There are situations where the phi coefficient is identical with the mathematical formula of the causal relationship k, but both are not identical in general.

3.6. Theorem. The Chi-Square Distribution And The Formula Of The Causal Relationship k.

Claim.

Under some assumptions, the mathematical formula of the causal relationship k is determined by the chi-square distribution as

$$k({}_R U_t, {}_0 W_t) = \sqrt{\frac{X^2_N}{N}} \quad (55)$$

Proof.

Starting with Axiom I it is

$$+1 = +1 . \quad (56)$$

Multiplying this equation by the causal relationship $\left| k({}_R U_t, {}_0 W_t) \right|$ it is

$$\left|k\left({}_R U_t, {}_0 W_t\right)\right| = \left|k\left({}_R U_t, {}_0 W_t\right)\right| \quad (57)$$

which is equivalent to

$$\frac{\left|k\left({}_R U_t, {}_0 W_t\right)\right| - E\left(\left|k\left({}_R U_t, {}_0 W_t\right)\right|\right)}{\sigma\left(\left|k\left({}_R U_t, {}_0 W_t\right)\right|\right)} = \frac{\left|k\left({}_R U_t, {}_0 W_t\right)\right| - E\left(\left|k\left({}_R U_t, {}_0 W_t\right)\right|\right)}{\sigma\left(\left|k\left({}_R U_t, {}_0 W_t\right)\right|\right)} \quad (58)$$

where $E\left(\left|k\left({}_R U_t, {}_0 W_t\right)\right|\right)$ denotes the expectation value of the causal relationship at each single Bernoulli trial t and $\sigma\left(\left|k\left({}_R U_t, {}_0 W_t\right)\right|\right)$ denotes the deviation of the causal relationship k at each Bernoulli trial t. At each Bernoulli trial t, the normal random variable of a standard normal distribution (called a standard score or a z-score) at each single Bernoulli trial t is determined

$$\text{as } Z\left(\left|k\left({}_R U_t, {}_0 W_t\right)\right|\right) = \frac{\left|k\left({}_R U_t, {}_0 W_t\right)\right| - E\left(\left|k\left({}_R U_t, {}_0 W_t\right)\right|\right)}{\sigma\left(\left|k\left({}_R U_t, {}_0 W_t\right)\right|\right)}.$$

Thus far we obtain

$$Z\left(\left|k\left({}_R U_t, {}_0 W_t\right)\right|\right) = \frac{\left|k\left({}_R U_t, {}_0 W_t\right)\right| - E\left(\left|k\left({}_R U_t, {}_0 W_t\right)\right|\right)}{\sigma\left(\left|k\left({}_R U_t, {}_0 W_t\right)\right|\right)} \quad (59)$$

Under conditions [1] where $E\left(\left|k\left({}_R U_t, {}_0 W_t\right)\right|\right) = 0$ and $\sigma\left(\left|k\left({}_R U_t, {}_0 W_t\right)\right|\right) = 1$ we obtain

$$Z\left(\left|k\left({}_R U_t, {}_0 W_t\right)\right|\right) = \frac{\left|k\left({}_R U_t, {}_0 W_t\right)\right| - 0}{1} \quad (60)$$

or

$$Z\left(\left|k\left({}_R U_t, {}_0 W_t\right)\right|\right) = \left|k\left({}_R U_t, {}_0 W_t\right)\right| \quad (61)$$

After the square root operation it is

$$Z\left(\left|k\left({}_R U_t, {}_0 W_t\right)\right|\right)^2 = \left|k\left({}_R U_t, {}_0 W_t\right)\right|^2 \quad (62)$$

Summarizing yields

$$\sum_{t=1}^N Z\left(\left|k\left({}_R U_t, {}_0 W_t\right)\right|\right)^2 = \sum_{t=1}^N \left|k\left({}_R U_t, {}_0 W_t\right)\right|^2 \quad (63)$$

which is equivalent with

$$Z^2 = \sum_{t=1}^N \left|k\left({}_R U_t, {}_0 W_t\right)\right|^2 \quad (64)$$

In other words, it is

$$Z^2 = \sum_{t=1}^N \left|k\left({}_R U_t, {}_0 W_t\right)\right|^2 = \left|k\left({}_R U_1, {}_0 W_1\right)\right|^2 + \left|k\left({}_R U_2, {}_0 W_2\right)\right|^2 + \dots + \left|k\left({}_R U_N, {}_0 W_N\right)\right|^2 = N \times \left|k\left({}_R U_t, {}_0 W_t\right)\right|^2 \quad (65)$$

Under conditions, where the causal relationship |k| is **constant** from trial to trial, we obtain

$$Z^2 = \sum_{t=1}^N \left|k\left({}_R U_t, {}_0 W_t\right)\right|^2 = \left|k\left({}_R U_1, {}_0 W_1\right)\right|^2 + \left|k\left({}_R U_2, {}_0 W_2\right)\right|^2 + \dots + \left|k\left({}_R U_N, {}_0 W_N\right)\right|^2 = N \times \left|k\left({}_R U_t, {}_0 W_t\right)\right|^2 \quad (66)$$

In statistics, it is known that $Z^2 = \sum_{t=1}^N k\left({}_R U_t, {}_0 W_t\right)^2 = X^2_N$ with N degrees of freedom, we obtain

$$Z^2 = \sum_{t=1}^N k\left({}_R U_t, {}_0 W_t\right)^2 = X^2_N \equiv N \times \overline{k\left({}_R U_t, {}_0 W_t\right)^2} \quad (67)$$

Where $\overline{k\left({}_R U_t, {}_0 W_t\right)^2}$ denotes the average value of the causal relationships |k| after N Bernoulli trials. We re-write this equation above as

$$X^2_N = N \times \overline{k\left({}_R U_t, {}_0 W_t\right)^2} \quad (68)$$

where X^2_N denotes the chi-squared distribution (also chi-square distribution) with N degrees of freedom. At the end, it follows that

$$\overline{k\left({}_R U_t, {}_0 W_t\right)^2} = \sqrt{\frac{X^2_N}{N}} \quad (69)$$

Quod erat demonstrandum.

Scholium.

This proof enable us to use the mathematical formula of the causal relationship for hypothesis testing with the possibility to calculate the p-values, the β -value et cetera even under conditions where the $|k|$ is not constant from trial to trial. Under these conditions, please recall the relationship $t_N = \frac{Z}{\sqrt{\frac{X^2_N}{N}}}$, where t_N denotes the t-distribution with N degrees of

freedom and Z denotes the Z value. For more details on this topic, I must refer the reader to primary literature.

4. Discussion

There is a long tradition of dualism [2] between causality and statistics. For reasons not relevant here, statistics seemed to exclude causality and vice versa. Especially, due to some quantum mechanical positions (Heisenberg's uncertainty, Bell's theorem, CHSH-Inequality) the deterministic relationship between a cause and its own effect became an impossibility. Meanwhile, the quantum mechanical no-go-theorems which excluded the deterministic relationship between cause and effect are already refuted [3], [4], [5]. While the mathematical methodology to extract cause and effect relationship out of (non-) experimental data is already published [6], [7], [8], [9] peer-reviewed [10] and presented to the scientific community, this highly original approach gives a new, unknown and exact mathematical derivation of the mathematical formula of the causal relationship k from a purely mathematical starting point.

In general, data can be gathered through an observational study, through an experiment et cetera. Afterwards, statistical inference allows the researcher to asses evidence in favor or some hypotheses about the population from which a sample has been drawn. The mathematical formula of the causal relationship k can be used as a test of significance to support or to reject hypotheses/claims based on data gathered. Especially, we are enabled to test whether there is a causal relationship between random variables investigated or not.

For example, in a clinical trial, the null hypothesis might be that there is *no causal relationship* between a random variable (i. e. Helicobacter pylori) and an effect (i. e. human gastric cancer). In other words, we would write **H0**: $k=0$. In other words, Helicobacter pylori and human gastric cancer are independent of each other. In the same clinical trial, the alternative hypothesis, HA, is a statement of what a statistical hypothesis test is set up to establish. The opposing hypothesis is the alternative hypothesis (HA). For example, in a clinical trial, the alternative hypothesis HA might be that there is *a causal relationship* between a random variable (i. e. Helicobacter pylori) and an effect (i. e. human gastric cancer). In other words, we would write **HA**: $k \neq 0$. The final conclusion is always given in terms of the null hypothesis as either "reject H0 in favor of Ha" or "do not reject H0".

Example.

Helicobacter pylori has been discussed [10], [11] as being associated with human gastric cancer. In several, previous (epidemiologic) studies and meta-analysis it has been reported that there is a close relation between H. pylori infection and gastric cancer. Still, the cause of human gastric cancer is not identified. Naomi Uemura et al. [12] conducted a long-term, prospective study of N=1526 Japanese patients, 1246 had H. pylori infection and 280 did not

(mean follow up 7.8 years, endoscopy at enrollment and then between one and three years after enrollment). None of the uninfected patients developed gastric cancer. Let us show this data in the following 2-2-table.

Fig.		Human gastric cancer		
		yes	no	
Helicobacter pylori infection of human stomach	yes	36	1210	1246
	no	0	280	280
		36	1490	1526

H0: $k=0$.
No significant causal relationship between Helicobacter pylori infection of human stomach and human gastric cancer. Alpha = 5 %.

HA: $k \neq 0$.
Significant causal relationship between Helicobacter pylori infection of human stomach and human gastric cancer.

Experiment.

An observational study or an experiment is performed, data are gathered.

Data analysis.

Calculation of the causal relationship k.

$$k \left({}_R U_t, {}_0 W_t \right) = \sqrt[2]{\frac{X^2_N}{N}} \equiv \frac{(1526 \times 36) - (36 \times 1246)}{\sqrt[2]{(1260 \times 280) \times (36 \times 1490)}} = +0,07368483 \tag{70}$$

Calculation of Pearson chi-square statistic, uncorrected for continuity, one degree of freedom and of the p-value. The Pearson chi-square statistic, uncorrected for continuity, is calculated as follows:

$$X^2_{Calculated} \equiv \frac{N \times (a \times d - b \times c)}{(a + b) \times (c + d) \times (a + c) \times (b + d)} \tag{71}$$

The following contingency table may provide some detailed information about this formula.

Fig.		Effect		
		yes	no	
Cause	yes	a	b	a+b
	no	c	d	c+d
		a+c	b+d	N

Due to the data of the study above, the Chi-Square can be calculated as

$$X^2_{\text{Calculated}} = \frac{1526 \times (36 \times 280 - 0 \times 1210)}{(1260 \times 280) \times (36 \times 1490)} = 8,28534801. \quad (72)$$

The Pearson chi-square statistic, uncorrected for continuity, is 8.2853. The P value is 0.00399664. This result is significant at $p < 0.05$.

Conclusion.

The data above do not support the null hypothesis H_0 , we must reject H_0 in favor of H_A (p-value 0.003997).

In other words, there is a highly significant causal relationship between an infection of human stomach with *Helicobacter pylori* and the development of human gastric cancer ($k=+0,073684834$, p value 0.003997). **Helicobacter pylori is the cause of human gastric cancer.** The methods above were already demonstrated and used to analyze the relationship between *Helicobacter pylori* and the gastric cancer. At XXIIIrd International Biometric Conference scheduled from July 16-21, 2006 in Montréal, Canada, a significant causal relationship between *Helicobacter pylori* and human gastric cancer using the methods above was presented [13] to the scientific community. *Helicobacter pylori* is the cause of gastric cancer.

5. Conclusion

This publication provides an exact mathematical derivation of the relationship between the cause and the effect. A new mathematical methodology for making causal inferences on the basis of (non-) experimental data for evaluating causal relationships from (non-) experimental data is presented in the simplest and most intelligible form. Anyone who wishes to elucidate cause effect relationships from (non-) experimental data will find this publication useful. Finally, a unified mathematical and statistical model of the relationship between the cause and the effect is available.

Acknowledgment

None.

Appendix

None.

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