Study on the Goldbach Conjecture

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"Every even integer greater than two can be expressed as the sum of two primes."

We know that, with a an integer and b an odd number, $2a + b$ will always be odd.

We know every prime greater than two is odd, therefore:

$$(2n + q) + (2m + k) = 2(n + m + \frac{q + k}{2})$$

With n and m integers and q and k odd integers. We can set $q = -k$ and the $\frac{q+k}{2}$ term becomes zero, leaving us with $2(m + n)$. To find an n and a m that will yield prime numbers when plugged into the equation, we can arrange these numbers into a table where the first row will be the value of q, the first column will list the primes p, and the entry (q, p) will be the value for n or for m for which either $2n + q$ and $2m - q$ will give us a prime number. The table below lists the first 18 primes.

	17	**15**	**13**	**11**	**9**	**7**	**5**	**3**	**1**	**-1**	**-3**	**-5**	**-7**	**-9**	**-11**	**-13**	**-15**	**-17**
3	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10
5	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11
7	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12
11	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
13	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
17	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
19	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
23	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
29	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
31	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
37	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27
41	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29
43	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
47	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32
51	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34
53	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35
57	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37
59	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38

An example to illustrate the method might help to explain the method. Say we want to find the two primes that add up to 30. We can say:

$$30 = 2(n + m)$$

$$15 = n + m$$

We need to pick two numbers from two different columns labeled with the same values but opposite signs that add up to 15. On the columns 1 and -1 we can find 8 and 7, which add to 15. Now, we have our values for n, m and q, mainly: $n = 8, m = 7$ and $q = 1$. Plugging these values into the original equation gives:

$$(2(8) + 1) + (2(7) - 1) = (16 + 1) + (14 - 1) = 17 + 13 = 30$$

Each value in the same row is the same as the value on the left plus one, so the array will span all the integers. But every row may be shifted from the row above by some amount, and if this amount becomes too big, adding numbers from opposite columns may not span all the natural numbers, because the n and m values will be either too big or too small to add up to an element of the naturals.

To better understand this table, we must generate another table with the same rules, but this time, with odd numbers on the first column.

	-17	**-15**	**-13**	**-11**	**-9**	**-7**	**-5**	**-3**	**-1**	**1**	**3**	**5**	**7**	**9**	**11**	**13**	**15**	**17**
3	10	9	8	7	6	5	4	3	2	1	0	-1	-2	-3	-4	-5	-6	-7
5	11	10	9	8	7	6	5	4	3	2	1	0	-1	-2	-3	-4	-5	-6
7	12	11	10	9	8	7	6	5	4	3	2	1	0	-1	-2	-3	-4	-5
9	13	12	11	10	9	8	7	6	5	4	3	2	1	0	-1	-2	-3	-4
11	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0	-1	-2	-3
13	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0	-1	-2
15	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0	-1
17	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0
19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1
21	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2
23	20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3
25	21	20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4
27	22	21	20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5
29	23	22	21	20	19	18	17	16	15	14	13	12	11	10	9	8	7	6
31	24	23	22	21	20	19	18	17	16	15	14	13	12	11	10	9	8	7
33	25	24	23	22	21	20	19	18	17	16	15	14	13	12	11	10	9	8
35	26	25	24	23	22	21	20	19	18	17	16	15	14	13	12	11	10	9
37	27	26	25	24	23	22	21	20	19	18	17	16	15	14	13	12	11	10
39	28	27	26	25	24	23	22	21	20	19	18	17	16	15	14	13	12	11
41	29	28	27	26	25	24	23	22	21	20	19	18	17	16	15	14	13	12
43	30	29	28	27	26	25	24	23	22	21	20	19	18	17	16	15	14	13
45	31	30	29	28	27	26	25	24	23	22	21	20	19	18	17	16	15	14
47	32	31	30	29	28	27	26	25	24	23	22	21	20	19	18	17	16	15
49	33	32	31	30	29	28	27	26	25	24	23	22	21	20	19	18	17	16
51	34	33	32	31	30	29	28	27	26	25	24	23	22	21	20	19	18	17
53	35	34	33	32	31	30	29	28	27	26	25	24	23	22	21	20	19	18
55	36	35	34	33	32	31	30	29	28	27	26	25	24	23	22	21	20	19
57	37	36	35	34	33	32	31	30	29	28	27	26	25	24	23	22	21	20
59	38	37	36	35	34	33	32	31	30	29	28	27	26	25	24	23	22	21

Here, finding numbers on opposite columns that generate all natural numbers is a trivial problem, because each row is a value of one away from the row before. However, we can take rows out of the table without damaging its capacity to generate all natural numbers greater than or equal to 3 because the only way in which a number might be skipped at a certain point is if the number of rows removed is greater than or equal to the number of rows above that certain point.

For example, we may not remove the row for number five, even if it is only one row, because there is only one more line above five: that of three, and this removal will make us unable to obtain a four out of the columns. Yet, we may remove line seven, because above line seven, there are two lines, those for three and five, thus, no numbers will be skipped.

To count how many lines D_b^a there are between any two given odd numbers a and b, with $b \geq a$ we have:

$$D_b^a = \frac{b-a}{2}$$

If the number of lines skipped after a given number "a" ($D_{skipped}$) must be greater than or equal to the number of lines above that number a, then we have:

$$D_{skipped} \geq D_a^1$$

Which, when talking about primes, becomes:

$$\frac{P_{n+1} - P_n}{2} \geq \pi(P_n) - 1$$

The negative one comes in because we do not count 2 as a prime in this table, being 2 an even number. Because of Bertrand's Postulate, we know the left hand side of the equation will be less than $\frac{P_n}{2}$ and what the inequality comes down to is whether more than half the numbers below P_n are prime.

Since $\frac{x}{\ln x} < \pi(x) < \frac{1.25506x}{\ln x}$ for $x \geq 17$ and the latter is bigger than $\frac{x}{2}$ for all values $x \geq 13$ the gap between consecutive rows will never be big enough for the addition between two numbers of opposite columns to yield all natural numbers above two. Therefore $n + m$ can be any integer between 2 and infinity, and $(2n + q)$ along with $(2m - q)$ will always be prime, because they come from the table, proving that every even integer greater than two can be expressed as the sum of at least two primes.

$$(2n + q) + (2m - q) = 2(n + m)$$