Frequency Modulation of 0S2-C

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Abstract: Precision measurements of the 0S2 quintet after the 2004-12-26 earthquake show that the middle spectral line near 309.3 µHz is frequency modulated. The strongest modulation frequencies are phase-locked and depend on the geographic location. By proper choice of the integration length the center frequency can be determined with high precision.

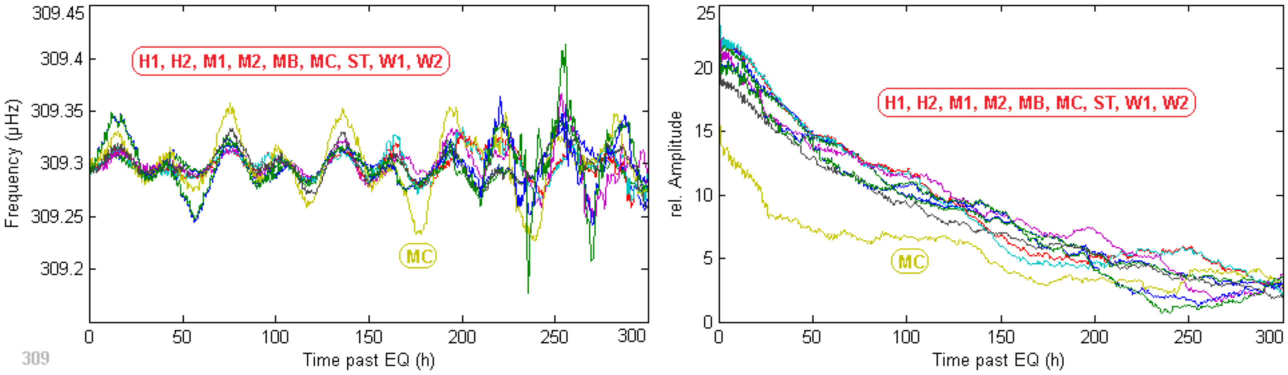
Introduction

After earthquakes, the Earth vibrates like a bell at different frequencies. The lowest ones near 300 µHz are particularly interesting because of their relative proximity to the rotation frequency of the earth. The remarkably wide error bars of all previous measurements are probably caused by the overlooked frequency modulation of these natural frequencies. High precision can only be achieved when the measurement period is adapted to the modulation frequency.

The Preparation of the data was described here[1]. The essential points: Before calculating the frequency, each data segment passed a narrow band Sinc filter with the bandwidth 0.8 µHz. To increase the resolution, a new method[2] was developed, eliminating the need for a window function and zero padding. In this way, any data corruption by the window function is avoided. FFT is replaced by the faster Goertzel algorithm, because this allows period lengths that are not a power of 2.

The Frequency of 0S2-C

As reasoned here[1], all following averages are calculated with the period length 3606 minutes. All European stations consistently show the same frequency modulation of this spectral line $_0S_2$ -C. Due to the small mutual distances, there is no noticeable phase shift.



During the first 230 hours after the earthquake, the records are almost undisturbed. Since the average frequency of a FM-oscillation is influenced by the length of the period, a random choice may generate an incorrect result. The period must be adapted to the "wavelength" of the modulation frequencies. If the product $L \cdot f_{MOD}$ is not an integer, systematic errors arise. At least for the modulation frequencies having the highest amplitudes, this condition must be met. Because both modulation frequencies are multiples of 4.58 µHz, a period of 60.6 hours (or multiples thereof) should be used. For each of all European stations (including MC), the average frequency of the first 909 measurements (corresponding to a period of 181.8 hours) was calculated. The jackknife method provides the mean frequency (309.29839 ± 0.00032) µHz.

In earlier measurements[3], significantly larger error bands were given. This may have been caused

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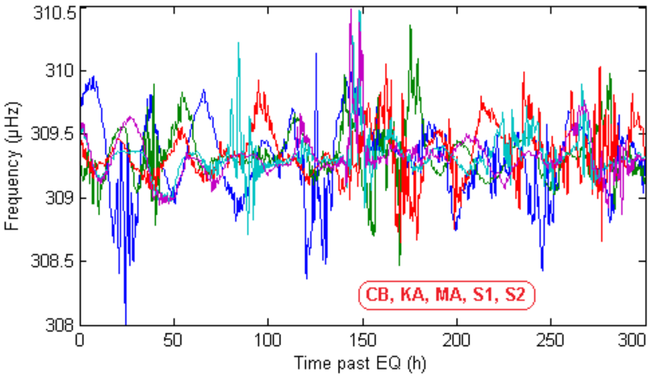
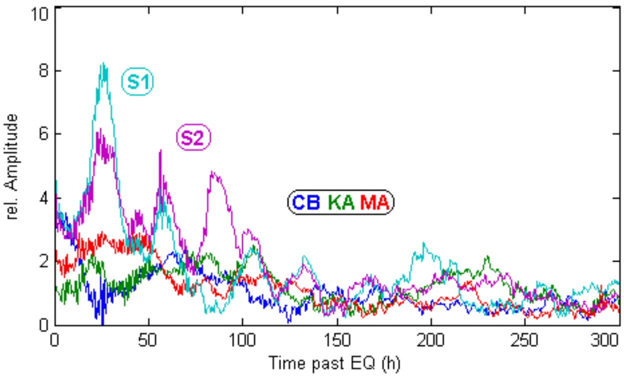
a) 13. June 2015, email: herbertweidner@gmx.de by the ignorance of the frequency modulation with its consequences.

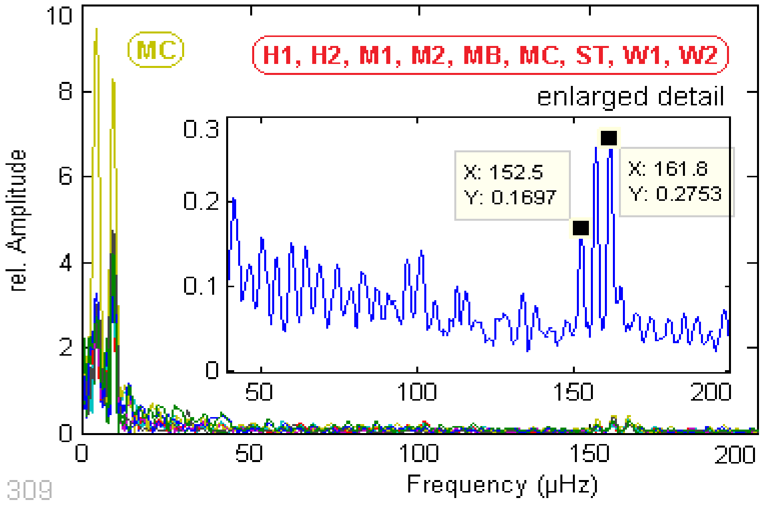
Outside Europe, the spectral line $_0S_2$ -C is usually too weak to allow for frequency determination.





The Modulation Frequencies

The spectral line $_0S_2$ -C is modulated with two different frequency groups. The low group includes two individual frequencies: $f_1 = 4.58$ µHz and $f_2 = 9.325$ µHz. Below is shown that these modulation frequencies are phase-locked. The higher frequency group around 160 µHz consists of three separate frequencies with similar spacings and much smaller amplitude. This high-frequency group seems to shift. The average frequency of this group depends on the frequency of the spectral line. Here, a simple formation law applies.



Spectral line $_0S_2$ -A (299.9 µHz) begins near 161.5 µHz[4]; Sum = 461.4 µHz

Spectral line $_0S_2$ -C (309.3 µHz) begins near 152.5 µHz; Sum = 461.8 µHz

Spectral line $_0S_2$ -E (318.4 µHz) begins near 143.1 µHz[1]; Sum = 461.5 µHz

The sum frequency coincides with the lowest frequency of the $_0S_3$ septet (m= −3)[5]. That may be a nice coincidence. Perhaps $_0S_2$ is influenced by $_0S_3$. The frequency of $_0S_2$ changes in the rhythm depending on $f(_0S_3) - f(_0S_2)$. Prerequisite of each intermodulation is a non-linearity because for strictly linear systems, the superposition principle applies.

Amplitude Decay and Q-Factor of 0S2-C

The amplitude reduction of the $_0S_2$ -C frequency near 309.3 µHz is expected to follow an exponential law that may depend on the geographic position of the measurement. The right picture above shows the superposition of the amplitude curves of all European SG stations. Ignoring the results recorded at MC, the decay during the first 220 hours past the earthquake may be described by the exponential function

$$A = A_0 \cdot e^{\frac{-t}{T}}$$

The time constant for the European stations (without MC) is $T_{0S2-C} = (140.99 \pm 5.09)\, hours$

The quality factor Q may be computed using the equation

$$A = A_0 \cdot e^{\frac{-t}{T}} \sin(\omega t + \varphi) = A_0 \cdot e^{\frac{-\omega t}{2Q}} \sin(\omega t + \varphi)$$

For $f_{0S2\text{-}C}$ =309.3 µHz, this equation yields

$$Q_{0S2-C} = 493.2 \pm 17.8$$

Amplitudes and Phases of 0S2-C

The spectral line near 309.3 µHz is frequency modulated with two main frequencies: f_1 = 4.58 µHz and f_2 = 9.325 µHz. At least during the first 200 hours after the earthquake, all European stations measure almost identical and synchronous frequency deviations around the average. This constancy of the waveform requires that the dominant modulation frequencies are phase-locked.

With sine waves of the two frequencies, the actual time-dependent frequency course of each station can be reconstructed with high accuracy. The required amplitudes and phases are tabulated below.

Station	$Ampl_1$	$Ampl_2$	$Phase_1$	$Phase_2$
F (µHz)	4.58	9.33	4.58	9.33
H1	70	97	6,06	5,1
H2	78	107	6,15	5,09
M1	73	104	5,7	5,2
M2	75	112	5,68	5,26
MB	70	110	6,28	4,93
ST	110	133	5,92	5,06
W1	176	138	5,97	5,21
W2	169	134	5,96	5,19

Acknowledgments

Thanks to the operators of the GGP stations for the excellent gravity data. The underlying data of this examination were measured by a net of about twenty SG distributed over all continents, the data are collected in the Global Geodynamic Project[6].

[1] H. Weidner, Frequency Modulation of 0S2-E, http://viXra.org/abs/1506.0084

[2] H. Weidner, A New method for High-resolution Frequency Measurements, http://vixra.org/abs/1506.0005

[3] R. Häfner, R. Widmer-Schnidrig, Signature of 3-D density structure in spectra of the spheroidal free oscillation 0S2, *Geophys. J. Int.,2012*

[4] H. Weidner, Frequency Modulation of 0S2-A, http://viXra.org/abs/1506.0094

[5] Rosat, Fukushima, Sato, Tamura, Application of a Non-Linear Damped Harmonic Analysis method to the normal modes of the Earth, *Journal of Geodynamics* (2007)

［6］The "Global Geodynamics Project", http://www.eas.slu.edu/GGP/ggphome.html