Frequency Modulation of 0S2-A

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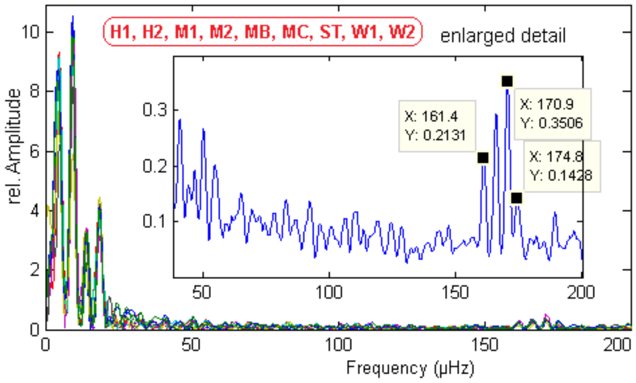
Abstract: Precision measurements of the 0S2 quintet after the 2004-12-26 earthquake show that the lowest spectral line near 299.9 µHz is frequency modulated. The different modulation frequencies are phase-locked and depend on the geographic location. By proper choice of the integration length the center frequency can be determined with high precision.

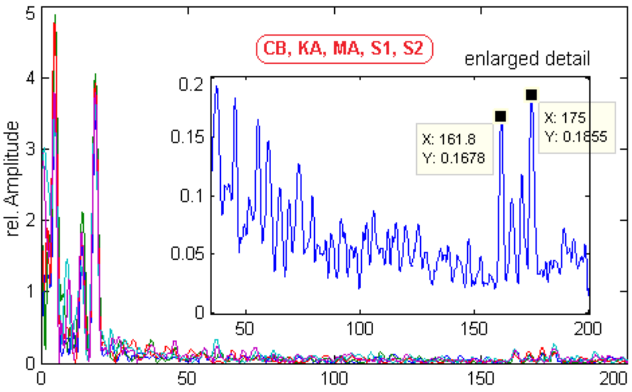
Introduction

After earthquakes, the Earth vibrates like a bell at different frequencies. The lowest ones near 300 µHz are particularly interesting because of their relative proximity to the rotation frequency of the earth. The remarkably wide error bars of all previous measurements are probably caused by the overlooked frequency modulation of these natural frequencies. High precision can only be achieved when the measurement period is adapted to the modulation frequency.

The Preparation of the data was described here[1] . The essential points: Before calculating the frequency, each data segment passed a narrow band Sinc filter with the bandwidth 0.8 µHz. To increase the resolution, a new method[2] was developed, eliminating the need for a window function and zero padding. FFT is replaced by the faster Goertzel algorithm.

The Modulation Frequencies





The spectral line ${}_0S_2$ -A is modulated with two different frequency groups. The low group includes four individual frequencies: $f_1 = 4.60$ µHz, 9.325 µHz, 13.94 µHz and 18.48 µHz. These are apparently multiples of the fundamental frequency f_1. Below is shown that these four modulation frequencies are phase-locked. The higher frequency group around 170 µHz consists also of four separate frequencies with similar spacings and much smaller amplitude. This high-frequency group was not further analyzed. Notably, the high-frequency group of spectral line ${}_0S_2$ -A begins near 161.5 µHz, against which in the modulation spectrum of ${}_0S_2$ -E, the group starts near 143 µHz.

Frequency Modulation of 0S2-A

As reasoned here[1], all following averages are calculated with the period length 3606 minutes. The European stations consistently show the same frequency modulation of this spectral line of ${}_0S_2$ -A. Due to the small mutual distances, there is no noticeable phase shift.

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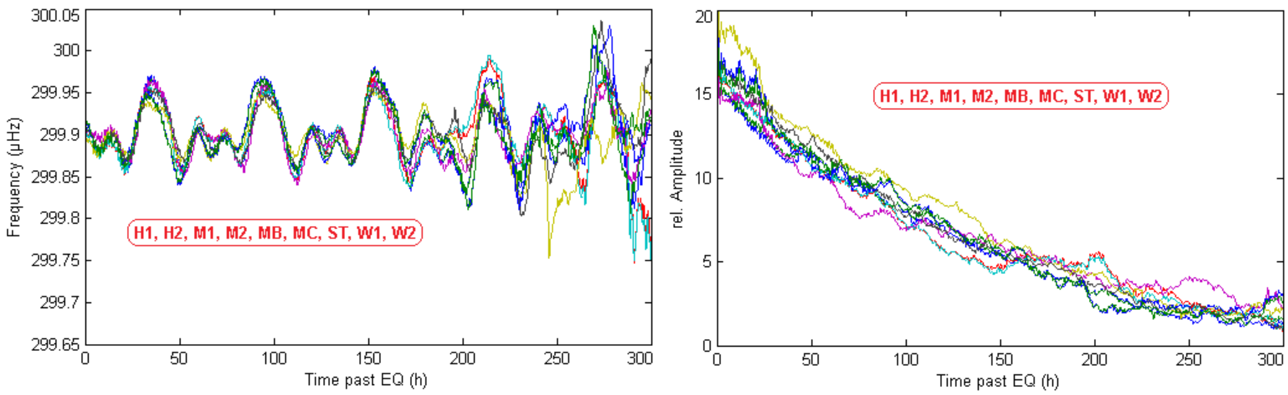
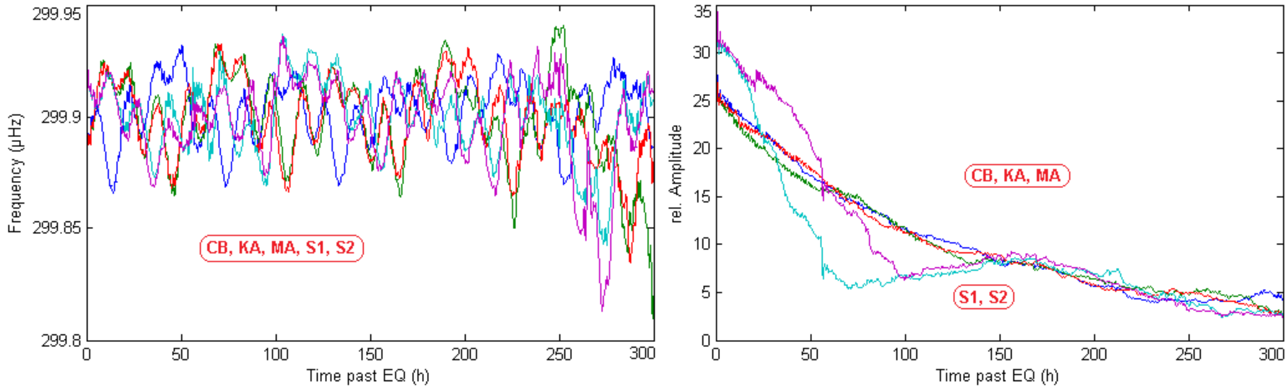


Outside Europe, there are fewer SG whose mutual distances are much greater. Nevertheless, the phase relationships are very clear, as shown below.



During the first 230 hours after the earthquake, all the records are almost undisturbed. Since the average frequency of a FM-oscillation is influenced by the length of the period, a random choice may generate an incorrect result. The period must be adapted to the "wavelength" of the modulation frequencies. If the product $L \cdot f_{\mathrm{MOD}}$ is not an integer, systematic errors arise. At least for the modulation frequencies having the highest amplitudes, this condition must be met. Because the most powerful modulation frequencies are multiples of 4.61 µHz, a period of 60.2 hours (or multiples thereof) should be used. For each of the fourteen stations, the average frequency of the first 903 measurements (corresponding to a period of 180.6 hours) was calculated. The jackknife method provides the mean frequency (299.90210 ± 0.00035) µHz.

In earlier measurements[3], significantly larger error bands were given. This may have been caused by the ignorance of the frequency modulation with its consequences.

Amplitude Decay and Q-Factor of 0S2-A

The amplitude reduction of the ${}_0S_2$ -A frequency near 299.9 µHz is expected to follow an exponential law that may depend on the geographic position of the measurement. The two (right) pictures above show the superposition of the amplitude curves of SG stations. It is noteworthy that the initial amplitudes measured by CB, KA and MA are about 50% higher than the measured values of European stations. S1 and S2 measured even higher initial amplitudes.

The decay during the first 200 hours past the earthquake may be described by the exponential function

$$A = A_0 \cdot e^{\frac{-t}{T}}$$

The time constant T for the European stations is (138.33 ± 3.43) hours. The time constant for the CA, KA and MA is (141.62 ± 2.06) hours. If each station (without S1 and S2) is assigned the same weight, the jackknife method returns the mean time constant

$$T_{0S2-A} = (139.14 \pm 2.63)\, hours$$

The quality factor Q may be computed using the equation

$$A = A_0 \cdot e^{\frac{-t}{T}} \sin(\omega t + \varphi) = A_0 \cdot e^{\frac{-\omega t}{2Q}} \sin(\omega t + \varphi)$$

For f_{0S2-A} =299.9 µHz, this equation yields

$$Q_{0S2-A} = 471.9 \pm 8.9$$

Amplitudes and Phases of 0S2-A

The spectral line near 299.9 µHz is frequency modulated with four main frequencies: f_1 = 4.60 µHz, 9.325 µHz, 13.94 µHz and 18.48 µHz. At least during the first 200 hours after the earthquake, all European stations measure almost identical and synchronous frequency deviations around the average. This constancy of the waveform requires that the dominant modulation frequencies are phase-locked.

With sine waves of the four frequencies, the actual time-dependent frequency course of each station can be reconstructed with high accuracy. The required amplitudes and phases are tabulated below.

Station	Ampl$_1$	Ampl$_2$	Ampl$_3$	Ampl$_4$	Phase$_1$	Phase$_2$	Phase$_3$	Phase$_4$
F (µHz)	4.60	9.33	13.94	18.48	4.60	9.33	13.94	18.48
H1	270	285	64	107	4,04	0,27	2,24	1,57
H2	263	276	67	107	4,1	0,29	2,26	1,57
M1	239	270	97	128	4,35	0,42	2,62	1,85
M2	226	259	106	130	4,42	0,45	2,62	1,88
MB	232	289	104	89	4,25	0,29	2,37	1,4
MC	210	141	94	126	4,06	0,71	2,75	1,99
ST	188	219	96	117	4,01	0,35	2,41	1,57
W1	273	271	79	111	4,21	0,72	2,65	2,08
W2	250	271	82	108	4,17	0,73	2,69	2,07
CB	136	11	61	118	3,23	2,66	0,41	5,19
KA	146	25	61	133	0,16	5,39	3,19	4,31
MA	161	34	60	116	0,23	4,82	3,19	4,43
S1	95	40	41	113	1,52	4,43	5,95	2,38
S2	94	25	52	109	1,35	4,79	5,92	2,54

Acknowledgments

Thanks to the operators of the GGP stations for the excellent gravity data. The underlying data of this examination were measured by a net of about twenty SG distributed over all continents, the data are collected in the Global Geodynamic Project[4].

[1] H. Weidner, Frequency Modulation of 0S2-E, http://viXra.org/abs/1506.0084

[2] H. Weidner, A New method for High-resolution Frequency Measurements, http://vixra.org/abs/1506.0005

［3］ R. Häfner, R. Widmer-Schnidrig, Signature of 3-D density structure in spectra of the spheroidal free oscillation 0S2, *Geophys. J. Int.,2012*

［4］ The "Global Geodynamics Project", http://www.eas.slu.edu/GGP/ggphome.html