Frequency Modulation of 0S2-E

Herbert Weidner [a]

Abstract: Precision measurements of the 0S2 quintet after the 2004-12-26 earthquake show that the highest spectral line near 318.4 µHz is frequency modulated. The different modulation frequencies are phase-locked and depend on the geographic location. By proper choice of the integration length the center frequency can be determined with high precision.

Introduction

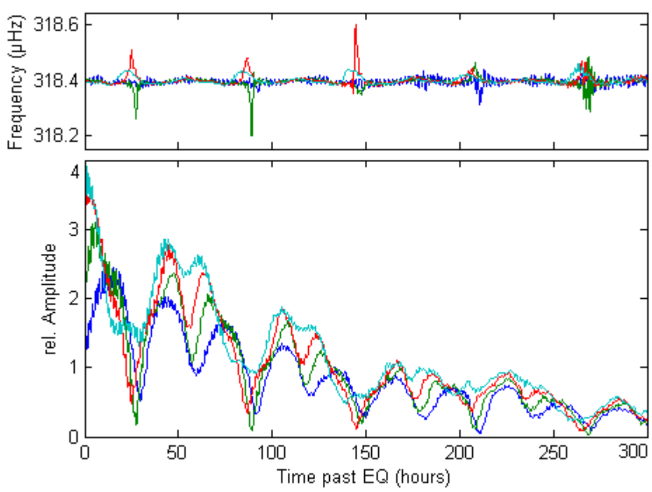
After earthquakes, the Earth vibrates like a bell at different frequencies. The lowest ones near 300 µHz are particularly interesting because of their relative proximity to the rotation frequency of the earth. The remarkably wide error bars of all previous measurements are probably caused by the overlooked frequency modulation of these natural frequencies. High precision can only be achieved when the measurement period is adapted to the modulation frequency.

The underlying data of this examination were measured by a net of about twenty SG distributed over all continents, the data are collected in the Global Geodynamic Project[1].

The Preparation of the data

All available CORMIN-data of SG-stations were bundled into separate two-year-clusters. To prevent intermodulation by numeric overload of the mathematical coprocessor inside the computer, the very strong spectral lines below 22 µHz were attenuated by narrow notch filters[2]. The measurement started 8725 hours after 1.1.2004 and ended 300 hours later because the signal quality was no longer satisfactory. At different times, the frequency and amplitude of $_0S_2$ -E near 318.4 µHz was measured, using different periods for the FFT. The shortest period was 1·256 minutes, the longest 14·256 minutes. The base period of 256 minutes was chosen to take advantage of the speed benefit of FFT, if the period is a power of 2. The start time was shifted in increments of 12 minutes in order to achieve a good overlap of neighboring time segments. To reduce the noise before standard FFT, each data segment passed a narrow band Sinc filter with the bandwidth 0.8 µHz. For each SG-Station, 14·1500 amplitudes and frequencies were calculated.

Puzzling Results



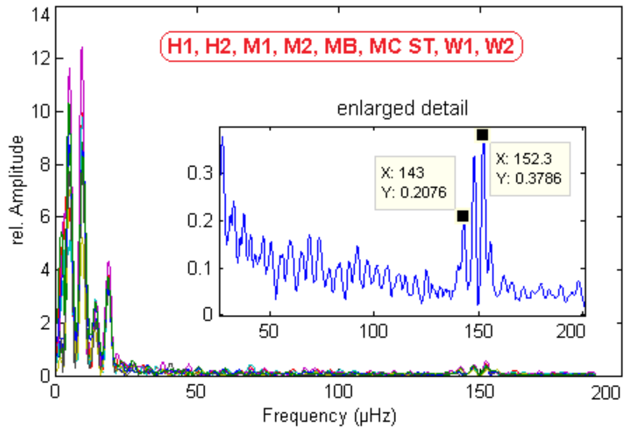
The upper picture shows the frequency based on periods containing 1024, 1280, 1536 or 1752 sampling points. At regular intervals of 60 hours, short sections repeat, in which the results of FFT differ very markedly from the average. In the picture below you can see that exactly at these times the amplitude of this spectral line falls below the 10% - threshold.

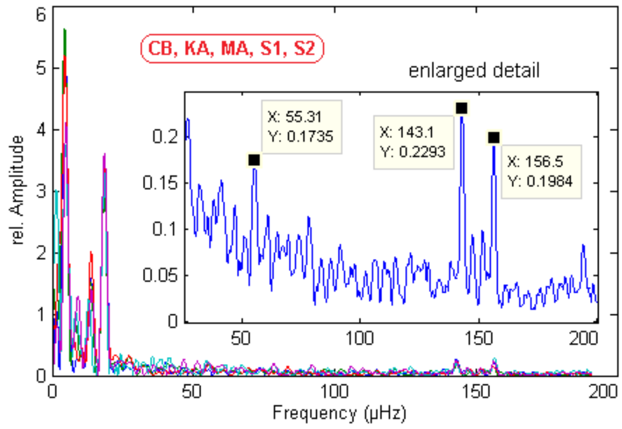
These very fast amplitude changes surprise for several reasons: They are significantly shorter than the sample periods (17 hours to 29.9 hours),

a) 10. June 2015, email: herbertweidner@gmx.de

they are independent of the selected bandwidth of the Sinc filter and they are almost independent of the choice of the SG-station.

The Modulation Frequencies





The search for the cause of this enigmatic variations of amplitudes was very time consuming. Numerous spectrograms of amplitude and frequency functions consistently showed that the spectral line $_0S_2$ -E is modulated with two different frequency groups. The low group includes four individual frequencies: f_1 = 4.62 µHz, 9.23 µHz, 13.86 µHz and 18.46 µHz. These are apparently multiples of the fundamental frequency f_1. Below is shown that these four modulation frequencies are phase-locked. The higher frequency group around 150 µHz consists also of four separate frequencies with similar spacings and much smaller amplitude. This higher-frequency group was not further analyzed.

A detailed study showed that the amplitudes of the lower four frequencies depend very much on the period length of the FFT. This is very unusual and untypical for an amplitude modulation, which can be detected without FFT.

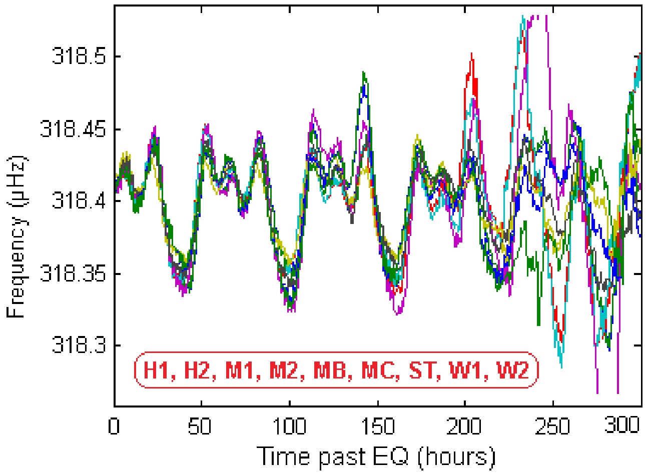
With a period length of 7·256 minutes, f_1 and f_3 are very weak and the amplitudes of f_2 and f_4 are particularly strong. The absence of two spectral components creates a strong distortion of the waveform.

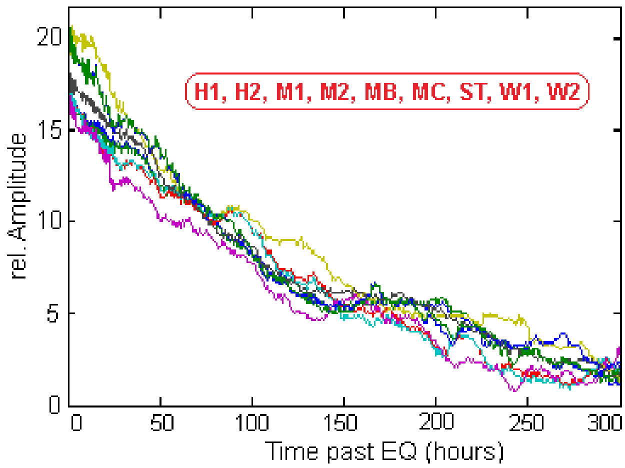
With the period length 14·256 minutes, all four amplitudes reach their maximum values and the rapid changes of the signal amplitude disappear almost completely.

The fact that the longer period is almost exactly equal to the oscillation period of the lowest modulation frequency, was the key to the solution. If the period is exactly 3606 minutes (or multiples thereof), the fast and regular amplitude fluctuations disappear completely. With this optimal period the spectrograms above were produced. Optimal means: In the left image below, the frequency deviation from the average is minimal and in the right image below, the difference from the exponential curve is not periodic. (Technical note: If the period is not a power of 2, FFT may be replaced by the faster Goertzel algorithm.) To increase the frequency resolution, a new method[3] was developed, eliminating the need for a window function and zero padding.

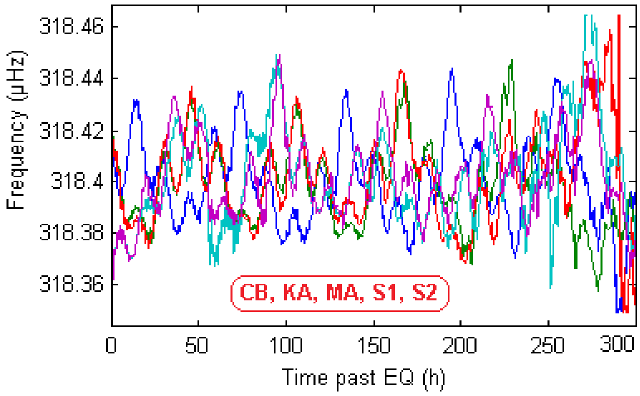
Frequency Modulation of 0S2-E

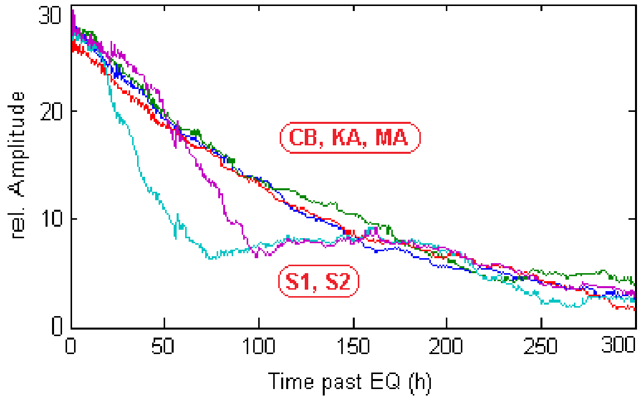
Using the optimal period of 3606 minutes, all European stations consistently show the same frequency modulation (FM) of this spectral line of $_0S_2$ -E. Due to the small mutual distances, there is no noticeable phase shift. The periodic variations of the amplitude decrease have disappeared.





Outside Europe, there are fewer SG whose mutual distances are much greater. Nevertheless, the phase relationships are very clear, as shown below.





During the first 230 hours after the earthquake, all the records are almost undisturbed. Since the average frequency of a FM-oscillation is influenced by the length of the period, a random choice may generate an incorrect result. The period must be adapted to the "wavelength" of the modulation frequencies. If the product $L \cdot f_{MOD}$ is not an integer, systematic errors arise. At least for the modulation frequencies having the highest amplitudes, this condition must be met. Because the most powerful modulation frequencies are multiples of 4.62 µHz, a period of 60.2 hours (or multiples thereof) should be used. For each of the fourteen stations, the average frequency of the first 903 measurements (corresponding to a period of 180.6 hours) was calculated. The jackknife method provides the mean frequency (318.40113 ± 0.00032) µHz.

In earlier measurements[[4]], significantly larger error bands were given. This may have been caused by the ignorance of the frequency modulation with its consequences. If the period of averaging greatly deviates from the optimum value, the results vary considerably.

Amplitude Decay and Q-Factor of 0S2-E

The amplitude reduction of the $_0S_2$ -E frequency is expected to follow an exponential law that may depend on the geographic position of the measurement. The two (right) pictures above show the superposition of the amplitude curves of SG stations. It is noteworthy that the initial amplitudes measured outside Europe are about 60% higher than the measured values of European stations.

The decay during the first 200 hours past the earthquake may be described by the exponential function

$$A = A_0 \cdot e^{\frac{-t}{T}}$$

The time constant T for the European stations is (135.55 ± 1.91) hours. The time constant for the non-European stations is (135.35 ± 5.12) hours. If each station is assigned the same weight, the jackknife method returns the mean time constant

$$T_{0S2-E} = (135.48 \pm 2.14)\, hours$$

The quality factor Q may be computed using the equation

$$A = A_0 \cdot e^{\frac{-t}{T}} \sin(\omega t + \varphi) = A_0 \cdot e^{\frac{-\omega t}{2Q}} \sin(\omega t + \varphi)$$

For f_{0S2-E} =318.4 µHz, this equation yields

$$Q_{0S2-E} = 487.9 \pm 7.7$$

Amplitudes and Phases of 0S2-E

The spectral line near 318.4 µHz is frequency modulated with four main frequencies: f_1 = 4.62 µHz, f_2 = 9.23 µHz, f_3 = 13.86 µHz and f_4 = 8.46 µHz. At least during the first 200 hours after the earthquake, all European stations measure almost identical and synchronous frequency deviations around the average. This constancy of the waveform requires that the dominant modulation frequencies are phase-locked.

With sine waves of the four frequencies, the actual time-dependent frequency course of each station can be reconstructed with high accuracy. The required amplitudes and phases are tabulated below.

Station	$Ampl_1$	$Ampl_2$	$Ampl_3$	$Ampl_4$	$Phase_1$	$Phase_2$	$Phase_3$	$Phase_4$
H1	257	262	66	120	0,74	2,76	5,39	4,64
H2	249	263	64	112	0,73	2,72	5,36	4,64
M1	249	250	69	105	0,77	3,1	5,65	5,06
M2	216	252	77	108	0,8	3,08	5,65	5,07
MB	317	319	85	142	0,74	2,68	5,61	4,73
MC	242	139	68	113	1	2,9	5,59	5,14
ST	267	217	64	111	0,75	2,73	5,44	4,7
W1	266	241	86	111	0,62	2,91	6,02	5,08
W2	279	237	87	114	0,56	2,9	5,98	5,04
CB	169	33	63	120	0,09	4,65	3,27	1,98
KA	164	21	48	91	2,95	1,34	5,96	1,14
MA	176	27	65	102	2,97	1,91	5,94	1,22
S1	162	18	27	97	4,06	2,61	3,05	5,48
S2	157	12	51	108	3,91	6,01	2,85	5,6

Acknowledgments

Thanks to the operators of the GGP stations for the excellent gravity data.

［1］The "Global Geodynamics Project", http://www.eas.slu.edu/GGP/ggphome.html
[2] H. Weidner, Unexplained Resonances in the Gravitation Field of the Earth, 2014, http://vixra.org/abs/1412.0225
[3] H. Weidner, A New method for High-resolution Frequency Measurements, http://viXra.org/abs/1506.0005
［4］ R. Häfner, R. Widmer-Schnidrig, Signature of 3-D density structure in spectra of the spheroidal free oscillation 0S2, *Geophys. J. Int.,2012*