Local Vacuum Pressure

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The existence of negative pressure of vacuum follows from the cosmological models, based on the results of observations. But, is it possible to detect the pressure of the vacuum as per the geometry of the space around the local bodies? The gravitational mass of bodies placed in confined volume, is less than the sum of the gravitational masses of these bodies, dispersed over infinite distance. It interprets into the transfer of energy to the vacuum, which becomes apparent from its deformation. We determine the gravitational impact of matter on the vacuum, equal in value and opposite in the sign of pressure, in case of weakly gravitating spherical bodies. We have evaluated a possibility to extend the obtained result to arbitrary gravitational systems.

The non-zero vacuum pressure is an element of cosmological models [1-3], resulting from the solution of Einstein's equations. He proposed that gravitation is the reason of space-time curvature. The matter, located more compactly, distorts the space in the local domain in a greater degree, however, creating smaller gravitational mass in comparison with the same amount of matter, distributed over a greater volume [1, 2]. The transfer of energy to the gravitational field, which results in the deformation of vacuum, explains this phenomenon. Its accumulation of energy during deformation demonstrates its elasticity. We will take its given characteristics into consideration, while determining the value of vacuum pressure.

Let us analyze a centrally symmetrical static gravitational field (using units with $c = G = 1$). In spherical coordinates $x^i = (t, r, \theta, \varphi)$ it is described by the metrics

$$ds^2 = e^{\upsilon(r)}\, dt^2 - e^{w(r)} dr^2 - r^2(d\theta^2 + \sin^2\theta\, d\varphi^2), \tag{1}$$

where - and ν, w - are functions of radial coordinates. The centrally symmetric stress-energy tensor T_i^i corresponds to the source of gravitation, which creates this type of field. Solution of Einstein's equation for a spherical body with a radius of a yields [1] the required functions

$$\begin{aligned} \nu = -w = \ln\left(1 - \frac{8\pi}{r}\int_0^a T_1^1 y^2 dy\right), \; r > a \\ w = -\ln\left(1 - \frac{8\pi}{r}\int_0^r T_1^1 y^2 dy\right), \qquad r \le a, \end{aligned} \tag{2}$$

where T_1^1 is the density of energy. The density of energy for the static source of gravitation is equivalent to the density of matter ρ, which is expressed by the following formula: $T_1^1 = \rho$.

In the external area the obtained functions ν, w correspond to the Schwarzschild metrics, and therefore the value

$$M = 4\pi \int_0^a \rho r^2 dr \tag{3}$$

is the gravitational mass of a spherical body with of radius a. Integration is performed here in case of the element of volume $dV_c = 4\pi r^2 dr$, which corresponds to the coordinate frame, whereas in its proper frame the given element of space volume will be $dV_p = 4\pi r^2 e^{w/2} dr$. Since w has a positive value, it means that the gravitational mass of bodies located in the confined volume is less than the sum of individual gravitational masses of these bodies. This interprets as the transfer of energy, as a source of gravitational field, to the vacuum.

The volume of spherical body in proper frame is obtained by integration of elements dV_p with (2) amounts to

$$\mathrm{V}_{int}^p(a) = \int_0^a 4\pi r^2 e^{w/2} dr = \int_0^a 4\pi r^2 \left(1 - \frac{8\pi\rho}{3} r^2\right)^{-1/2} dr. \tag{4}$$

For small space curvature inside the sphere, i.e. with $\rho \mathrm{a}^2 << 1$, representation of the expression under the integral into a formal power series turns out to be

$$\mathrm{V}_{int}^p(a) = \frac{4\pi}{3} a^3 + \frac{16\pi^2}{15} a^5 \rho. \tag{5}$$

Since the density of matter is constant, the mass of body in this frame or the proper mass will be $M^p = \mathrm{V}_{int}^p(a)\rho$. A proper energy of static source of gravitation is defined as $\mathrm{E}^p = M^p$.

The gravitational impact on the vacuum is determined as the relation of difference between proper energies of two spherical bodies with identical gravitational mass to the change of proper volume of space. With constant densities ρ_1,ρ_2 and radiuses a_1, a_2, $(a_1 < a_2)$ this mass is $M = (4/3)\pi\rho_1 a_1^3 = (4/3)\pi\rho_2 a_2^3$. The difference of proper masses of two bodies is written as follows:

$$\Delta M^p = M_1^p - M_2^p = \frac{16\pi^2}{15} a_1^6 \rho_1^2 \left(\frac{1}{a_1} - \frac{1}{a_2}\right). \tag{6}$$

Due to equality of gravitational masses of both bodies, the space distortion in the area $r > a_2$, created by them, will be identical. Let's find the difference between the volumes in the proper frame, which are set in coordinate frame by the condition $r \leq a_2$. This volume for the first body is the sum of this body's own volume and the peripheral area $a_1 < r \leq a_2$, namely,

$$\mathrm{V}_1^p = \mathrm{V}_{int}^p(a_1) + \mathrm{V}_{ext}^p(a_1, a_2), \tag{7}$$

where the second term is given by

$$\mathrm{V}^p_{ext}(a_1, a_2) = \int_{a_1}^{a_2} 4\pi r^2 e^{w/2} dr. \tag{8}$$

Breaking the expression under integral into the formal power series, in case of $M/r << 1$ we obtain $\mathrm{V}^p_{ext}(a_1, a_2) = (4/3)\pi(a_2^3 - a_1^3) + 2\pi(a_2^2 - a_1^2)M$. As a result, the volume (7) will amount to $\mathrm{V}^p_1 = (4/3)\pi a_2^3 + (8/15)\pi^2 \rho_1 a_1^3 (5a_2^2 - 3a_1^2)$. The area $r \leq a_2$ restricts the second body, whose proper volume for the weak gravitational field according to (5) is $\mathrm{V}^p_2 = \mathrm{V}^p_{int}(a_2) = (4/3)\pi a_2^3 + (16/15)\pi^2 a_2^5 \rho_2$. The difference between the proper volumes, confined within the radius a_2 in coordinate frame, will be

$$\Delta \mathrm{V}^p = \mathrm{V}^p_1 - \mathrm{V}^p_2 = \frac{8\pi^2 \rho_1 a_1^3}{5}(a_2^2 - a_1^2). \tag{9}$$

The ratio of change in the energy of the spherical body $\Delta \mathrm{E}^p = \Delta M^p$ to the change of its volume for small $\Delta a = a_2 - a_1$ retaining its gravitational mass taking (6) into consideration yields

$$\wp = \frac{\Delta \mathrm{E}^p}{\Delta \mathrm{V}^p} = \frac{1}{3}\rho. \tag{10}$$

Provided that $\wp$ corresponds to the pressure, this expression coincides with the equation of the state of photon gas [4]. Being positive it characterizes the gravitational impact of matter on the vacuum, which lies in its constraint. Accordingly, the direction of vacuum pressure is opposite to it:

$$p_v = -\wp \tag{11}$$

and one may be considered as mean vacuum pressure in case of low gravitation inside the static sphere.

Let us examine arbitrary space-time, containing a source of gravitation with density ρ, which is described by the metric $ds^2 = g_{ij} dx^i dx^j$. We allocate a small area, in which metrical coefficients and density can be considered as constant in the first approximation and whose boundary is a sphere in the proper frame. It is assumed also that the gravitational field produced by matter in it corresponds to one described above. Under these conditions, we determine, setting the radius of the sphere as a_2, the proper pressure of vacuum p_v for gravitational field of arbitrary configuration, which according to (10) and (11) is

$$p_v = -\frac{1}{3}\rho. \tag{12}$$

Direct transfer of obtained result for vacuum pressure for the weakly gravitating sphere into the arbitrary point of space requires observation of the strong principle of equivalence.

Standard cosmological models propose existence of some positive energy density of the vacuum [5,6]. Therefore, the pressure of vacuum is negative and proportional to the sum of density of its energy and matter located within it.

References

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