# Disproof of the No-communication Theorem by Decoherence Theory

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**Abstract**

The No-communication Theorem has been seen as the bar to communication by quantum state collapse. The essence of this theory is the procedure of taking the partial trace on an entangled, hence inseparable multi-particle system. This mathematical procedure applied unthinkingly, strikes out the off-diagonal elements from the ensemble density matrix and renders the reduced trace matrix representative of a mixed state. Decoherence theory is able to justify this mathematical procedure and we review it to show: the partial trace results for both unitary and non-unitary processes (hence measurement) on one, several or all particles of the ensemble; and that a unitary process keeps interference terms in the trace reduced matrix.

## 1. Introduction

Cornwall[1] introduced a scheme for communication by collapse of a joint wavefunction of a two particle entangled system. It followed on from the well known Bell thought experiment[2] and Aspect's coincidence counting experiment[3]. The original motivation for this topic dates to Schrodinger's thoughts on the extension of quantum mechanics to multi-particle systems and Bohr's interpretation of the wavefunction[4]. Einstein, Podolsky and Rosen expressed concerns that wavefunction collapse of entangled systems was in abeyance of Relativity[5].

One could argue, for a *single* particle from a spherical source, that conservation of probability alone would be enough to challenge Relativity[1]; this is beyond the scope of this paper but the author is currently investigating ways to use a single particle source and path entanglement to affect his communication scheme[6]. The discussion is timely in the centenary year of General Relativity, as the belief that information cannot be sent faster-than-light appears to be straining at the leash[7], we aim to show using standard quantum theory that something has to break. The author has already provided a framework for metrology with faster-than-light signals[1, 8, 9].

There are several descriptions of the process of wavefunction collapse but the only one that appeals to Objective Reality and what actually occurs in the laboratory, is Decoherence Theory[10, 11]. Thus we shall use it to elucidate the flaw in the "No-communication Theorem"[12-14]. Let us first consider how a state vector is extended for multi-particle systems:

$$\begin{aligned}&\frac{1}{\sqrt{2}}\left(\left|H_1\right\rangle+\left|V_1\right\rangle\right)\otimes\frac{1}{\sqrt{2}}\left(\left|H_2\right\rangle+\left|V_2\right\rangle\right)\\&=\\&\frac{1}{4}\left(\left|H_1\right\rangle\left|H_2\right\rangle+\left|H_1\right\rangle\left|V_2\right\rangle+\left|V_1\right\rangle\left|H_2\right\rangle+\left|V_1\right\rangle\left|V_1\right\rangle\right)\end{aligned} \qquad \text{eqn. 1}$$

Here, for example, two photons in the diagonal basis are combined into one vector spanning both Hilbert spaces by the tensor product[10]. Such a system we call factorisable or separable and we'd expect no operation performed on subsystem 1 or 2 to affect the other. For instance, if system 1 is projected into the horizontal or vertical states, we'd factorise as,

$$\left|H_1\right\rangle\otimes\left(\left|H_2\right\rangle+\left|V_2\right\rangle\right)\ or\ \left|V_1\right\rangle\otimes\left(\left|H_2\right\rangle+\left|V_2\right\rangle\right)$$

This leaves the other system unaffected.

However when a system is prepared subject to some conservation rule (in the following example with polarisation, the conservation of angular momentum and energy[15]), the possibilities for the product space are curtailed, giving the Bell States for instance. We might write,

$$\left|H_1\right\rangle\left|H_2\right\rangle \quad +0 \quad +0 \quad \pm\left|V_1\right\rangle\left|V_2\right\rangle$$

and realise that this cannot be factored, leading to the inescapable conclusion that a measurement on system will affect the other. However, with wavefunction collapse being a strictly indeterminate process[16, 17], projection into a state would not lead to certainty of that state (there is only certainty with repeated measurements if the state is not given sufficient time to evolve, the so-called "Quantum Zeno" principle), thus any communication scheme by distant measurement would seem to be thwarted by the randomness inherent in quantum measurement but *a posteriori,* we could discern correlations by pooling experimental results[3] and comparing local and distant measurement events.

Let us now use the density matrix formalism and write the tensor product of our (for example) two particles system and the environment. The environment also will include the measurement apparatus, but we write before measurement:

$$\rho_{total} = |start\rangle\langle start| = |\psi\rangle\langle\psi| \otimes |e\rangle\langle e| \quad \text{eqn. 2}$$

Our two component system is entangled and can't be separated, let each particle have n states,

$$\rho_{12} = |\psi\rangle\langle\psi| = \sum_{i,j=1,n^2} \psi_i \psi_j^* \quad \text{eqn. 3}$$

To consider our two particle entangled system in isolation, we take the reduced trace which is defined as:

$$\rho_A = tr_B(\rho_{AB}) \quad \text{eqn. 4}$$

For a two component system with any two vectors (the extension to more is obvious), we take the partial trace as:

$$\begin{aligned} &tr_B(|a_1\rangle\langle a_2| \otimes |b_1\rangle\langle b_2|) \\ &= |a_1\rangle\langle a_2| tr(|b_1\rangle\langle b_2|) \\ &= |a_1\rangle\langle a_2| \langle b_1|b_2\rangle \end{aligned}$$

Returning to our entangled system and the environment, this can be traced out thus:

$$\begin{aligned} \rho_{12} &= tr_e(\rho_{total}) \\ &= |\psi_{12}\rangle\langle\psi_{12}| \otimes \langle e|e\rangle \quad \text{eqn. 5} \\ &= |\psi_{12}\rangle\langle\psi_{12}| \end{aligned}$$

## 2. Joint Unitary Evolution

Our entangled system can evolve in isolation subject to unitary operators acting on each particle $U_1$ and $U_2$ respectively:

$$\begin{aligned} &|\psi_{12}\rangle_{t+1} = (U_1 \otimes U_2)|\psi_{12}\rangle_t \\ &or \quad \text{eqn. 6} \\ &\rho_{12}|_{t+1} = (U_1 \otimes U_2)\rho_{12}|_t (U_1 \otimes U_2)^\dagger \end{aligned}$$

In particular, with reference to the entangled communication scheme of Cornwall[1], $U_2$ is "Bob's" interferometer apparatus and it can distinguish the pure entangled state of the entangled system.

Indeed, considering the expansion of $|\psi\rangle$ into its constituent basis, the transition probability can be written:

$$\begin{aligned} P(\psi_t \to \psi_{t+1}) &= \langle\psi_{t+1}|\rho_{12}|\psi_{t+1}\rangle \\ &= \langle\psi_{t+1}|\psi_t\rangle\langle\psi_t|\psi_{t+1}\rangle \\ &= \sum_{i=1,n^2} |\psi_{i,t}^* \psi_{i,t+1}|^2 + \sum_{\substack{i,j=1,n^2 \\ i\neq j}} \psi_{i,t}^* \psi_{j,t} \psi_{j,t+1}^* \psi_{i,t+1} \end{aligned}$$

eqn. 7

This clearly shows interference terms and thus a combination of diagonal and off-diagonal elements in the density matrix.

## 3. Interaction with the Environment, Measurement and the Partial Trace

As the experiment ends, the entangled system begins to interact with the environment (that includes the measurement apparatus):

$$\begin{aligned} \rho_{total} &= |end\rangle\langle end| \\ &= |\psi\rangle\langle\psi| \cdot |i\rangle\langle j| \otimes |e_i\rangle\langle e_j| \quad \text{eqn. 8} \\ &= \sum_{i,j=1,n^2} \psi_i \psi_j^* |i\rangle\langle j| \otimes |e_i\rangle\langle e_j| \end{aligned}$$

The partial trace is taken again to isolate our two particle system:

$$\begin{aligned} \rho_{12} &= tr_e(\rho_{total}) \\ &= \sum_{i,j=1,n^2} \psi_i \psi_j^* |i\rangle\langle j| \otimes \langle e_i|e_j\rangle \\ &= \sum_{i,j=1,n^2} \psi_i \psi_j^* |i\rangle\langle j| \delta_{ij} \quad \text{eqn. 9} \\ &= \sum_{i=1,n^2} |\psi_i|^2 |i\rangle\langle i| \end{aligned}$$

Once again we can compute the probability of transition but given the circumstance of interaction with the environment:

$$\begin{aligned} P(\psi_t \to \psi_{t+1}) &= \langle\psi_{t+1}|\rho_{12}|\psi_{t+1}\rangle \\ &= \sum_{i,j=1,n^2} |\psi_{i,t}|^2 |\psi_{i,t+1}|^2 \langle j|i\rangle\langle i|j\rangle \quad \text{eqn. 10} \\ &= \sum_{i=1,n^2} |\psi_{i,t}^* \psi_{i,t+1}|^2 \end{aligned}$$

Comparing eqn. 10 to eqn. 7 we see the lack of interference terms and so quantum superpositions have given way to classical probabilities, when the system interacts with the environment. We note too that in eqn. 9, although the original system was entangled, that the diagonal density matrix indicates that our two particle system is now separable – that is, not entangled.

Initially we spoke of the entangled system evolving through the tensor product of two unitary operators:

$$\left|\psi_{12}\right\rangle_{t+1} = \left(U_1 \otimes U_2\right)\left|\psi_{12}\right\rangle_t$$
$$or$$
$$\rho_{12}\big|_{t+1} = \left(U_1 \otimes U_2\right)\rho_{12}\big|_t \left(U_1 \otimes U_2\right)^\dagger$$

The decoherence analysis performed the evolution thus:

$$\begin{aligned}&\left|\psi_{12}\right\rangle_{t+1} = \left(M_1 \otimes M_2\right)\left|\psi_{12}\right\rangle_t\\ &\Rightarrow \\ &M_1\left|\psi_1\right\rangle_{t+1} \text{ and } M_2\left|\psi_2\right\rangle_{t+1}\end{aligned} \qquad \text{eqn. 11}$$

That is, both particles interacted with the environment over the 2n basis vectors of the joint state vector, which then collapsed into two separable systems. This is also equivalent to:

$$\begin{aligned}&\left|\psi_{12}\right\rangle_{t+1} = \left(M_1 \otimes U_2\right)\left|\psi_{12}\right\rangle_t\\ &\Rightarrow \\ &M_1\left|\psi_1\right\rangle_{t+1} \text{ and } \mathrm{U}_2\left|\psi_2\right\rangle_{t+1}\end{aligned} \qquad \text{eqn. 12}$$

It is sufficient that only one measurement be performed – "Alice" can communicate to "Bob" via her measurement. Bob does a unitary transform with his interferometer and measures the resulting mixed state and not the pure, interfering, entangled state before her measurement. To see that this is so, that only one measurement is needed to decohere/de-entangle the two particle system, consider the tensor product of two state vectors (the coefficients have been left out) :-

$$\begin{aligned}&\left(\left|\mathbf{e}_1\right\rangle + \left|\mathbf{e}_2\right\rangle + \ldots + \left|\mathbf{e}_n\right\rangle\right) \otimes \left(\left|\mathbf{f}_1\right\rangle + \left|\mathbf{f}_2\right\rangle + \ldots + \left|\mathbf{f}_n\right\rangle\right)\\ &= \\ &\quad \left(\left|\mathbf{e}_1\right\rangle\left|\mathbf{f}_1\right\rangle + \left|\mathbf{e}_1\right\rangle\left|\mathbf{f}_2\right\rangle + \ldots + \left|\mathbf{e}_1\right\rangle\left|\mathbf{f}_n\right\rangle\right)\\ &\quad + \ldots + \\ &\quad \left(\left|\mathbf{e}_n\right\rangle\left|\mathbf{f}_1\right\rangle + \left|\mathbf{e}_n\right\rangle\left|\mathbf{f}_2\right\rangle + \ldots + \left|\mathbf{e}_n\right\rangle\left|\mathbf{f}_n\right\rangle\right)\end{aligned} \qquad \text{eqn. 13}$$

Although the product has $n^2$ terms, the density matrix in eqn. 8 has $n^4$ terms, as does the interaction matrix $\left|i\right\rangle\left\langle j\right|$, the form of each term $|\mathbf{e}_i\mathbf{f}_j>$ means that every term has a single state interaction term and indeed, in the case of an entangled system, this term $e_if_j$ isn't separable (i.e. $e_i$ or $f_j$) and a measurement on one system is a measurement on the other system too. The end result is still the same: whether it is one or both entangled particles interacting, eqn. 9 shows the partial trace.

## 4. The Flaw in the No-communication theorem

Current wisdom believes that the act of taking the partial trace will leave both systems in a mixed state. Clearly from the proceeding analysis this isn't so:

- The scenario of unitary evolution *and then* the trace-out of the environment: eqn. 5, eqn. 6 and eqn. 7, allowed interference terms to be kept,
- Whilst the scenario of interaction with the environment and trace-out of the environment: eqn. 8, eqn. 9 and eqn. 10 didn't.

In other words: It is not the act of taking the partial trace that causes the mixed state and loss of interference but the interaction with the environment. This interaction, through the loss of the interference terms, is obviously non-unitary. A non-unitary operation is synonymous with measurement.

## 5. Conclusion

It has been proven in this paper by Decoherence Theory that unitary evolution preserves interference terms in the reduced density matrix. The No-communication argument[12-14] is simply flawed resting on this premise.

We have argued before[1] that the pure and mixed states of an entangled system can be discerned by an interferometer, which itself has a unitary evolution operator. The issue isn't really just a problem in multi-particle entangled systems but is applicable too, in its purest sense, to single particle wavefunction collapse and path entanglement[6].

Given Aspect et al[3] and Zbinden et al[7] results the most fundamental questions are being asked about Objective Reality. Faster-than-light signalling can find no bar in the No-communication theory, it is just wrong.

One has to admit that the picture of reality presented by Relativity is on shaky ground when it comes to the transmission of mass-energy-less signals (pure quantum state information). True Science will apply Occam's Razor to questions of multiple universes, observer manifested reality or even retro-causality. Nature abhors unnecessary artificial constructs and complication; we suggest that the scheme of dropping the retarded time terms in the Lorentz transform[1, 8, 9] (figure 7 from Cornwall[1] reproduced at the end of this paper) is the way to go.

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Figure 7 reproduced from Cornwall[1].

<u>The Lorentz transform</u>: $x = \gamma\left(x' + vt'\right) \quad t = \gamma\left(t' + \frac{vx'}{c^2}\right)$

Describes the transformation between inertial frames for different observers of mass-energy phenomena. All information about the co-ordinates is sent as mass-energy <u>too</u> so inevitably our measurement of space and time is affected (a bit like kicking a soccer ball whilst the goal posts are moving!).

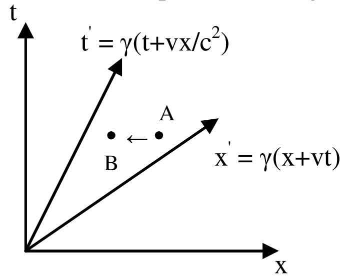
This view point leads to the space-time construct, destruction of simultaneity in space and time (events A and B below) and the consideration of co-ordinate transformations as hyperbolic rotations in 4-space (hyperbolic 'angle' α in analogy to θ in 3-space rotations).

$$u = \left(x_1 \ x_2 \ x_3 \ ict\right) \qquad u' = L(\alpha)u$$

*Hyperbolic rotation matrix*

$$L = \begin{matrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \cosh\alpha & i\sinh\alpha \\ 0 & 0 & -i\sinh\alpha & \cosh\alpha \end{matrix} \qquad \textit{Where } \alpha = \tanh^{-1}\frac{v}{c}$$

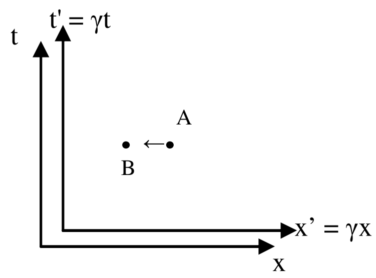
Thus we obtain the familiar space-time diagram:




Space-time diagram

The terms in the Lorentz transform Δx = γvΔt' and Δt = γvΔx'/$c^2$ can simply be understood as the <u>delay</u> in sending the information about the co-ordinates to the non-primed frame. For instance if it takes the primed frame Δt' seconds to perform a measurement then the frame will have moved a distance vΔt' which we correct back to the un-primed frame, γvΔt' in addition to any other distance measurement. As regards the time: the frame will have moved vΔt' once again so the light signal will require an extra vΔt'/c seconds to reach the source, now Δt' = Δx'/c so the extra time is γvΔx'/$c^2$ in the un-primed frame.

Sending information superluminally knocks out the terms Δx = γvΔt' and Δt = γvΔx'/$c^2$ in the Lorentz transform giving the following transformation diagram:




Lorentz's original view