The Quark Gluon Plasma Conundrum - Liquid or Gas ?

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Abstract

The behaviour of Quark Gluon Plasma (QGP) as a dense fluid with very low viscosity, exhibiting hydrodynamic flow and complete absorption of high momentum probes, demands a paradigmatic shift in our view of QGP, vis-a-vis the earlier held view of it being a gas of weakly interacting quarks and gluons. This would have been a major setback for the quark gas model of QGP, but for the discovery of a new empirical information. The study of the v_2 parameter of the elliptic flow as a function of KE_T, the transverse kinematic energy, displays evidence of scaling by n_q, the number of constituent quarks in baryons and mesons. This is an incontrovertible evidence of the underling role of the quark degrees of freedom in establishing the elliptic flow. So is the QGP a liquid of a gas? This is the new conundrum. Here we provide a resolution of this puzzle through a consistent application of the symmetry structure of the full $SU(3)_c$ group itself, rather than just its local Lie group algebra.

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The experimental study of the Quark Gluon Plasma (QGP) at CERN and BNL has been a story of grand success in the last two decades or so. On the theoretical side there have been setbacks though. For example, the very first idea that the QGP would be a gas of quarks, antiquarks and gluons, was found not to be up to the mark.

"The success of the hydrodynamic model in describing the bulk of hadronic production in Au + Au collisions at the RHIC has led to paradigmatic shift in our view of the QGP; instead of behaving like a gas of weakly interacting quarks and gluons, as naively expected on the basis of asymptotic freedom in QCD, it's collective properties rather reflect those of a "perfect fluid" with almost vanishing viscosity [1]. Actually it is the most perfect fluid created in the laboratory. It is also highly opaque to colored probes, as indicated by the observed large parton energy loss. Actually these two phenomena are fundamentally related to each other, and complement each other to bring about a strongly coupled plasma [2].

The above demolishes the free gas model of QGP. This would have been the end of the gas model, but nay, like the Phoenix it rises again. This is due to the new surprising result of the observation of the quark number scaling of the elliptic flow [3] (interestingly this was discovered by the appropriately named PHENIX Collaboration). They studied how the elliptic flow parameter v_2 depends upon the "transverse kinematic energy" $KE_T = \sqrt{m^2 + {p_T}^2} - m$. They found that $v_2(KE_T)$ displayed two branches, the upper one consisting of baryons and the lower one having only mesons. Most importantly, they found a further scaling of both KE_T and $v_2(KE_T)$ by n_q, the number of constituent quarks. This is a very strong empirical evidence for quark number scaling of the flow phenomenon. This point is further strengthened by the observation that ϕ - meson, although more massive than the nucleon, still follows the curve for mesons, and not that of the baryons. "While it is tempting to conclude that the strict scaling according to constituent quark content provides incontrovertible evidence for the underlying role of the quark degrees of freedom in establishing the elliptic flow, such a conclusion appears to be at odds with the observation of perfect fluidity" [4].

So is the QGP a fluid or a gas? This is the new conundrum. In this paper we present a resolution for this conundrum by studying the symmetry of the full color group $SU(3)_c$, rather than just relying upon the local Lie algebraic structure only.

It is believed that in QCD "transition from hadronic matter to the quark gluon matter is a transition from local color confinement (on the scale of 1 fm) to global cloy confinement" [5]. It is not clear as to what maintains long-range correlations implicit in global color-confinement, for example for sizes as large as the quark stars [6]. To better understand the role of the color degree of freedom we use the color projection technique which uses the complete structure of the full $SU(3)_c$ group.

With the mathematical development of the consistent inclusion of internal symmetries in a statistical thermodynamical description of quantum gases [7] the idea was applied to the color $SU(3)_c$ group [8,9]. Therein the group theoretical projection technique was used to project out color-singlet representation for a bulk system consisting of Quark Gluon Plasma at finite temperature. The requirement of imposition of color-singletness for these systems has been found to be of a great significance and work has been done using this technique of color projection. Several interesting results were obtained but perhaps the most significant was that if one were to compare a color unprojected bulk QGP system with a color-singlet projected QGP system then important finite size corrections are introduced [8,9]. These finite size corrections arising from the imposition of color- singletness disappear as the size and/or temperature of the system increases. This was taken to mean that for large-sized QGP systems, which may have been relevant in the early universe QCD phase transition scenarios one may automatically assume global color-singletness [5] of the system without any significant modifications. This allowed for the possible existence of large size stable quark stars (which were trivially assumed to be color-singlet [6] in the early universe QCD phase transition. These scenarios continue to dominate the hadronization ideas in the big bang models.

The orthogonality relation for the associated characters $\chi_{(p,\ q)}$ of the (p, q) multiplet of the group $SU(3)_c$ with the measure function $\zeta(\phi,\ \psi)$ is

$$\int_{SU(3)_c} d\phi\ d\psi\ \zeta\,(\phi,\psi)\,\chi^{\star}_{(p,\ q)}\,(\phi,\psi)\,\chi_{(p',\ q')}\,(\phi,\psi)\ =\ \delta_{p\ p'}\ \delta_{q\ q'} \tag{1}$$

Let us now introduce the generating function Z^G as

$$Z^G(T,V,\phi,\psi)\ =\ \sum_{p,q}\frac{Z_{(p,\ q)}}{d(p,q)}\chi_{(p,\ q)}(\phi,\psi) \tag{2}$$

with

$$Z_{(p,\ q)} \ = \ tr_{(p,q)} \left[\exp\left(-\beta \hat{H}_0 \right) \right] \tag{3}$$

$Z_{(p,q)}$ is the canonical partition function. The many-particle states which belong to a given multiplet (p, q) are used in the statistical trace with the free hamiltonian $\hat{H}_0$, d(p, q) is its dimensionality and β is the inverse of the temperature T. The projected partition function $Z_{(p,q)}$ can be obtained by using the orthogonality relation for the characters. Hence the projected partition function for any representation (p, q) is

$$Z_{(p,\ q)} \ = \ d(p,q) \int_{SU(3)_c} d\phi \ d\psi \ \zeta\left(\phi,\psi\right) \chi^{\star}_{(p,\ q)}(\phi,\psi) \ Z^G\left(T,V,\phi,\psi\right) \tag{4}$$

The characters of the different representations are as follows:

$$\begin{aligned}
\chi_{(1,\ 0)} &= exp(2i\psi/3) + 2\ exp(-i\psi/3)\ cos(\phi/2) && (5)\\
\chi_{(0,\ 1)} &= \chi^{\star}_{(1,\ 0)} && (6)\\
\chi_{(1,\ 1)} &= 2\ +\ 2\ \left[cos\phi + cos\left(\phi/2+\psi\right) + cos\left(-\phi/2+\psi\right)\right] && (7)\\
\chi_{(2,\ 2)} &= 2\ +\ 2\ \left[cos\phi + cos(3\phi/2)cos(\phi/2)\right] + \\
& \quad 2\ \left(1 + 2\ cos\phi\right)\left\{cos\left(\phi/2+\psi\right) + cos\left(-\phi/2+\psi\right) + cos2\psi + (1/2)\, cos(8)\right\} &&
\end{aligned}$$

The expressions of the generating function used in (4) is

$$Z^G(T,V,\phi,\psi) \ = \ tr\left[exp(-\beta\hat{H}_0 + i\phi\hat{I}_z + i\psi\hat{Y})\right] \tag{9}$$

where $\hat{I}_z$ and $\hat{Y}$ are the diagonal generators of the maximal abelian Cartan subgroup of $SU(3)_c$. Our plasma consists of light spin 1/2 (anti) quarks in the (anti) triplet representation (0, 1) and (1, 0) respectively, and massless spin one gluons in the octet representation (1,1). Note that the non-interacting hamiltonian $\hat{H}_0$ is diagonal in the occupation-number representation. In the same representation one can write the charge operators $\hat{I}_z$ and $\hat{Y}$ as linear combinations of particle-number operators. Hence Z^G can be easily calculated in the occupation-number representation. With an imaginary 'chemical potential' this is just like a grand canonical partition function for free fermions and bosons. One obtains

$$Z^G_{quark} = \prod_{q=l,m,n} \prod_k [1 + exp(-\beta\epsilon_k - i\alpha_q)]\,[1 + exp(-\beta\epsilon_k + i\alpha_q)] \quad (10)$$

$$Z^G_{glue} = \prod_{g=\mu,\nu,\rho,\sigma} \prod_k [1 - exp(-\beta\epsilon_k + i\alpha_g)]^{-1}\,[1 - exp(-\beta\epsilon_k - i\alpha_g)]^{-1} \quad (11)$$

Here the single-particle energies are given as ϵ_k. For (1, 0), (0, 1) and (1, 1) multiplets, the eigenvalues of $\hat{I}_z$ and $\hat{Y}$ gives the expression for different angles as:

$$\alpha_l = (1/2)\phi + (1/3)\psi,\ \alpha_m = (-1/2)\phi + (1/3)\psi,\ \ \alpha_n = (-2/3)\psi \quad (12)$$

$$\alpha_\mu = \alpha_l - \alpha_m,\ \alpha_\nu = \alpha_m - \alpha_n,\ \alpha_\rho = \alpha_l - \alpha_n,\ \alpha_\sigma = 0 \quad (13)$$

We neglect the masses of the light quarks. At large volume the spectrum of single particle becomes a quasi-continuous one and $\Sigma ... \rightarrow V/(2\pi)^3 \int d^3p ...$ Then one gets

$$Z^G(T, V, \phi, \psi) = Z^G_{quark}(T, V, \phi, \psi)\ Z^G_{glue}(T, V, \phi, \psi) \quad (14)$$

This then enables us to obtain the partition function for any representation ie. $Z_{(p,\ q)}$. One may thus obtain any thermodynamical quantity of interest for a particular representation. For example the energy

$$E_{(p,\ q)} = T^2 \frac{\partial}{\partial T} \ln Z_{(p,\ q)}. \quad (15)$$

Work was done earlier by several groups to impose color-singletness on the system [8,9]. One believes that the main consequence of the color interaction is to cause global color-confinement of quarks and gluons and this is automatically taken care of by restricting the partition function to color-singlet states [5]. One finds that

$$E_{(0,\ 0)} = E_0 + E_{corr} \quad (16)$$

where E_0 was the unprojected energy (ie. with no color restriction whatsoever) given by

$$E_0 = 3\ a_q\ V\ T^4 \quad (17)$$

with $a_q = (37\ \pi^2/90)$ and E_{corr} was the correction introduced due to the imposition of color-singletness. They found that [9] E_{corr} was significant only for the finite size i.e. when $TV^{1/3}/\hbar c$ was small (< 2) and vanished when $TV^{1/3}/\hbar c$ became large (> 2). "One may call this "phase transformation" of the system... In nuclear collisions relatively small droplets will be formed,just of the order $R \sim 2/T \sim 2.5 fm$" [9]. This would mean that color-singletness restriction only affects for these sizes while for large size and higher temperatures one need not perform explicit color projection calculation because the consequent corrections are negligible therein [9]. But below we shall show that this is not the whole story.

Next we projected out different representations like octet (1, 1), 27-plet (2, 2) etc. on this QGP. Now the idea is that for ground state one knows that the singlet state is bound and the higher representations are expelled to infinite energies. Also for the ground states the role of the higher representations may be significant and that is also quite well studied [10]. The point to be emphasized is that the role of global color-singletness at high temperatures is only an assumption and has never been explicitly demonstrated even in a model calculations. Here we would like to study the basis of this assumption and also the role, if any, of higher representations like octet, 27-plet etc. Therefore we projected out different representations [11] like octet (1,1). 27-plet (2,2) etc. for these QGP calculations. We shall study the further significance of the same in this paper.

Let us look at octet, 27-plet etc. projection. We take $\mu = 0$ case with 2 flavors. We plot in Fig. 1

$$D^{eff}_{(p,\ q)} \;=\; E_{(p,\ q)}/E_0 \;=\; 1 \;+\; E^{corr}_{(p,\ q)}/E_0 \tag{18}$$

Now puzzle arises when we look at the octet and the 27-plet representations. We note that for small values of $TV^{1/3}/\hbar c$ the octet and the 27-plet energies shoot up. Thus for our $\mu = 0$ case there is a clear distinction between the global color singlet states and the global color octet and 27-plet states. As the octet and the 27-plet states have moved to infinity, these are inaccessible to the ground states and where the global color singlet state dominates. It turns out that this is independent of the number, 0-, 2- or 3-flavors [11]. The fact that the global color singlet state gets favored over the color octet and 27-plet etc. representations, at low temperatures and/or small sizes, supports the global color symmetry concept.

But at higher values of $TV^{1/3}/\hbar c$ all, unprojected, singlet, octet and 27-plet states become degenerate. There is nothing to distinguish one from the other. Clearly one would expect that the higher representations like, (3,3) representation of dimension 64, (4,4) representation of dimension 125 and (5,5) representation of dimension 216 etc. would behave similarly - i.e. blowing up for small values of $TV^{1/3}/\hbar c$ and merging with the others for higher values of $TV^{1/3}/\hbar c$. We may extrapolate to the infinite dimensional self-conjugate representation and the same conclusion would hold.

Thus we conclude that all states, unprojected, color singlet, 27-plet, 64-plet, all the way to the infinite dimensional color projected representations for $\mu = 0$ case for QGP, are degenerate for higher values of $TV^{1/3}/\hbar c$. There is nothing that distinguishes one state from the other. What does it mean?

The answer is that this is providing us with an infinite dimensional Hilbert space. If color singlet state is the minimal invariant state then this one is the maximal color projected state. One can add nothing to it and one cannot take anything away from it. Thus it is an invariant of the $SU(3)_c$ group.

Hence what we find is that this infinite dimensional color projected state is what QPG exists in. This holds uniformly for each and every point of QGP. Thus QGP forms fully homogeneous and isotropic system in color space. The situation is similar to the homogeneity and isotropy of the real 3-dimensional space in cosmology. Thus in this QGP only two forms of motion are possible - uniform expansion or uniform contraction. This is so as, if any other motion were to occur, then some part of QGP would look different from the other regions and that is forbidden by the color isotropy and homogeneity of the system. This gives us our perfect fluid!

Now about the complete absorption of the high momentum partons. Clearly as being infinite dimensional color space, any colored entity cannot pass through it and is dissolved in it. Thus complete absorption of color in this fluid shall occur.

Hence the above is the "perfect fluid" state of the QGP. This also absorbs any color as well. This solves one part of the QGP conundrum. How about the other part.

For this we look at the mathematical nature of the of the character of a particular representation $\chi_{(p,\ q)}$ of the group $SU(3)_c$. It is an important mathematical property of character of a particular representation, that once constructed it forgets as to what and how many microscopic entities created it. "The important property of the definition (of group character) for the invariant group function is that it is independent of the microscopic structure of the states, which transform under the irreducible representation (p,q), i.e. it does not matter of how many particles the multiplet is made up". [4].

Thus we build up the color singlet state (for small values of $TV^{1/3}/\hbar c$) from point like current quarks, antiquarks and gluons, as these are what contribute to it as $3 \otimes \bar{3} = 1 + ...$ and $8 \otimes 8 = 1 + ...$ But for higher values of $TV^{1/3}/\hbar c$, it is degenerate with octet, 27-plet, 64-plet and all the representations right up to infinity. Clearly all these will contribute to the colors singlet state existing there. Now the quarks which give $3 \otimes \bar{3} = 1 +$ .. are current quarks no more. This, as all the infinity of color states would contribute to fatten this point like current quark as say, $3 \otimes (27 \otimes 27) \rightarrow 3 \otimes (1 + ..) \rightarrow 3$. This is now a quasi-particle or a constituent quark.

Now as the temperature of the QGP drops, all the highly colored representations 264-plet, 125-plet, 64-plet, 27-plet and octet, go out of contention by disappearing to infinite energies. Only color singlet state, now made up of constituent quarks, moves down to manifest as physical hadrons.

Now this has only constituent quarks and no gluons left in it. Therefor when this leads to creation of baryon-antibaryons and mesons, it will be pure constituent quark property of these hadrons and quark scaling (as seen by the PHENIX group [3]) shall occur. Thus we explain this mysterious property of the QGP as well.

Thus this model is able to explain the QGP conundrum of whether it is a liquid or a gas. The explanation arises due to a consistent application of the global aspect of the full color group $SU(3)_c$.

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Figure 1: D_{eff} (see text) for the color representations singlet, octet and 27-plet (with two flavors) as a function of $TV^{1/3}/\hbar c$