The relation between counting primes and twin primes

by Islem Ghaffor*

ghaffor.prime@outlook.com

0.1 Abstract

In this paper we give a new formula for the relation between counting primes and twin primes, we use in this formula the arithmetic progressions and the cardinal of the set.

0.2 Notations

\( p_n \): The \( n \)-th prime number \( p_1 = 2; p_2 = 3; \ldots \) and so on.

\( \pi(p_n^2 - 3) \): The number of primes less than \( p_n^2 - 3 \).

\( \pi_2(p_n^2 - 3) \): The number of twin primes less than \( p_n^2 - 3 \).

\( I_n \) and \( G_n \): Two sets.

\( l \): The length of the arithmetic progression.

0.3 Formula

\[ 4 \leq n \]

\[ \pi_2(p_n^2 - 3) = \pi(p_n^2 - 3) + Card(I_n \cap G_n) - \left( \frac{p_n^2 - 1}{6} \right) \]

with

\[ I_n = \bigcup_{a=3}^{n-1} \left( \left( \frac{p_a^2 - 13}{6} \right) + bp_a \right) \]

\[ b = \left[ \frac{(p_n^2) - (p_a^2)}{6p_a} \right] \]

and

\[ G_n = \bigcup_{a=3}^{n-1} \left( \left( \frac{p_a^2 + (6 \left\lfloor \frac{p_a}{6} \right\rfloor + 9)p_a - 22}{12} \right) + bp_a \right) \]

\[ b = \left[ \frac{(2p_n^2 - 4) - (p_a^2 + (6 \left\lfloor \frac{p_a}{6} \right\rfloor + 9)p_a)}{12p_a} \right] \]

* My biography: [www.primepuzzles.net/thepuzzlers/Ghaffor.htm](http://www.primepuzzles.net/thepuzzlers/Ghaffor.htm)
0.4 Some examples

<table>
<thead>
<tr>
<th>$n$</th>
<th>$p_n$</th>
<th>$\pi(p_n^2 - 3)$</th>
<th>$\text{Card}(I_n \cap G_n)$</th>
<th>$p_n^2 - 1/6$</th>
<th>$\pi_2(p_n^2 - 3)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>7</td>
<td>$\pi(46) = 14$</td>
<td>0</td>
<td>8</td>
<td>$\pi_2(46) = 6$</td>
</tr>
<tr>
<td>5</td>
<td>11</td>
<td>$\pi(118) = 30$</td>
<td>0</td>
<td>20</td>
<td>$\pi_2(118) = 10$</td>
</tr>
<tr>
<td>6</td>
<td>13</td>
<td>$\pi(166) = 38$</td>
<td>2</td>
<td>28</td>
<td>$\pi_2(166) = 12$</td>
</tr>
<tr>
<td>7</td>
<td>17</td>
<td>$\pi(286) = 61$</td>
<td>6</td>
<td>48</td>
<td>$\pi_2(286) = 19$</td>
</tr>
<tr>
<td>8</td>
<td>19</td>
<td>$\pi(358) = 71$</td>
<td>10</td>
<td>60</td>
<td>$\pi_2(358) = 21$</td>
</tr>
<tr>
<td>9</td>
<td>23</td>
<td>$\pi(526) = 99$</td>
<td>14</td>
<td>88</td>
<td>$\pi_2(526) = 25$</td>
</tr>
<tr>
<td>10</td>
<td>29</td>
<td>$\pi(838) = 145$</td>
<td>28</td>
<td>140</td>
<td>$\pi_2(838) = 33$</td>
</tr>
<tr>
<td>11</td>
<td>31</td>
<td>$\pi(958) = 162$</td>
<td>33</td>
<td>160</td>
<td>$\pi_2(958) = 35$</td>
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<tr>
<td>12</td>
<td>37</td>
<td>$\pi(1366) = 218$</td>
<td>56</td>
<td>228</td>
<td>$\pi_2(1366) = 46$</td>
</tr>
<tr>
<td>13</td>
<td>41</td>
<td>$\pi(1678) = 263$</td>
<td>70</td>
<td>280</td>
<td>$\pi_2(1678) = 53$</td>
</tr>
<tr>
<td>14</td>
<td>43</td>
<td>$\pi(1846) = 282$</td>
<td>82</td>
<td>308</td>
<td>$\pi_2(1846) = 56$</td>
</tr>
<tr>
<td>15</td>
<td>47</td>
<td>$\pi(2206) = 328$</td>
<td>107</td>
<td>368</td>
<td>$\pi_2(2206) = 67$</td>
</tr>
<tr>
<td>16</td>
<td>53</td>
<td>$\pi(2806) = 409$</td>
<td>139</td>
<td>468</td>
<td>$\pi_2(2806) = 80$</td>
</tr>
<tr>
<td>17</td>
<td>59</td>
<td>$\pi(3478) = 487$</td>
<td>186</td>
<td>580</td>
<td>$\pi_2(3478) = 93$</td>
</tr>
</tbody>
</table>

$n = 4$

\[ \pi_2(46) = \pi(46) + \text{Card}(I_4 \cap G_4) - 8 = 6 + \text{Card}(I_4 \cap G_4) \]

\[ I_4 = (2 + b5 \ l = 1) = (2) \]

\[ G_4 = (4 + b5 \ l = 1) = (4) \]

\[ \pi_2(46) = 6 + \text{Card}((2) \cap (4)) = 6 + \text{Card}(\emptyset) = 6 \]

$n = 5$

\[ \pi_2(118) = \pi(118) + \text{Card}(I_5 \cap G_5) - 20 = 10 + \text{Card}(I_5 \cap G_5) \]

\[ I_5 = (2 + b5 \ l = 4) \cup (6 + b7 \ l = 2) \]

\[ I_5 = (2 \ 7 \ 12 \ 17) \cup (6 \ 13) = (2 \ 6 \ 7 \ 12 \ 13 \ 17) \]

\[ G_5 = (4 + b5 \ l = 3) \cup (11 + b7 \ l = 1) \]

\[ G_5 = (4 \ 9 \ 14) \cup (11) = (4 \ 9 \ 11 \ 14) \]

\[ \pi_2(118) = 10 + \text{Card}(I_5 \cap G_5) = 10 + \text{Card}((2 \ 6 \ 7 \ 12 \ 13 \ 17) \cap (4 \ 9 \ 11 \ 14)) \]

\[ \pi_2(118) = 10 + \text{Card}(\emptyset) = 10 \]
\[ n = 6 \]

\[ \pi_2(166) = \pi(166) + \text{Card}(I_6 \cap G_6) - 28 = 10 + \text{Card}(I_6 \cap G_6) \]

\[ I_6 = (2 + b5 \ l = 5) \cup (6 + b7 \ l = 3) \cup (18 + b11 \ l = 1) \]

\[ I_6 = (2 \ 7 \ 12 \ 17 \ 22) \cup (6 \ 13 \ 20) \cup (18) \]

\[ G_6 = (4 + b5 \ l = 5) \cup (11 + b7 \ l = 3) \cup (22 + b11 \ l = 1) \]

\[ G_6 = (4 \ 9 \ 14 \ 19 \ 24) \cup (11 \ 18 \ 25) \cup (22) \]

\[ G_6 = (4 \ 9 \ 11 \ 14 \ 18 \ 19 \ 22 \ 24 \ 25) \]

\[ \pi_2(166) = 10 + \text{Card}((2 \ 6 \ 7 \ 12 \ 13 \ 17 \ 18 \ 20 \ 22) \cap (4 \ 9 \ 11 \ 14 \ 18 \ 19 \ 22 \ 24 \ 25)) \]

\[ \pi_2(166) = 10 + \text{Card}(18 \ 22) = 12 \]

0.5 The result

Calculate \( \lim_{n \to \infty} \pi(p_n^2 - 3) + \text{Card}(I_n \cap G_n) - \left(\frac{p_n^2 - 1}{6}\right) \) means twin prime conjecture proof.

0.6 The problem

We must study \( \text{Card}(I_n \cap G_n) \) for calculate the limit.

References


The end of the paper

My biography

www.primepuzzles.net/thepuzzlers/Ghaffor.htm

Self-educated with no home, no study and no job.

My conference

Participation in the 3rd International Conference on Applied Algebra:

virtuelcampus.univ-msila.dz/faculte-mi/index.php/2-features/109-the-3rd-international-conference-on-applied-algebra

My paper -the number of twin primes-: www.primepuzzles.net/conjectures/conj_072.htm

was accepted by the scientific committee.

My dream is just study mathematics in every second of my life and continue my research in university outside Algeria.

Can you help me just by send to me an invitation from your university to my e-mail?

ghaffor.prime@outlook.com