The Physical Meaning Of The Wavefunction

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Abstract: Under some well-defined conditions the mathematical formalism of quantum mechanics enables physicists, chemists and other to calculate and predict the outcome of a vast number of experiments. In fact, especially the Schrödinger equation which involves an imaginary quantity describes how a quantum state of a physical system changes with time and is one of the main pillars of modern quantum mechanics. The **wave function** itself is a determining part of the Schrödinger equation, but the physical meaning of the wave function is still not clear. Altogether, does the wave function represent a new kind of reality? This publication will solve the problem of the physical meaning of the wave function by investigating the relationship between the wave function and the theory of special relativity. It is shown that the wave function is determined by notion co-ordinate **time** of the special theory of relativity. Moreover, the result of this investigation suggests a new understanding of the wave function, according to which the wave function and co-ordinate time of the theory of special relativity are equivalent. Apparently, based upon the close relationship between time and gravitational field and the normalized relativistic energy-momentum relation, this contribution provides a way to calculate the **"mass-equivalent" of a photon** in SI units as 7.372 503 726 490 51 * 10^{-51} and the **"mass-equivalent" of a graviton** in SI units as 1.346053370*10^{-136} . A necessary mathematical formalism for **the quantization of the gravitational field** is developed.

Key words: Quantum theory, relativity theory, unified field theory, causality.

1. Introduction

The Schrödinger equation, published in 1926 by the Austrian physicist Erwin Rudolf Josef Alexander Schrödinger (1887-1961), is determined by Newton's (1642 – 1727) second law, in its original form known as

"Mutationem motus proportionalem esse vi motrici impressae, et fieri secundum lineam rectam qua vis illa imprimitur." [1]

and to some extent an analogue of Newton's second law in quantum mechanics. Leonhard Euler (1707-1783), a pioneering Swiss mathematician and physicist, formulated in 1752 Newton lex secunda [2] in its mathematical form as

$$\vec{F} = m \times \vec{a}. \tag{1}$$

The famous Schrödinger equation [3], a partial differential equation which describes how a quantum state of a system changes with time. The Schrödinger equation **for any system**, **no matter whether relativistic or not**, no matter how complicated, has the form

$$H \times {}_{R}\Psi(t) = i\hbar \frac{\partial}{\partial t}\, {}_{R}\Psi(t), \tag{2}$$

where i is the imaginary unit, $\hbar = \frac{h}{2 \times \pi}$ is Planck's constant divided by 2xπ, the symbol $\frac{\partial}{\partial t}$ indicates a partial derivative with respect to time t, ${}_{R}\Psi$ is the wave function of the quantum system, and H is the Hamiltonian operator.

In quantum mechanics, the Hamiltonian operator is a quantum mechanical operator which characterizes the total energy of a quantum mechanical system and is usually denoted by H. The Hamiltonian operator H takes different forms depending upon situation. The form of the Schrödinger equation itself depends on the physical situation and is determined by the wavefunction ${}_{R}\Psi$. The wavefunction itself is one of the most fundamental concepts of quantum mechanics.

Schrödinger himself states, that he has "**not attached a definite physical meaning to the wavefunction Ψ**." [4] The physical meaning of the wave function is in dispute in the alternative interpretations of quantum mechanics. The de Broglie-Bohm theory or the many-worlds interpretation has another view on the physical meaning of the wave function then the Copenhagen interpretation of the wave function.

In view of this unsatisfactory situation, it seems to be necessary to put some light on the problem of the physical meaning of the wave function from the standpoint of the theory of special relativity. In a similar way, hereafter, we shall restrict ourselves to a one-dimensional treatment in order to decrease the amount of notation needed, since in all cases, the generalization to *four* (i. e. quantum mechanics) or *n-dimensions* (i. e. quantum field theory) will be equally simple.

2. Definitions

2.1. **Definition. Einstein's Mass-Energy Equivalence Relation**

According to Albert Einstein [5], it is

$${}_{0}m = {}_{R}m * \sqrt[2]{1 - \frac{v^2}{c^2}} \tag{3}$$

or equally

$$\frac{{}_{0}E}{{}_{R}E} = \frac{{}_{0}m \times c^2}{{}_{R}m \times c^2} = \sqrt[2]{1 - \frac{v^2}{c^2}} \tag{4}$$

where ${}_{0}m$ denotes the "rest" mass, ${}_{R}m$ denotes the "relativistic" mass, v denotes the relative velocity and c denotes the speed of light in vacuum.

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Scholium.
In general, let **E($_0$m)** denote the expectation value of $_0$m (as determined by the co-moving observer), let **E($_R$m)** denote the expectation value of $_R$m as determined by the stationary observer. A (co-) *moving observer* is defined by being at rest (relative velocity **v = 0**) relative to a moving (quantum mechanical) object. There are circumstances where the relative velocity **v** between a *stationary observer* and a moving (quantum mechanical) object can be equal to **v=0**. In contrast to the stationary observer, the relative velocity **v** between a (co-) *moving observer* and a moving (quantum mechanical) object is under any circumstances equal to **v=0**. In general, the "rest-mass" can be treated as an eigenvalue as obtained by a (co-) *moving observer* after the collapse of the wave function into an eigenfunction.

2.2. **Definition. The Normalized Relativistic Energy Momentum Relation**

Based on Einstein's mass-energy equivalence, we define *the normalized relativistic energy momentum relation* [6], a probability theory consistent formulation of Einstein's energy momentum relation, as

$$\frac{{}_0m^2}{{}_Rm^2}+\frac{v^2}{c^2}=1 \tag{5}$$

where $_0$k denotes a kind of *a complex coefficient* with

$$ {}_0k^2=\frac{{}_0m^2}{{}_Rm^2} \quad \text{i.e.} \left|{}_0k\right|=\frac{{}_0m}{{}_Rm} \tag{6}$$

and *anti $_0k$* denoted as $_0\underline{k}$ is defined as

$$ {}_0\underline{k}^2\equiv 1-\frac{{}_0m^2}{{}_Rm^2}\equiv\frac{v^2}{c^2} \tag{7}$$

2.3. **Definition. The Principle Of Superposition**

A (quantum mechanical) system can sometimes exist as a linear combinations of (two) other (eigen-) states, for example

$$ {}_R\Psi\equiv{}_0k\times{}_0\psi+{}_0\underline{k}\times{}_0\underline{\psi}\,. \tag{8}$$

Under conditions where all three functions are normalized and where $_0\psi$ and $_0\underline{\psi}$ are orthogonal we find

$$\int\Psi\times\Psi^*d\tau\equiv{}_0k^2+{}_0\underline{k}^2\equiv 1 \tag{9}$$

The *particle-wave duality* [7] is defined by

$$\frac{{}_0m^2}{{}_Rm^2}+\frac{v^2}{c^2}=\frac{{}_0m^2\times c^2}{{}_Rm^2\times c^2}+\frac{v^2\times c^2\times{}_R\,m^2}{c^2\times c^2\times{}_R\,m^2}=\frac{{}_0E^2}{{}_RE^2}+\frac{\left(v^2\times{}_R\,m^2\right)\times c^2}{{}_RE^2}=\frac{{}_0E^2}{{}_RE^2}+\frac{\left({}_Rp^2\right)\times c^2}{{}_RE^2}=\frac{{}_0E^2}{{}_RE^2}+\frac{{}_{Wave}E^2}{{}_RE^2}=1 \tag{10}$$

where ${}_{R}p$ denotes the momentum and ${}_{Wave}E$ denotes the energy of an associated electromagnetic wave. In general, due to quantum mechanics, it is ${}_{Wave}E = {}_{r}p*c$.

2.4. **Definition. The Eigenfunction and the Anti-Eigenfunction.**

The wavefunction ${}_{R}\Psi$ itself is not an eigenfunction of an operator. However, every wavefunction can be expressed as *a superposition of eigenfunctions* of an operator such that

$${}_R\Psi = {}_0k \times_0 \psi + {}_0\underline{k} \times_0 \underline{\psi} \tag{11}$$

where ${}_{0}\psi$ denotes the corresponding *eigenfunction* (as determined by a co-moving observer) and ${}_{0}\underline{\psi}$ denotes the corresponding *anti-eigenfunction*. The *complex coefficient* ${}_{0}k$ represent the degree to which the full wavefunction possesses the character of an eigenfunction, the degree to which the full wavefunction is determined by an eigenfunction. The *anti complex coefficient* ${}_{0}\underline{k}$ represent the degree to which the full wavefunction does not possesses the character of a special eigenfunction ${}_{0}\psi$, the degree to which the full wavefunction is not determined by a special eigenfunction ${}_{0}\psi$.

Scholium.
The wavefunction ${}_{R}\Psi$ corresponding to the system state can change (out of itself or by a third [i.e. measurement]) into an eigenfunction ${}_{0}\psi$. The change of the wavefunction into an eigenfunction of an operator (corresponding to the measured quantity) is called *wavefunction collapse*. In general, the superposition principle follows as

$${}_R\Psi = \sum_{i=0}^{n} {}_ik \times {}_i\psi = {}_0k \times_0 \psi + \sum_{i=1}^{n} {}_ik \times {}_i\psi = {}_0k \times_0 \psi +_0 \underline{k} \times_0 \underline{\psi} \tag{12}$$

where the corresponding eigenfunction of the moving observer is denoted as ${}_{0}\psi$ and the corresponding anti eigenfunction is denoted as ${}_{0}\underline{\psi}$. Corresponding to each eigenvalue is an 'eigenfunction'. Only certain eigenvalues with associated eigenfunctions are able to satisfy Schrödinger's equation. The *eigenvalue* as measured by a co-moving observer is one of the eigenvalues of the quantum mechanical observable. The eigenvalue (corresponding to some scalar) concept as such is not limited only to energy. Finding a specific function (i.e. eigenfunction) which describes an energy state (i.e. a solution to the Schrodinger equation) is very important. Under conditions where the eigenvalues are discrete, a physical variable is said to be 'quantized' and an index i plays the role of a 'quantum number' which is characterizing a specific state.

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2.5. **Definition. Einstein's Relativistic Time Dilation Relation**

An accurate **clock in motion slow down** with respect a stationary observer (observer at rest). The proper time ${}_0t$ of a clock moving at constant velocity v is related to a stationary observer's coordinate time ${}_Rt$ by Einstein's relativistic time dilation [8] and defined as

$$ {}_0t = {}_Rt \times \sqrt[2]{1-\frac{v^2}{c^2}} \tag{13} $$

where ${}_0t$ denotes the "proper" time, ${}_Rt$ denotes the "relativistic" (i. e. stationary or coordinate) time, v denotes the relative velocity and c denotes the speed of light in vacuum. Equally, it is

$$ \frac{{}_0t}{{}_Rt} = \sqrt[2]{1-\frac{v^2}{c^2}} \tag{14} $$

or

$$ \frac{{}_0t}{c^2} \times \frac{c^2}{{}_Rt} = \sqrt[2]{1-\frac{v^2}{c^2}} \,. \tag{15} $$

Scholium.
Coordinate systems can be chosen freely, deepening upon circumstances. In many coordinate systems, an event can be specified by one time coordinate and three spatial coordinates. The time as specified by the time coordinate is denoted as coordinate time. Coordinate time is distinguished from proper time. The concept of proper time, introduced by Hermann Minkowski in 1908 and denoted as ${}_0t$, incorporates Einstein's *time dilation effect.* In principle, Einstein is defining time exclusively for every place where a watch, measuring this time, is located.

"... Definition ... der ... Zeit ... für den Ort, an welchem sich die Uhr … befindet ..." [9]

In general, a watch is treated as being at rest relative to the place, where the same watch is located.

"Es werde ferner mittels der **im ruhenden System** befindlichen **ruhend**en Uhren die Zeit t [i. e. ${}_Rt$, author] des ruhenden Systems ... bestimmt, ebenso werde die Zeit τ [${}_0t$, author] des **bewegten System**s, in welchen sich relativ zu letzterem **ruhend**e Uhren befinden, bestimmt..." [10]

Only, the place where a watch at rest is located can move together with the watch itself. Therefore, due to Einstein, it is necessary to distinguish between clocks as such which are qualified to mark the time ${}_Rt$ when at rest relatively to the stationary system R, and the time ${}_0t$ when at rest relatively to the moving system O.

"Wir denken uns ferner eine der Uhren, welche **relativ zum ruhenden System ruhend** die Zeit t [${}_Rt$, author], **relativ zum bewegten System ruhend** die Zeit τ [${}_0t$, author] anzugeben befähigt sind ..." [11]

In other words, we have to take into account that both observers have at least one point in common, the stationary observer R and the moving observer O are at rest, but at rest relative to what? The stationary observer R is at rest relative to a stationary co-ordinate system R, the moving observer O is at rest relative to a moving co-ordinate system O. Both co-ordinate systems can but must not be at rest relative to each other. The time $_{R}t$ of the stationary system R is determined by clocks which are at rest relatively to that stationary system R. Similarly, the time $_{0}t$ of the moving system O is determined by clocks which are at rest relatively to that the moving system O. In last consequence, due to Einstein's theory of special relativity, an accelerated clock ($_{0}t$) will measure a smaller elapsed time between two events than that measured by a non-accelerated (inertial) clock ($_{R}t$) between the same two events.

2.6. **Definition. The Normalized Time Dilation Relation**

As defined above, due to Einstein's special relativity, it is

$$\frac{_{0}t}{_{R}t} = \sqrt[2]{1 - \frac{v^2}{c^2}} . \tag{16}$$

The normalized time dilation relation follows as

$$\frac{_{0}t^2}{_{R}t^2} + \frac{v^2}{c^2} = 1 . \tag{17}$$

2.7. **Definition. The Energy Operator H**

In quantum mechanics, the Hamiltonian, named after the Irish mathematician Hamilton, is a quantum mechanical operator corresponding to the total energy of a quantum mechanical system rather than Newton's second law F=m*a and usually denoted by H. By analogy with classical mechanics and special relativity, the Hamiltonian is i. e. the sum of operators corresponding to the potential and kinetic energies (of all the particles) associated with a quantum mechanical system and can take different forms depending on the situation. The Hamiltonian operator H is Hermitian. According to the expansion postulate, the wavefunction can be expanded as a series of its eigenfunctions where an eigenfunction belongs to an eigenvalue of H. The Hamiltonian function is equal to the total energy of the system. Consequently, an eigenstate of the operator H is one in which the energy is perfectly defined and equal to i. e. $_{0}E$. Thus far, an important property of Hermitian operators is that their eigenvalues are real. The total energy operator H is determined as

$$H = i\hbar \frac{\partial}{\partial t} . \tag{18}$$

For our purposes, the (non-relativistic or relativistic) Hamiltonian is corresponding to the total energy of a quantum mechanical object or system. Thus far, it is

$$_{R}E = H = i\hbar \frac{\partial}{\partial t} \tag{19}$$

where E_R is identical with the notion "relativistic" energy of a (quantum mechanical) system.

2.8. **Definition. The Quantum Mechanical Operator Of Matter**

In quantum mechanics, the Hamiltonian, named after the Irish mathematician Hamilton, is the total energy operator. Thus far we define the quantum mechanical operator of matter as

$$_{R}M \equiv \hat{M} \equiv \frac{_{R}E}{c^{2}} = \frac{H}{c^{2}} = \frac{i\hbar}{c^{2}} \times \frac{\partial}{\partial t} \tag{20}$$

where $_{R}M$ denotes the quantum mechanical operator of matter (and not only of mass) form the standpoint of the stationary (i.e. relativistic) observer R.

2.9. **Definition. The Relationship Between Energy And Time**

Let

$$_{R}S \equiv _{R}E + _{R}t \tag{21}$$

where $_{R}E$ denotes the 'relativistic' energy and $_{R}t$ denotes the 'relativistic' time.

Scholium.
The notion $_{R}S$ can but must not be the equivalent of space. Following Aristotele's principle of the excluded middle, *tertium no datur*, it is important to stress out, that **all but energy** is denoted as time. Consequently, there is no third between energy and time, a third is not given. Let

$$_{O}S \equiv _{O}E + _{O}t \tag{22}$$

where $_{O}E$ denotes the 'rest' energy and $_{O}t$ denotes the 'proper' time. Due to special relativity it is

$$_{O}S \equiv \sqrt[2]{1 - \frac{v^{2}}{c^{2}}} \times _{R}S \,. \tag{23}$$

2.10. **Definition. The Relationship Between Matter And Gravitational Field**

From the standpoint of a stationary observer R let us define

$$_{R}U \equiv _{R}M + _{R}g \tag{24}$$

where $_{R}M$ denotes the matter and $_{R}g$ denotes the gravitational field. From the standpoint of a moving observer O we define

$$_{O}U \equiv _{O}M + _{O}g \tag{25}$$

where $_{O}M$ denotes the matter from the standpoint of a moving observer and $_{O}g$ denotes the gravitational field from the standpoint of a moving observer.

Scholium.

In the context of general relativity, Einstein himself demands that everything but the gravitational field has to be treated as matter. Thus far, matter as such includes matter in the ordinary sense and the electromagnetic field as well. In other words, there is no third between matter and gravitational field. Einstein himself wrote:

"Wir unterscheiden im folgenden zwischen 'Gravitationsfeld' und 'Materie', in dem Sinne, daß alles außer dem Gravitationsfeld als 'Materie' bezeichnet wird, also nicht nur die 'Materie' im üblichen Sinne, sondern auch das elektro-magnetische Feld. " [12]

Einstein's writing translated into English:

>> We make a distinction hereafter between 'gravitational field' and 'matter' in this way, that we denote everything but the gravitational field as 'matter', the word matter therefore includes not only matter in the ordinary sense, but the electromagnetic field as well. <<

This definition of the relationship between matter and gravitational field is based on Einstein's definition of **matter** (i. e. not only mass) **ex negativo.** Clearly, it is $_R\mathbf{U} = {_R\mathbf{U}} + \mathbf{0}$ or in other words $_R\mathbf{U} = {_R\mathbf{U}} - {_R\mathbf{M}} + {_R\mathbf{M}}$. The observable $_RU$ is determined by matter and something else. Due to Einstein, the other of matter is the gravitational field or $_RU - {_RM} = {_Rg}$. **But there is matter even under conditions of the special theory of relativity. One feature of matter as such is its own gravitational field, whatever the strength (i. e. acceleration) of such a gravitational field may be. The existence of a gravitational field cannot be restricted to or identified with the strength of the gravitational field itself.**

2.11. Definition. The Relationship Between $_R$S And $_R$U

Let us define

$$c^2 \equiv \frac{_RS}{_RU} \,. \tag{26}$$

Due to special relativity it is

$$_OU \equiv \frac{_OS}{c^2} \equiv \sqrt[2]{1-\frac{v^2}{c^2}} \times \frac{_RS}{c^2} \equiv \sqrt[2]{1-\frac{v^2}{c^2}} \times {_RU} \,. \tag{27}$$

This relation yields

$$_Og \equiv \sqrt[2]{1-\frac{v^2}{c^2}} \times {_Rg} \quad . \tag{28}$$

2.12. Definition. The Square Of The Wavefunction (i. e. Born's rule).

In general, the wavefunction as such represents the probability amplitude for finding a particle *at a given point* in space *at a given time*. Due to special theory of relativity, time as measured by a stationary observer can be different from time as determined in the same respect by a moving observer. Thus far, let us define the following.

Let the wavefuntion ${}_R\Psi({}_RX, {}_Rt)$ denote the single-valued probability amplitude at $({}_RX, {}_Rt)$ where ${}_RX$ is position and ${}_Rt$ is time from the standpoint of *a stationary observer*. Let ${}_R\Psi^*({}_RX, {}_Rt)$ denote *the complex conjugate* of the wavefuntion ${}_R\Psi({}_RX, {}_Rt)$ from the standpoint of a stationary observer.

Let the wavefuntion ${}_0\Psi({}_0X, {}_0t)$ denote the single-valued probability amplitude at $({}_0X, {}_0t)$ where ${}_0X$ is position and ${}_0t$ is time from the standpoint of *a co-moving observer*. Let ${}_0\Psi^*({}_0X, {}_0t)$ denote *the complex conjugate* of the wavefuntion ${}_0\Psi({}_0X, {}_0t)$ from the standpoint of *a co-moving observer*.

Let ${}_0p({}_0X, {}_0t)$ denote the probability from the standpoint of *a co-moving observer* that a "particle" will be found at $({}_0X, {}_0t)$.

Let ${}_Rp({}_RX, {}_Rt)$ denote the probability from the standpoint of a stationary observer that a "particle" will be found at $({}_RX, {}_Rt)$.

In general, due the Born's rule, named after Max Born [13], it is

$$ {}_R p({}_R X, {}_R t) = \left| {}_R\Psi({}_R X, {}_R t) \right|^2 = {}_R\Psi({}_R X, t) \times {}_R\Psi^*({}_R X, {}_R t) \tag{29} $$

and

$$ {}_O p({}_O X, {}_O t) = \left| {}_O\Psi({}_O X, {}_O t) \right|^2 = {}_O\Psi({}_O X, {}_O t) \times {}_O\Psi^*({}_O X, {}_O t) \quad . \tag{30} $$

From these definitions (i. e. Born's rule) follows that

$$ {}_R\Psi({}_R X, {}_R t) = \frac{{}_R p({}_R X, {}_R t)}{{}_R\Psi^*({}_R X, {}_R t)} \; or \; that \; \frac{{}_R\Psi({}_R X, {}_R t) \times {}_R\Psi^*({}_R X, {}_R t)}{{}_R p({}_R X, {}_R t)} = 1 \tag{31} $$

and that

$$ {}_O\Psi({}_O X, {}_O t) = \frac{{}_O p({}_O X, {}_O t)}{{}_O\Psi^*({}_O X, {}_O t)} \; or \; that \; \frac{{}_O\Psi({}_O X, {}_O t) \times {}_O\Psi^*({}_O X, {}_O t)}{{}_O p({}_O X, {}_O t)} = 1 \tag{32} $$

Scholium.
In general, the wavefuntion ${}_0\Psi({}_0X, {}_0t)$ denotes an eigenfunction.

2.13. Definition. The Wavefunction As Such Is Reference Frame Dependent.

Theoretically, it is possible that the *probability* as such is *reference frame independent* (i. e. the moving and the stationary observer will agree on the numerical value of probability) while the *wavefunction* as such is *reference frame dependent*. Under these conditions let us consider the following. In general, let

$$\frac{{}_{O}\Psi({}_{O}X,{}_{O}t)}{{}_{R}\Psi({}_{R}X,{}_{R}t)}=\frac{{}_{R}\Psi^{*}({}_{R}X,{}_{R}t)}{{}_{O}\Psi^{*}({}_{O}X,{}_{O}t)}=\sqrt[2]{1-\frac{v^{2}}{c^{2}}} \tag{33}$$

where the wavefuntion ${}_{R}\Psi({}_{R}X, {}_{R}t)$ denotes the single-valued probability amplitude at $({}_{R}X,{}_{R}t)$ where ${}_{R}X$ is position and ${}_{R}t$ is time from the standpoint of *a stationary observer*, ${}_{R}\Psi^{*}({}_{R}X, {}_{R}t)$ denotes the complex conjugate of the wavefuntion ${}_{R}\Psi({}_{R}X, {}_{R}t)$ from the standpoint of a stationary observer, the wavefuntion ${}_{0}\Psi({}_{0}X, {}_{0}t)$ denotes the single-valued probability amplitude at $({}_{0}X, {}_{0}t)$where ${}_{0}X$ is position and ${}_{0}t$ is time from the standpoint of *a co-moving observer* (i. e. an eingenfunction) and ${}_{0}\Psi^{*}({}_{0}X, {}_{0}t)$ denotes the complex conjugate of the wavefuntion ${}_{0}\Psi({}_{0}X, {}_{0}t)$ from the standpoint of *a co-moving observer*.

2.14. **Definition. The Expectation Value Of X.**

In mathematical statistics and probability theory, the expectation value at one point in space-time ${}_{R}t$ from the standpoint of *a stationary observer*, denoted as $E({}_{R}X_{t})$, is defined as

$$E({}_{R}X_{t})={}_{R}\ p({}_{R}X,{}_{R}t)\times{}_{R}X_{t}={}_{R}\ \Psi({}_{R}X,{}_{R}t)\times{}_{R}\Psi^{*}({}_{R}X,{}_{R}t)\times{}_{R}X_{t} \tag{34}$$

The expectation value at one point in space-time ${}_{0}t$ from the standpoint of *a co-moving observer*, denoted as $E({}_{0}X_{t})$, is defined as

$$E({}_{O}X_{t})={}_{O}p({}_{O}X,{}_{O}t)\times{}_{O}X_{t}={}_{O}\Psi({}_{O}X,{}_{O}t)\times{}_{O}\Psi^{*}({}_{O}X,{}_{O}t)\times{}_{O}X_{t} \tag{35}$$

In general it is,

$${}_{O}p({}_{O}X,{}_{O}t)=\frac{{}_{O}p({}_{O}X,{}_{O}t)\times{}_{O}X_{t}\times{}_{O}\ p({}_{O}X,{}_{O}t)\times{}_{O}X_{t}}{{}_{O}p({}_{O}X,{}_{O}t)\times{}_{O}X_{t}\times{}_{O}X_{t}}=\frac{E({}_{O}X_{t})^{2}}{E({}_{O}X_{t}{}^{2})} \tag{36}$$

or

$${}_{R}p({}_{R}X,{}_{R}t)=\frac{{}_{R}p({}_{R}X,{}_{R}t)\times{}_{R}X_{t}\times{}_{R}\ p({}_{R}X,{}_{R}t)\times{}_{R}X_{t}}{{}_{R}p({}_{R}X,{}_{R}t)\times{}_{R}X_{t}\times{}_{R}X_{t}}=\frac{E({}_{R}X_{t})^{2}}{E({}_{R}X_{t}{}^{2})} \tag{37}$$

2.15. **Definition. The Expectation Value Of X^2.**

In mathematical statistics and probability theory, the expectation value ${}_{R}X_{t}^{2}$ at one point in space-time ${}_{R}t$ from the standpoint of *a stationary observer*, denoted as $E({}_{R}X_{t}^{2})$, is defined as

$$E({}_{R}X_{t}{}^{2})={}_{R}\ p({}_{R}X,{}_{R}t)\times{}_{R}X_{t}{}^{2}={}_{R}\ \Psi({}_{R}X,{}_{R}t)\times{}_{R}\Psi^{*}({}_{R}X,{}_{R}t)\times{}_{R}X_{t}{}^{2} \tag{38}$$

The expectation value of ${}_{O}X_{t}^{2}$ at one point in space-time ${}_{O}t$ from the standpoint of *a co-moving observer*, denoted as $E({}_{O}X_{t}^{2})$, is defined as

$$E\left({}_{O}X_{t}^{\,2}\right) = {}_{O}p\left({}_{O}X,\ {}_{O}t\right) \times {}_{O}X_{t}^{\,2} = {}_{O}\Psi\left({}_{O}X,\ {}_{O}t\right) \times {}_{O}\Psi^{*}\left({}_{O}X,\ {}_{O}t\right) \times {}_{O}X_{t}^{\,2} \quad (39)$$

2.16. Definition. The Variance.

In mathematical statistics and probability theory, the variance at one Bernoulli trial *t*, at one run of an experiment *t* from the standpoint of a stationary observer is defined as

$$\sigma\left({}_{R}X_{t}\right)^{2} = E\left({}_{R}X_{t}^{\,2}\right) - E\left({}_{R}X_{t}\right)^{2}. \quad (40)$$

The variance at one Bernoulli trial *t*, at one run of an experiment *t* from the standpoint of a co-moving observer (i.e. after the collapse of the wavefunction), is defined as

$$\sigma\left({}_{O}X_{t}\right)^{2} = E\left({}_{O}X_{t}^{\,2}\right) - E\left({}_{O}X_{t}\right)^{2}. \quad (41)$$

In contrast to the above definitions, the variance of a population is defined in general as

$$\sigma\left({}_{R}X\right)^{2} = E\left({}_{R}X^{2}\right) - E\left({}_{R}X\right)^{2}. \quad (42)$$

or as

$$\sigma\left({}_{O}X\right)^{2} = E\left({}_{O}X^{2}\right) - E\left({}_{O}X\right)^{2}. \quad (43)$$

2.17. Definition. The Logical Contradiction And The Inner Contradiction.

Let $\Delta({}_{R}X_{t})^{2}$ denote *the logical contradiction* of a random variable ${}_{R}X_{t}$ from the standpoint of *a stationary observer*. Let ${}_{R}p_{t}$ denote the probability of the random variable ${}_{R}X_{t}$ from the standpoint of *a stationary observer*. Let $\Delta({}_{O}X_{t})^{2}$ denote the logical contradiction of "same" random variable ${}_{O}X_{t}$ as determined from the standpoint of *a co-moving observer*. Let ${}_{O}p_{t}$ denote the probability of the random variable ${}_{O}X_{t}$ from the standpoint of *a stationary observer*. In general, we define

$$\Delta\left({}_{R}X_{t}\right)^{2} = \left({}_{R}p_{t}\right) - \left({}_{R}p_{t}\right)^{2} = {}_{R}p_{t} \times \left(1 - {}_{R}p_{t}\right). \quad (44)$$

and

$$\Delta\left({}_{O}X_{t}\right)^{2} = \left({}_{O}p_{t}\right) - \left({}_{O}p_{t}\right)^{2} = {}_{O}p_{t} \times \left(1 - {}_{O}p_{t}\right). \quad (45)$$

Let $\Delta({}_R X_t)$ denote *the inner contradiction* of a random variable ${}_R X_t$ from the standpoint of *a stationary observer*. Let $\Delta({}_0 X_t)$ denote the inner contradiction of "same" random variable ${}_0 X_t$ as determined from the standpoint of *a co-moving observer*. In general we define

$$\Delta\left({}_R X_t\right) \equiv \sqrt[2]{\Delta\left({}_R X_t\right)^2} = \sqrt[2]{\left({}_R p_t\right) - \left({}_R p_t\right)^2} = \sqrt[2]{{}_R p_t \times \left(1 - {}_R p_t\right)}. \tag{46}$$

and

$$\Delta\left({}_O X_t\right) \equiv \sqrt[2]{\Delta\left({}_O X_t\right)^2} = \sqrt[2]{\left({}_O p_t\right) - \left({}_O p_t\right)^2} = \sqrt[2]{{}_O p_t \times \left(1 - {}_O p_t\right)}. \tag{47}$$

Scholium
Under conditions where the probability of an event is equal to op=1, the logical contradiction is equivalent with 1*(1 – 1) = 0, which is known as the form of the logical contradiction in classical logic and in Boolean algebra. As soon as ${}_0 p < 1$, we are no longer under conditions of classical logic and Boolean algebra, a multi-valued (*dialectical*) logic is necessary. In general, the logical contradiction can take the values $\mathbf{\Delta({}_0 X_t)^2 = {}_0 p^*(1 - {}_0 p) \leq (1/4)}$ or in terms of quantum mechanics $\mathbf{\Delta({}_0 X_t)^2 = {}_0 p^*(1 - {}_0 p)} \leq (\hbar \times \pi / h)^2$. The logical and the inner contradiction is reference frame independent.

2.18. Definition. The Wavefunction Of The Gravitational Field Γ

In general, let the wavefunction of the gravitational field represent something like a probability amplitude in accordance with special theory of relativity.

Let ${}_R\Gamma$ denote the wavefunction of the gravitational field from the standpoint of *a stationary observer*. Let ${}_R\Gamma^*$ denote *the complex conjugate* of the wavefunction of the gravitational field from the standpoint of *a stationary observer*.
Let ${}_0\Gamma$ denote the wavefunction of the gravitational field from the standpoint of *a co-moving observer*. Let ${}_0\Psi^*$ denote *the complex conjugate* of the wavefunction of the gravitational field from the standpoint of *a co-moving observer.* Let

$${}_R g \equiv {}_R\Gamma \equiv \frac{{}_R\Psi}{c^2} = \sum_{i=0}^{n} {}_i k \times \frac{{}_i\psi}{c^2} = {}_0 k \times \frac{{}_0\psi}{c^2} + \sum_{i=1}^{n} {}_i k \times \frac{{}_i\psi}{c^2} \tag{48}$$

and

$${}_O g \equiv {}_O\Gamma = \frac{{}_O\Psi}{c^2} \tag{49}$$

where ${}_0\Psi$ denotes something like an eigenfunction.

3. Theorems

3.1. Theorem. The Expectation Value Of A Random Variable Is Reference Frame Dependent.

Special relativity implies some consequences for the calculation of expectation values.

Claim.

Under conditions of special theory of relativity, the expectation value is reference frame dependent (assumed that an expectation value exists). We obtain

$$\frac{E({}_{O}m)}{E({}_{R}m)} = \sqrt[2]{1-\frac{v^2}{c^2}} \tag{50}$$

Proof.

Starting with Axiom I it is

$$+1 = +1 . \tag{51}$$

Assuming that an expectation value of "rest- mass" ${}_{0}m$ exists, we obtain equally

$$E({}_{O}m) = E({}_{O}m) \tag{52}$$

Due to special relativity it follows that

$$E({}_{O}m) = E\left({}_{R}m \times \sqrt[2]{1-\frac{v^2}{c^2}}\right) \tag{53}$$

Under conditions of special theory of relativity, the term $\sqrt[2]{1-\frac{v^2}{c^2}}$ is constant. Due to the rules of mathematics it is **E(constant * X) = constant * E(X).** Thus far, the equation before can be simplified as

$$E({}_{O}m) = \sqrt[2]{1-\frac{v^2}{c^2}} \times E({}_{R}m) \tag{54}$$

In general, it follows that

$$\frac{E({}_{O}m)}{E({}_{R}m)} = \sqrt[2]{1-\frac{v^2}{c^2}} \tag{55}$$

Quod erat demonstrandum.

Scholium.
The expectation value of a random variable is **reference frame dependent**. Only under conditions where the relative velocity **v=0,** both observer (the moving observer O and the stationary observer R) will agree on the expectation value, otherwise not.

3.2. **Theorem. The Measure Of Probability p Is Reference Frame Independent.**

Coordinate systems are used in describing nature and physical laws. But does a coordinate system exist *a priori* in nature? What is the relationship between a coordinate system and physical law? Are the physical laws independent of the choice of a coordinate system related to each other by *any kind* of relative motion? Can a physical law take the same mathematical form in all coordinate systems (*Einstein's principle of general covariance*)?

Claim.
Under conditions of the special theory of relativity, the probability measure p is reference frame or coordinate system independent. We obtain

$$ {}_{O}p = {}_{R}p \tag{56}$$

Proof.
It is

$$ E({}_{O}m) = E({}_{O}m) \tag{57}$$

or due to special relativity

$$ E({}_{O}m) = E\left(\sqrt[2]{1-\frac{v^2}{c^2}} \times {}_{R}m\right) \tag{58}$$

Under conditions of special theory of relativity, the term $\sqrt[2]{1-\frac{v^2}{c^2}}$ = constant. In general, due to mathematics, it is **E(constant * X) = constant * E(X)** and we obtain

$$ E({}_{O}m) = \sqrt[2]{1-\frac{v^2}{c^2}} \times E({}_{R}m) \ . \tag{59}$$

Rearranging equation yields

$$ \frac{E({}_{O}m)}{E({}_{R}m)} = \sqrt[2]{1-\frac{v^2}{c^2}} \tag{60}$$

and equally

$$\frac{E({}_O m)}{E({}_R m)} = \frac{{}_O p \times {}_O m}{{}_R p \times {}_R m} = \sqrt[2]{1 - \frac{v^2}{c^2}} \tag{61}$$

where ${}_0p$ denotes the probability of ${}_0m$ as determined by the co-*moving* observer (observer at rest relative to ${}_0m$) and ${}_Rp$ denotes the probability of ${}_Rm$ as determined by the *stationary* observer. We obtain

$$\frac{{}_O p \times {}_O m}{{}_R p \times {}_R m} = \sqrt[2]{1 - \frac{v^2}{c^2}} \,. \tag{62}$$

Due to special relativity it is

$$\frac{{}_O p \times {}_O m}{{}_R p} \times \frac{\sqrt[2]{1 - \frac{v^2}{c^2}}}{{}_O m} = \sqrt[2]{1 - \frac{v^2}{c^2}} \tag{63}$$

The most terms cancel out. We obtain

$$\frac{{}_O p}{{}_R p} = 1 \tag{64}$$

which completes our proof. Under conditions of the special theory of relativity it is

$${}_O p = {}_R\, p \tag{65}$$

Quod erat demonstrandum.

Scholium.
This proof is of far reaching and general importance. Under conditions of the theory of special relativity the *stationary* and the *moving* observer will agree on the probability *p* of a random variable while both observers will disagree at the same time on the expectation value (in principle). We see from the proof above that in relativistic quantum theory, the probability is left unchanged if it is measured in a co-ordinate system moving with some other constant relative velocity. In attempts to extend the quantum theory to the relativistic domain, serious difficulties have arisen. Thus far, is this theorem valid under conditions of the general theory of relativity too? Under conditions of the general theory of relativity at every space-time point there exist a locally inertial reference frames in which the physics of general theory of relativity is locally indistinguishable from that of special relativity (*Einstein's famous strong equivalence principle*). Due to our proof above, it is reasonable to expect that probability theory is of use even under conditions of the general theory of relativity. A reference frame independent account of probability is appropriate for causal inference. Any subjective interpretation of probability advocated by some prominent philosopher and psychologist has no place in science.

3.3. **Theorem. The Wavefunction From The Standpoint Of A Stationary Observer R**

The Hamiltonian, usually denoted by H of $\hat{H}$ is a quantum mechanical operator corresponding to the total energy of a (quantum mechanical) system. In most of the cases, **the spectrum of the Hamiltonian H** is the set of possible outcomes when one measures the total energy of a system. In special relativity, the total energy of the system is denoted by $_{R}E$ and is equally the set of possible outcomes at least $_{0}E$ + $_{\Delta}E$ = $_{R}E$.

Claim.

Under conditions of the special theory of relativity from the standpoint of a stationary observer R it is

$$_{R}t = {}_{R}\Psi(t). \tag{66}$$

Proof.

Under conditions of the special theory of relativity and from the standpoint of a stationary observer R, a system is completely described by the equation

$$_{R}E \times {}_{R}t = {}_{R}E \times {}_{R}t \tag{67}$$

The same system, no matter how complicated, is equally described by Schrödinger equation. It is

$$_{R}E \times {}_{R}t = \hat{H} \times {}_{R}\Psi(t) \tag{68}$$

The Hamiltonian is a quantum mechanical equivalent of the total energy of a system, the set of all possible outcomes. In special relativity, the total energy of the system is denoted by $_{R}E$. Therefore, we equate both notions as $_{R}E = \hat{H}$. Rearranging the equation above, it follows that

$$_{R}E \times {}_{R}t = {}_{R}E \times {}_{R}\Psi(t) \quad . \tag{69}$$

At the end, it is

$$_{R}t = {}_{R}\Psi(t). \tag{70}$$

which completes our proof.

Quod erat demonstrandum.

Scholium.

On the first sight, $_{R}t$ has nothing in common with $_{R}\Psi(t)$. We can rearrange $_{R}t$ and obtain an equivalent relationship as $_{R}\Psi(t)$=**a*(1/a)*** $_{R}t$. Let it be that $b*e^{-d}=((1/a)*{}_{R}t)$ and that f=a*b. We obtain at the end $\mathbf{_{R}\Psi(t)= f * e^{-d}}$. All the changes have no influence on the fact that $\mathbf{_{R}t = {}_{R}\Psi(t)}$.

3.4. **Theorem. The Wavefunction From The Standpoint Of A Moving Observer O**

A moving observer does not measure the superposition of several eigenstates but measures a specific energy state, an observable associated with an eigenbasis. In other words, from the standpoint of the moving observer, the wave function, usually a linear superposition of its eigenstates, has collapsed from the full to just one of the basis eigenstates. The probability of a wave function to collapse into a given eigenstate is called the Born probability.

Claim.

Under conditions of the special theory of relativity from the standpoint of a moving observer O it is

$$ {}_{O}t = {}_{O}\Psi \,. \tag{71}$$

Proof.

Under conditions of the special theory of relativity and from the standpoint of a moving observer O, a system is completely described by the equation

$$ {}_{O}E \times {}_{O}t = {}_{O}E \times {}_{O}t \tag{72}$$

where ${}_{0}E$ denotes the rest energy and ${}_{0}t$ denotes the time as determined by the moving observer. The measurement of a moving observer describes a very specific energy state, an eigenstate or an eigenvalue. Corresponding to such an eigenvalue ${}_{0}\hat{H}$ is an 'eigenfunction' ${}_{O}\Psi$. The system of a moving observer, no matter how complicated, is equally described by Schrödinger equation.

$$ {}_{O}E \times {}_{O}t = {}_{0}\hat{H} \times {}_{O}\Psi \tag{73}$$

The specific energy state obtained by the moving observer is identical with an eigenvalue, an energy state after the collapse of the wavefunction. We equate ${}_{O}E = {}_{0}\hat{H}$. Rearranging equation we obtain

$$ {}_{O}E \times {}_{O}t = {}_{O}E \times {}_{O}\Psi \tag{74}$$

At the end, it is

$$ {}_{O}t = {}_{O}\Psi \quad . \tag{75}$$

which completes our proof.

Quod erat demonstrandum.

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Scholium.
A wave function is initially in **a superposition of several eigenstates**. To obtain a specific eigenvalue of a physical parameter (for example energy), it is necessary to operate on the wavefunction with a quantum mechanical operator associated with a parameter. Corresponding to each eigenvalue is an 'eigenfunction'. The solution of the Schrodinger equation for a given energy involves also finding the specific function, denoted as eigenfunction, which describes a specific energy state.

3.5. **Theorem. The Wavefunction Is Reference Frame Dependent**

Claim.
Under conditions of the special theory of relativity it is

$$\frac{{}_{O}\Psi}{{}_{R}\Psi} = \frac{{}_{R}\Psi^{*}}{{}_{O}\Psi^{*}} = \frac{{}_{O}t}{{}_{R}t} = \sqrt[2]{1-\frac{v^2}{c^2}}\,. \tag{76}$$

Proof.
Under conditions of the special theory of relativity, the moving and the stationary observer will agree on the probability. In general, it is

$$ {}_{O}p = {}_{R}p \tag{77}$$

which due to Born's rule is equivalent with

$$ {}_{O}p \equiv {}_{O}\Psi \times {}_{O}\Psi^{*} = {}_{R}\Psi \times {}_{R}\Psi^{*} \equiv {}_{R}p\,. \tag{78}$$

Rearranging this equation, we obtain

$$\frac{{}_{O}\Psi}{{}_{R}\Psi} = \frac{{}_{R}\Psi^{*}}{{}_{O}\Psi^{*}} \tag{79}$$

As proofed before, it is ${}_{O}t = {}_{O}\Psi$ and equally ${}_{R}t = {}_{R}\Psi(t)$. Thus far, we obtain

$$\frac{{}_{O}\Psi}{{}_{R}\Psi} = \frac{{}_{R}\Psi^{*}}{{}_{O}\Psi^{*}} = \frac{{}_{O}t}{{}_{R}t}\,. \tag{80}$$

Due to special relativity theory, this is equivalent to

$$\frac{{}_{O}\Psi}{{}_{R}\Psi} = \frac{{}_{R}\Psi^{*}}{{}_{O}\Psi^{*}} = \frac{{}_{O}t}{{}_{R}t} = \sqrt[2]{1-\frac{v^2}{c^2}}\,. \tag{81}$$

Quod erat demonstrandum.

Scholium.
Due to our definition above it is equally

$$\frac{{}_{O}\Psi}{{}_{R}\Psi} = \frac{{}_{R}\Psi^{*}}{{}_{O}\Psi^{*}} = \sqrt[2]{1-\frac{v^2}{c^2}} \tag{82}$$

or in other words it is

$${}_{O}\Psi = {}_{R}\Psi \times \sqrt[2]{1-\frac{v^2}{c^2}} . \tag{83}$$

The relationship between an eigenfunction ${}_{0}\Psi$ of a wavefunction and the wavefunction ${}_{R}\Psi$ itself is determined by the needs of special theory of relativity. Consequently, the description of physical reality given by the wave function in quantum mechanics must not be regarded as being incomplete with the theory of special relativity. Under conditions where the relative velocity squared is $v^2 > 0$, the moving observer O and the stationary observer R will obtain contradictory results, if both observer use the same wave function to describe a concrete physical reality. A correction factor $\sqrt[2]{1-\frac{v^2}{c^2}}$ is needed to achieve correct results. This relation is of great importance especially under circumstances where ${}_{0}\Psi$ is identical with an *eigenfunction*.

3.6. **Theorem. The Variance Is Reference Frame Dependent**

Claim.
Under conditions of the special theory of relativity the variance and the standard deviation of a random variable is reference frame dependent. In general it is

$$\frac{\sigma\left({}_{0}X_{t}\right)^2}{\sigma\left({}_{R}X_{t}\right)^2} = 1-\frac{v^2}{c^2} \quad . \tag{84}$$

Proof.
In general it is

$$\sigma\left({}_{0}X_{t}\right)^2 = \sigma\left({}_{0}X_{t}\right)^2 . \tag{85}$$

Under conditions of special relativity where ${}_{0}X_{t} = {}_{R}X_{t} \times \sqrt[2]{1-\frac{v^2}{c^2}}$ and it follows that

$$\sigma\left({}_{0}X_{t}\right)^2 = \sigma\left({}_{R}X_{t} \times \sqrt[2]{1-\frac{v^2}{c^2}}\right)^2 . \tag{86}$$

Under conditions of the special theory of relativity, **the term** $\sqrt[2]{1-\frac{v^2}{c^2}}$ **is constant**. Due to the rules of mathematics it is **σ(constant * X)² = constant² * σ(X)²** . Thus far, the equation before can be simplified as

$$\sigma\left({}_0X_t\right)^2 = \sqrt[2]{1-\frac{v^2}{c^2}} \times \sqrt[2]{1-\frac{v^2}{c^2}} \times \sigma\left({}_RX_t\right)^2 . \tag{87}$$

Rearranging equation yields

$$\frac{\sigma\left({}_0X_t\right)^2}{\sigma\left({}_RX_t\right)^2} = \left(\sqrt[2]{1-\frac{v^2}{c^2}} \times \sqrt[2]{1-\frac{v^2}{c^2}}\right)^2 = 1-\frac{v^2}{c^2} \quad . \tag{88}$$

Quod erat demonstrandum.

Scholium.

Due to the theorem above and under conditions where ${}_0X_t = {}_RX_t \times \sqrt[2]{1-\frac{v^2}{c^2}}$, **the standard deviation** is reference frame dependent too and we obtain

$$\frac{\sigma\left({}_0X_t\right)}{\sigma\left({}_RX_t\right)} = \sqrt[2]{1-\frac{v^2}{c^2}} \tag{89}$$

or

$$\sigma\left({}_0X_t\right) = \sigma\left({}_RX_t\right) \times \sqrt[2]{1-\frac{v^2}{c^2}} . \tag{90}$$

In contrast to the variance and the standard deviation, the general and the inner contradiction are reference frame independent.

3.7. **Theorem. Chebyshev's inequality**

The Chebyshev's inequality, named after the Russian mathematician Pafnuty Lvovich Chebyshev (1821-1894), plays a fundamental role in various modern fields of probability theory. Especially, due to Chebyshev's inequality it is possible to estimate the probability (i. e. of the deviation of a random variable from its mathematical expectation) in terms of the variance of the random variable. In this setting, let us derive an exact equality as an alternative to Chebyshev's inequality.

Claim.
Under conditions of the special theory of relativity we obtain

$$1 - {}_0p_t = \frac{\sigma\left({}_0X_t\right)^2}{E\left({}_0X_t{}^2\right)} . \tag{91}$$

Proof.
In general it is

$$\sigma\left({}_0X_t\right)^2 = \sigma\left({}_0X_t\right)^2 \tag{92}$$

or

$$\sigma\left({}_0X_t\right)^2 = E\left({}_0X_t{}^2\right) - E\left({}_0X_t\right)^2. \tag{93}$$

Rearranging equation, we obtain

$$\sigma\left({}_0X_t\right)^2 + E\left({}_0X_t\right)^2 = E\left({}_0X_t{}^2\right). \tag{94}$$

or

$$\frac{\sigma\left({}_0X_t\right)^2}{E\left({}_0X_t{}^2\right)} + \frac{E\left({}_0X_t\right)^2}{E\left({}_0X_t{}^2\right)} = +1. \tag{95}$$

In general it is *per definitionem* $\frac{E\left({}_0X_t\right)^2}{E\left({}_0X_t{}^2\right)} \equiv \frac{{}_0p_t \times {}_0X_t \times {}_0p_t \times {}_0X_t}{{}_0p_t \times {}_0X_t \times {}_0X_t} \equiv {}_0p_t$. We obtain

$$\frac{\sigma\left({}_0X_t\right)^2}{E\left({}_0X_t{}^2\right)} + {}_0p_t = +1. \tag{96}$$

In other words, it is

$$1 - {}_0p_t = \frac{\sigma\left({}_0X_t\right)^2}{E\left({}_0X_t{}^2\right)}. \tag{97}$$

Quod erat demonstrandum.

Scholium.
Due to Chebyshev's inequality it is $p\left(\left|{}_0X_t - E\left({}_0X_t\right)\right| \geq \sqrt[2]{E\left({}_0X_t{}^2\right)}\right) \leq \frac{\sigma\left({}_0X_t\right)^2}{E\left({}_0X_t{}^2\right)}$. Thus far, we equate both equations. It follows that $p\left(\left|{}_0X_t - E\left({}_0X_t\right)\right| \geq \sqrt[2]{E\left({}_0X_t{}^2\right)}\right) = 1 - {}_0p_t$. Under conditions, where $\sqrt[2]{E\left({}_0X_t{}^2\right)} = k \times \sigma\left({}_0X_t\right)$ we obtain $p\left(\left|{}_0X_t - E\left({}_0X_t\right)\right| \geq \sqrt[2]{E\left({}_0X_t{}^2\right)}\right) \leq \frac{1}{k^2}$.

3.8. **Theorem. The Wavefunction Is Determined By The Variance**

Claim.
In general, the Schrödinger equation is determined by

$$\frac{\sigma\left({}_{O}X_{t}\right)^{2}}{\sqrt[2]{1-\frac{v^{2}}{c^{2}}\times{}_{O}X_{t}\times\left(1-{}_{O}p_{t}\right)\times{}_{O}\Psi^{*}}}=H\times_{O}\Psi\ . \tag{98}$$

Proof.
The variance as determined by the co-moving observer is defined as

$$\sigma\left({}_{O}X_{t}\right)^{2}=E\left({}_{O}X_{t}{}^{2}\right)-E\left({}_{O}X_{t}\right)^{2}. \tag{99}$$

Due the definition of *the expectation value* above this equation is equivalent to

$$\sigma\left({}_{O}X_{t}\right)^{2}=\left({}_{O}p_{t}\times{}_{O}X_{t}{}^{2}\right)-\left({}_{O}p_{t}\times{}_{O}X_{t}\right)^{2} \tag{100}$$

which is equivalent to

$$\sigma\left({}_{O}X_{t}\right)^{2}={}_{O}X_{t}{}^{2}\times\left(\left({}_{O}p_{t}\right)-\left({}_{O}p_{t}\right)^{2}\right) \tag{101}$$

or to

$$\sigma\left({}_{O}X_{t}\right)^{2}={}_{O}X_{t}{}^{2}\times\left({}_{O}p_{t}\times\left(1-{}_{O}p_{t}\right)\right). \tag{102}$$

Rearranging equation, we obtain

$$\frac{\sigma\left({}_{O}X_{t}\right)^{2}}{{}_{O}X_{t}\times\left(1-{}_{O}p_{t}\right)}={}_{O}X_{t}\times{}_{O}p_{t}=E\left({}_{O}X_{t}\right). \tag{103}$$

Based on Born's rule, it is ${}_{O}p\equiv{}_{O}\Psi\times{}_{O}\Psi^{*}={}_{R}\Psi\times{}_{R}\Psi^{*}\equiv{}_{R}p$. Rearranging the equation, we obtain

$$\frac{\sigma\left({}_{O}X_{t}\right)^{2}}{{}_{O}X_{t}\times\left(1-{}_{O}p_{t}\right)}={}_{O}X_{t}\times{}_{O}\Psi\times{}_{O}\Psi^{*} \tag{104}$$

or

$$\frac{\sigma\left({}_{O}X_{t}\right)^{2}}{{}_{O}X_{t}\times\left(1-{}_{O}p_{t}\right)\times{}_{O}\Psi^{*}}={}_{O}X_{t}\times{}_{O}\Psi \quad . \tag{105}$$

Under conditions of special relativity where ${}_{0}X_{t}={}_{R}X_{t}\times\sqrt[2]{1-\frac{v^{2}}{c^{2}}}$ it follows that

$$\frac{\sigma\left({}_{O}X_{t}\right)^{2}}{{}_{O}X_{t}\times\left(1-{}_{O}p_{t}\right)\times{}_{O}\Psi^{*}}={}_{R}X_{t}\times\sqrt[2]{1-\frac{v^{2}}{c^{2}}}\times{}_{O}\Psi \tag{106}$$

or that

$$\frac{\sigma\left({}_{O}X_{t}\right)^{2}}{{}_{O}X_{t}\times\sqrt[2]{1-\frac{v^{2}}{c^{2}}}\times\left(1-{}_{O}p_{t}\right)\times{}_{O}\Psi^{*}}={}_{R}X_{t}\times{}_{O}\Psi \quad . \tag{107}$$

Under conditions where ${}_{R}X_{t}=H$, the Schrödinger equation follows as

$$\frac{\sigma\left({}_{O}X_{t}\right)^{2}}{{}_{O}X_{t}\times\sqrt[2]{1-\frac{v^{2}}{c^{2}}}\times\left(1-{}_{O}p_{t}\right)\times{}_{O}\Psi^{*}}={}_{R}X_{t}\times{}_{O}\Psi=H\times{}_{O}\Psi \tag{108}$$

Since ${}_{O}X_{t}\times\left(1-{}_{O}p_{t}\right)=\left({}_{O}X_{t}-E\left({}_{O}X_{t}\right)\right)$ we obtain

$$\frac{\sigma\left({}_{O}X_{t}\right)^{2}}{\sqrt[2]{1-\frac{v^{2}}{c^{2}}}\times\left({}_{O}X_{t}-E\left({}_{O}X_{t}\right)\right)\times{}_{O}\Psi^{*}}={}_{R}X_{t}\times{}_{O}\Psi=H\times{}_{O}\Psi \quad . \tag{109}$$

Let us assume that *the standard normal variable* Z is defined as

$$z\left({}_{O}X_{t}\right)=\frac{\left({}_{O}X_{t}-E\left({}_{O}X_{t}\right)\right)}{\sigma\left({}_{O}X_{t}\right)} .$$

It follows that

$$\frac{\sigma\left({}_{O}X_{t}\right)}{\sqrt[2]{1-\frac{v^{2}}{c^{2}}}\times Z\left({}_{O}X_{t}\right)\times {}_{O}\Psi^{*}} = {}_{R}X_{t}\times_{O}\Psi = H\times_{O}\Psi \quad . \tag{110}$$

which completes our proof. In general it is

$$\frac{\sigma\left({}_{O}X_{t}\right)^{2}}{\sqrt[2]{1-\frac{v^{2}}{c^{2}}}\times_{O}X_{t}\times\left(1-{}_{O}p_{t}\right)\times {}_{O}\Psi^{*}} = H\times_{O}\Psi \quad . \tag{111}$$

Quod erat demonstrandum.

3.9. **Theorem. The Frist Basic Law Of Special Relativity**

Special relativity implies a basic physical law of far reaching consequences.

Claim.
In general, the basic law of special relativity is determined as

$$_{0}E\times{}_{R}t = {}_{R}E\times{}_{0}t \tag{112}$$

Proof.
Starting with

$$+1=+1 \tag{113}$$

it is equally

$$1*\sqrt[2]{1-\frac{v^{2}}{c^{2}}} = 1*\sqrt[2]{1-\frac{v^{2}}{c^{2}}}. \tag{114}$$

Based on Einstein's mass-energy equivalence we obtain

$$\frac{_{0}m\times c^{2}}{_{R}m\times c^{2}} = \sqrt[2]{1-\frac{v^{2}}{c^{2}}}. \tag{115}$$

Rearranging this equation due to the relativistic time dilation, it follows that

$$\frac{{}_{0}m \times c^{2}}{{}_{R}m \times c^{2}} = \frac{t_{0}}{t_{R}} \tag{116}$$

which is equivalent to

$${}_{0}m \times c^{2} \times {}_{R}t = {}_{R}m \times c^{2} \times {}_{0}t \tag{117}$$

or to

$${}_{0}E \times {}_{R}t = {}_{R}E \times {}_{0}t . \tag{118}$$

Quod erat demonstrandum.

Scholium.
Any relativistic or non-relativistic (i. e. v = 0) (quantum mechanical) system, no matter how complicated, is completely described by this equation. The same system is described by Schrödinger's equation too.

3.10. Theorem. The Second Basic Law Of Special Relativity

Special relativity implies another basic physical law of no less reaching consequences. In general, it is,

$${}_{0}t + {}_{\Delta}t = {}_{R}t \tag{119}$$

and equally

$${}_{0}E + {}_{\Delta}E = {}_{R}E . \tag{120}$$

Claim.
Under these circumstances, the second basic law of special relativity is determined as

$${}_{\Delta}E \times {}_{R}t = {}_{R}E \times {}_{\Delta}t \tag{121}$$

Proof.
From

$${}_{0}E + {}_{\Delta}E = {}_{R}E . \tag{122}$$

follows that

$$\frac{{}_0E}{{}_RE}+\frac{{}_\Delta E}{{}_RE}=1 \tag{123}$$

Equally, from

$$ {}_0t+{}_\Delta t={}_R\, t \tag{124}$$

follows that

$$\frac{{}_0t}{{}_Rt}+\frac{{}_\Delta t}{{}_Rt}=1. \tag{125}$$

In general, it is

$$+1=+1 \tag{126}$$

and we obtain the equivalence of

$$\frac{{}_0E}{{}_RE}+\frac{{}_\Delta E}{{}_RE}=\frac{{}_0t}{{}_Rt}+\frac{{}_\Delta t}{{}_Rt}\,. \tag{127}$$

After multiplication with (${}_RE$ x ${}_Rt$), it follows that

$$ {}_0E\times {}_Rt+{}_\Delta E\times {}_Rt={}_0\, t\times {}_RE+{}_\Delta t\times {}_RE\,. \tag{128}$$

According to the first basic law of special relativity, it is

$$ {}_0E\times {}_Rt={}_R\, E\times {}_0t\,. \tag{129}$$

Based on this insight, we rearrange the equation above and obtain

$$ {}_0E\times {}_Rt+ {}_\Delta E\times_R t={}_0\, E\times {}_Rt+ {}_\Delta t\times {}_RE \tag{130}$$

or at the end similar the law of the lever as provided and proven by Archimedes (~287 BC - ~212 BC)

$$ {}_\Delta E\times_R t={}_\Delta t\times_R E\,. \tag{131}$$

Quod erat demonstrandum.

Scholium.
The straightforward question is, are there conditions where

$$ {}_{\Delta}E \times {}_{R}t = {}_{\Delta}t \times {}_{R}E = h \times (f = 1) \tag{132} $$

with all the consequences for quantum and special relativity theory. Under these or similar circumstances, a "quantization" of time would be possible in principle. The Planck constant h is related to the quantization of light and matter and named after Max Planck, is the quantum of action in quantum mechanics. In *SI units*, the Planck constant h is expressed in *Joule seconds* (J·s) or (N·m·s) i.e., **energy multiplied by time**.

3.11. Theorem. The Wavefunction Is Reference Frame Or Observer Dependent

Claim.
In general, it is

$$ {}_{0}E \times {}_{R}\Psi = {}_{R}E \times {}_{0}\Psi \,. \tag{133} $$

Proof.
Starting with

$$ +1 = +1 \tag{134} $$

According to the first basic law of special relativity, it is equally

$$ {}_{0}E \times {}_{R}t = {}_{R}E \times {}_{0}t \,. \tag{135} $$

Based on this insight, we obtain

$$ {}_{0}E \times {}_{R}t = {}_{R}E \times {}_{0}t \times 1 \,. \tag{136} $$

Based on Born's rule ${}_{O}p \equiv {}_{O}\Psi \times {}_{O}\Psi^{*} = {}_{R}\Psi \times {}_{R}\Psi^{*} \equiv {}_{R}p$, it is

$$ {}_{0}E \times {}_{R}t = {}_{R}E \times {}_{0}t \times \frac{{}_{0}\Psi \times {}_{0}\Psi^{*}}{{}_{0}p} \,. \tag{137} $$

Rearranging equation, it follows that

$$ {}_{0}E \times \frac{{}_{0}p \times {}_{R}t}{{}_{0}\Psi^{*} \times {}_{0}t} = {}_{R}E \times {}_{0}\Psi \,. \tag{138} $$

Simplifying equation, we obtain

$$ {}_0E \times \frac{{}_0p}{{}_0\Psi^*} \times \frac{1}{\sqrt[2]{1-\frac{v^2}{c^2}}} = {}_RE \times {}_0\Psi \,. \tag{139} $$

In other words, based on Born's rule it follows that $\frac{{}_Op}{{}_O\Psi^*} \equiv_O \Psi$ and we obtain

$$ {}_0E \times {}_0\Psi \times \frac{1}{\sqrt[2]{1-\frac{v^2}{c^2}}} = {}_RE \times {}_0\Psi \,. \tag{140} $$

Due to our definition ${}_0\Psi \times \frac{1}{\sqrt[2]{1-\frac{v^2}{c^2}}} =_R \Psi$ we obtain

$$ {}_0E \times {}_R\Psi = {}_RE \times {}_0\Psi \,. \tag{141} $$

Quod erat demonstrandum.

3.12. Theorem. The Normalization Of Energy And Time

Claim.
In general, it is

$$ \frac{{}_RE}{{}_RS} + \frac{{}_Rt}{{}_RS} \equiv 1 \tag{142} $$

Proof.
Form the definition

$$ {}_RS \equiv_R E +_R t \tag{143} $$

follows that

$$ \frac{{}_RE}{{}_RS} + \frac{{}_Rt}{{}_RS} \equiv 1 \tag{144} $$

Quod erat demonstrandum.

3.13. Theorem. The Normalization Of Matter And Gravitational Field

Claim.
In general, it is

$$\frac{{}_R M}{{}_R U} + \frac{{}_R g}{{}_R U} \equiv 1 \tag{145}$$

Proof.
Form the definition

$${}_R U \equiv {}_R M + {}_R g \tag{146}$$

follows that

$$\frac{{}_R M}{{}_R U} + \frac{{}_R g}{{}_R U} \equiv 1 \tag{147}$$

Quod erat demonstrandum.

Time and gravitational field are related. Following Einstein's special theory of relativity, both are equivalent [13].

3.14. Theorem. The Equivalence Of Time And Gravitational Field

Claim.
In general it is

$${}_R t = c^2 \times {}_R g \,. \tag{148}$$

Proof.
It is

$$+1 = +1 \tag{149}$$

and thus far

$$\frac{{}_R E}{{}_R S} + \frac{{}_R t}{{}_R S} \equiv 1 \tag{150}$$

and equally

$$\frac{{}_R E}{{}_R S} + \frac{{}_R t}{{}_R S} = \frac{{}_R M}{{}_R U} + \frac{{}_R g}{{}_R U} = 1 \ . \tag{151}$$

Rearranging equality, we obtain

$${}_R E + {}_R t = \frac{{}_R S \times {}_R M}{{}_R U} + \frac{{}_R S \times {}_R g}{{}_R U} = {}_R S \tag{152}$$

or equally

$${}_R E + {}_R t = \frac{{}_R S}{{}_R U} \times {}_R M + \frac{{}_R S}{{}_R U} \times {}_R g = {}_R S \tag{153}$$

Based on the definition $c^2 \equiv \frac{{}_R S}{{}_R U}$ we obtain

$${}_R E + {}_R t = c^2 \times {}_R M + c^2 \times {}_R g = {}_R S \tag{154}$$

Due to Einstein's special relativity matter and energy are equivalent. Thus far, it is ${}_R E = c^2 \times {}_R M$. The equation can be rearranged as

$${}_R E + {}_R t = {}_R E + c^2 \times {}_R g \tag{155}$$

which completes our proof. In general it is

$${}_R t = c^2 \times {}_R g \ . \tag{156}$$

Quod erat demonstrandum.

Scholium.

This proof before is based on the assumption that $c^2 \equiv \frac{{}_R S}{{}_R U}$ and that ${}_R E = c^2 \times {}_R M$.

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3.15. Theorem. The Schrödinger Equation In Relation To The Gravitational Field

Claim.
In general, wave equation of the gravitational field is determined as

$$ {}_{R}M \times {}_{R}\Gamma = \frac{i\hbar}{c^2} \frac{\partial}{\partial t} \times \frac{{}_{R}\Psi}{c^2} \tag{157} $$

Proof.
The Schrödinger equation is defined as

$$ H \times {}_{R}\Psi = i\hbar \frac{\partial}{\partial t} {}_{R}\Psi \tag{158} $$

Dividing the equation by $c^2 * c^2$ we obtain

$$ \frac{1}{c^2 \times c^2} \times H \times {}_{R}\Psi = \frac{1}{c^2 \times c^2} \times i\hbar \frac{\partial}{\partial t} \times {}_{R}\Psi \tag{159} $$

or

$$ \frac{H}{c^2} \times \frac{{}_{R}\Psi}{c^2} = \frac{i\hbar}{c^2} \frac{\partial}{\partial t} \times \frac{{}_{R}\Psi}{c^2} \tag{160} $$

Due to our definition above it is ${}_{R}M = \frac{H}{c^2}\times$ and ${}_{R}\Gamma = \frac{{}_{R}\Psi}{c^2}$. The wave equation of the gravitational field follows as

$$ {}_{R}M \times {}_{R}\Gamma = \frac{i\hbar}{c^2} \frac{\partial}{\partial t} \times \frac{{}_{R}\Psi}{c^2} \tag{161} $$

Quod erat demonstrandum.

3.16. Theorem. The Generally Covariant Form Of The Schrödinger Equation Claim.

Under some well defined circumstances, Schrödinger's equation can be expressed in a generally covariant form as

$$(G_{ae} + \Lambda \times g_{ae}) \times \Psi^{ae} = \left(\frac{2 \times \pi \times 4 \times \gamma}{c^2 \times c^2} \times T_{ae} \right) \times \Psi^{ae} \tag{162}$$

Proof.
Due to our understanding it is

$${}_{R}S = {}_{R}S \tag{163}$$

or in other words

$${}_{R}S = {}_{R}E + {}_{R}t \,. \tag{164}$$

Rearranging equation, we obtain

$${}_{R}S - {}_{R}E = + {}_{R}t \tag{165}$$

or equally

$${}_{R}S + {}_{R}t - {}_{R}E = 2 \times {}_{R}t \,. \tag{166}$$

Dividing by 2, it is

$$\frac{{}_{R}S + {}_{R}t}{2} - \frac{{}_{R}E}{2} = {}_{R}t \tag{167}$$

Let us define $R \equiv {}_{R}S + {}_{R}t$ and $\Lambda \equiv \frac{{}_{R}E}{2}$. Thus far, it is

$$\frac{R}{2} - \Lambda = {}_{R}t \tag{168}$$

Multiplying with the metric tensor of general relativity theory, g_{ae}, we obtain

$$\frac{R}{2} \times g_{ae} - \Lambda \times g_{ae} = {}_{R}t \times g_{ae} \tag{169}$$

Due to our prove above, it is ${}_R\Psi ={}_R t$. Let us define a **wave-function tensor** as $\Psi_{ae} \equiv_R t \times g_{ae}$ or as $\Psi^{ae} \equiv_R t \times g^{ae}$. In general it is

$$\frac{R}{2} \times g_{ae} - \Lambda \times g_{ae} =_R t \times g_{ae} \equiv \Psi_{ae} \tag{170}$$

Multiplying Einstein's field equation $G_{ae} + \Lambda \times g_{ae} = \left(\frac{2 \times \pi \times 4 \times \gamma}{c^2 \times c^2}\right) \times T_{ae}$ by the wave-function tensor Ψ^{ae}, we obtain **the generally covariant form of Schrödinger's equation** as

$$\left(G_{ae} \times \Psi^{ae}\right) + \left(\Lambda \times g_{ae} \times \Psi^{ae}\right) = \left(\frac{2 \times \pi \times 4 \times \gamma}{c^2 \times c^2} \times T_{ae}\right) \times \Psi^{ae} \tag{171}$$

Quod erat demonstrandum.

Scholium.
While electromagnetic, weak and strong force act in a given space-time, gravitation itself is far more difficult. According to the general theory of relativity (GRT), gravitation is identified with the curvature of space-time itself. Consequently, quantizing gravitation could be equivalent with quantizing space-time, and it is not at all clear what that could mean. To put it another way, it is very difficult to incorporate gravitation into a setting of a unified field theory. There are many approaches to reconcile quantum theory and general relativity, String theory, for instance, is one of them. String theory as one of the many candidates for bridging the gap between quantum field theories (QFT) and general relativity theory(GRT) is supplying a unified theory of all natural forces, including gravitation, in terms of strings. Strings are able to interact on an extended distance and not only at a point. In contrast to String theory, quantum field theories (QFT) takes particles as fundamental objects but not strings. Thus far, in quantum field theories (QFT), it is complicated to quantize the gravitational field. The above proposal shows a way how to quantize the gravitational field.
From our definition ${}_RS = {}_RE + {}_Rt$ follows that ${}_RS - {}_RE = {}_Rt$. Adding time ${}_Rt$ to this equation we obtain ${}_RS + {}_Rt - {}_RE = {}_Rt + {}_Rt$ which is equivalent to ${}_RS + {}_Rt - {}_RE = 2 * {}_Rt$. It is possible to divide this equation by 2, we obtain $({}_RS + {}_Rt)/2 - ({}_RE/2) = {}_Rt$. Multiplying this equation by the metric tensor g_{ae} of the general theory of relativity, it is $(({}_RS + {}_Rt)/2) * g_{ae} - ({}_RE/2) * g_{ae} = {}_Rt * g_{ae}$. Under conditions of the general theory of relativity where $({}_RS + {}_Rt)/2 = R/2$ and where the term $({}_RE/2) = \Lambda$ (R denotes the Ricci scalar and Λ denotes cosmological constant Λ) Einstein's field equation follows in as $R_{ae} = (8 * \pi * \gamma / c^4) * T_{ae} + {}_Rt * g_{ae}$. In general, due to our proof above the notion ${}_Rt * g_{ae}$ can be regarded as being equivalent with ${}_R\Psi * g_{ae}$ or as ${}_R\Psi * g^{ae}$, which can be regarded as something like a *wave-function tensor*. Thus far, multiplying Einstein's field equation by the wavefunction tensor leads to a generally covariant form of the Schrödinger equation as

$$\left(G_{ae} + \Lambda \times g_{ae}\right) \times_R \Psi \times g^{ae} = \left(\frac{4 \times 2 \times \pi \times \gamma}{c^2 \times c^2} \times T_{ae}\right) \times_R \Psi \times g^{ae} \quad . \tag{172}$$

Since $2 \times \pi = \frac{h}{\hbar}$ we obtain equally

$$\left(G_{ae} + \Lambda \times g_{ae}\right) \times_R \Psi \times g^{ae} = \frac{h}{c^2} \times \left(\frac{4 \times \gamma}{\hbar \times c^2} \times T_{ae}\right) \times_R \Psi \times g^{ae} \tag{173}$$

or

$$\left(G_{ae} + \Lambda \times g_{ae}\right) \times_R \Psi \times g_{ae} = \frac{h}{c^2} \times \left(\frac{4 \times \gamma}{\hbar \times c^2} \times T_{ae}\right) \times_R \Psi \times g_{ae} \quad . \tag{174}$$

A possible conclusion is that we must accept that $\frac{1}{c^2 \times c^2} \times H \times_R \Psi = \left(\frac{2 \times \pi \times 4 \times \gamma}{c^2 \times c^2} \times T_{ae}\right) \times g^{ae} \times_R \Psi = \frac{1}{c^2 \times c^2} \times i\hbar \frac{\partial}{\partial t} \times_R \Psi$. Under these assumption, the Hamiltonian follows as $H = \left(2 \times \pi \times 4 \times \gamma \times T_{ae}\right) \times g^{ae}$ or of course as $H = \left(2 \times \pi \times 4 \times \gamma \times T_{ae}\right) \times g_{ae}$ too.

3.17. Theorem. The Gravitational Waves Under Conditions Of Special Relativity

One central feature of the theory of General Relativity is the existence of gravitational waves. For the usual, in contrast to the theory of General Relativity, Einstein's Special Theory of Relativity is of use especially for systems which are not accelerating. Most commonly, today's academic positions in physics are more or less that the spacetime of special relativity is a spacetime where there is no gravity at all. Under these circumstances, it becomes important to note that within the spacetime of special relativity, a **line** (i. e. from the standpoint of a moving observer O) is **straight** while the same line in the same respect is curved (from the standpoint of a stationary observer R). The **curved line**, as a simple form of curved spacetime, is possible even under conditions of special relativity. Any distortion of spacetime geometry can be regarded as gravity. Thus far, gravity under conditions of the special theory of relativity must be treated different form gravity under conditions of the general theory of relativity. Even under conditions of special relativity, it is possible (and necessary) to distinguish between **the gravitational field** itself and **the strength of a gravitational field**. Under conditions of the special theory of relativity the strength of a gravitational field can be equivalent to zero while the gravitational field itself is different from zero. Thus far, let us consider the structure of spacetime under conditions of the theory of special relativity.

Claim.

The probability theory consistent normalization of the relationship between the gravitational field and the gravitational wave follows as

$$\frac{{}_{0}g^{2}}{{}_{R}g^{2}} + \frac{{}_{W}g^{2}}{{}_{R}g^{2}} = 1 . \tag{175}$$

Proof.
Due to Axiom I it is

$$+1=+1 \tag{176}$$

The normalized relativistic time dilation relationship is determined as

$$\frac{{}_{0}t^{2}}{{}_{R}t^{2}}+\frac{v^{2}}{c^{2}}=1. \tag{177}$$

As we found before, it is equally

$$\frac{c^{2}\times c^{2}\times {}_{0}g^{2}}{c^{2}\times c^{2}\times {}_{R}g^{2}}+\frac{v^{2}\times c^{2}\times {}_{R}g^{2}}{c^{2}\times c^{2}\times {}_{R}g^{2}}=1. \tag{178}$$

We **define** ${}_{W}t \equiv v\times c\times {}_{R}g \equiv {}_{P}t\times c$ where ${}_{P}t = v\times {}_{R}g$ and where v denotes the relative velocity. The equation above can be simplified as

$$\frac{{}_{0}g^{2}}{{}_{R}g^{2}}+\frac{{}_{W}t^{2}}{c^{2}\times c^{2}\times {}_{R}g^{2}}=1. \tag{179}$$

We **define** ${}_{W}g=\frac{{}_{W}t}{c^{2}}$. Thus far, it is ${}_{W}g^{2}=\frac{{}_{W}t^{2}}{c^{2}\times c^{2}}$ which completes our proof. Under conditions of the special theory of relativity it is

$$\frac{{}_{0}g^{2}}{{}_{R}g^{2}}+\frac{{}_{W}g^{2}}{{}_{R}g^{2}}=1. \tag{180}$$

Quod erat demonstrandum.

Scholium.
In our understanding ${}_{W}g$ represents *the gravitational waves* under conditions of the special theory of relativity. Consequently, it is ${}_{W}g=\frac{{}_{W}t}{c^{2}}=\frac{v\times {}_{R}g\times c}{c^{2}}=\frac{{}_{P}t}{c}=\frac{v}{c}\times {}_{R}g$. This theorem is based on the difference between the strength of a gravitational field and the gravitational field itself.

3.18. Theorem. Newton's second law

The *gravitational field* itself and *the gravitational field strength* are not identical. The gravitational field strength, denoted as $_ag$, is not identical with the gravitational field, denoted as $_Rg$, itself. The gravitational field strength in the international system of units is measured in meters per second squared [m/s^2], the gravitational field due to our proof above is measured in seconds times (second squared per meters squared) or as [s^3/m^2]. Mathematically, acceleration itself is defined as change in velocity (Δv) divided by the duration of the period (Δt), the SI unit for acceleration is the meters per second squared [m/s^2].

Claim.
Under some well defined conditions, the gravitational field $_Rg$ itself and the gravitational field strength $_ag$ are equivalent. We obtain

$$ {}_a g = c^2 \times c^2 \times {}_R g \quad . \tag{181} $$

Proof.
Newton's second law from the standpoint of the stationary observer R is defined as

$$ {}_R\vec{F} = {}_R m \times {}_R\vec{a} \, . \tag{182} $$

In general, vector quantities can be substituted by scalars in the equations as soon as motion is in a straight line. Thus far, let us consider motion is in a straight line. According to the equivalence principle the gravitational mass of a test particle is equal to the inertial mass of this particle and we obtain

$$ {}_R F = {}_R m \times {}_R a = {}_R m \times {}_a g \tag{183} $$

Multiplying Newton's second law by an unknown parameter $_RX$, we obtain

$$ {}_R F \times {}_R X = {}_R m \times {}_a g \times {}_R X \, . \tag{184} $$

We equate this equation with $_RE$*$_Rt$ and obtain

$$ {}_R E \times {}_R t = {}_R m \times {}_a g \times {}_R X \, . \tag{185} $$

Due to special relativity and our proof above, this equation can be rearranged as

$$ {}_R m \times c^2 \times {}_R g \times c^2 = {}_R m \times {}_a g \times {}_R X \, . \tag{186} $$

The unknown parameter $_RX$ follows as

$$_{R}X = c^2 \times c^2 \times \frac{_{R}g}{_{a}g} \quad . \tag{187}$$

Under conditions where $_{R}X = 1$ it follows that

$$_{a}g = c^2 \times c^2 \times {_{R}g} \quad . \tag{188}$$

Quod erat demonstrandum.

Scholium.
There may exist circumstances where acceleration and gravitational field are equivalent but this must not be regarded as being given in general. In general relativity, the gravitational field is associated with the metric tensor g_{ae}. The metric tensor g_{ae} of the general theory of relativity is not completely identical with the gravitational field. By the way, it should be stressed that the metric tensor g_{ae} is more or less a kind of a generalization of **the Newtonian gravitational potential** and not of the gravitational field.

3.19. Theorem. The "mass-equivalent" of a photon m_p

Let $_{p}m$ denote the "mass-equivalent" of a photon. In general it is

$$_{p}m = \frac{h}{c^2} \approx 7{,}372503726\,49051 \times 10^{-51} \times \left(_{R}f = 1 \right) \tag{189}$$

Proof.
Due to Axiom I it is

$$+1 = +1 \tag{190}$$

or

$$_{Wave}E = {_{Wave}E} \; . \tag{191}$$

The energy of a (electro-magnetic) wave, denoted as $_{Wave}E$, is or can be treated a being massless. But due to special relativity, energy, even if mass-less, is "equivalent" to a certain amount of mass too. We obtain

$$_{p}m \times c^2 = {_{Wave}E} \; . \tag{192}$$

where $_{p}m$ denotes the mass-equivalent of an electro-magnetic wave. The same wave energy (from the standpoint of a stationary observer R) can be quantized and is determined as

$$_{p}m \times c^2 = h \times {_{R}f} \tag{193}$$

where $_{R}f$ denotes the frequency as associated with a certain electro-magnetic field. Rearranging equation, we obtain

$$_{p}m = \frac{h}{c^{2}} \times_{R} f \ . \tag{194}$$

This relationship is valid in general but equally for a photon with a frequency of $_{R}$**f = 1**. Under these circumstances ($_{R}f = 1$), we obtain

$$_{p}m = \frac{h}{c^{2}} \times \left(_{R} f = 1 \right). \tag{195}$$

In SI-Units it is

$$_{p}m = \frac{h}{c^{2}} \times 1 = \frac{6.6260755 \times 10^{-34}}{\left(299792458\right)^{+2}} = 7{,}372503726490051 \times 10^{-51} \tag{196}$$

Quod erat demonstrandum.

Scholium.
A photon, an elementary particle and the force carrier for the electromagnetic force, is the quantum of all forms of electromagnetic radiation and of light too. In empty space, a photon moves at the speed of light c. The photon has an energy of the amount E_{Wave}. In fact, due to special theory of relativity even photon's energy is equivalent to certain amount of mass $_{p}m$ and we obtain $E_{Wave} = {}_{p}m * c^{2}$. This does not imply, that a photon must possess a (rest-)mass. The energy (and momentum) of a photon is depending only on its frequency or inversely on its wavelength. The lowest frequency possible a photon can have is equal to $_{R}$**f=1**. Consequently, the mass-equivalent follows as $_{p}m$ = 7,372 503 726 490 51 * 10 $^{-51}$.

3.20. Theorem. The "mass-equivalent" of a photon $_{p}$m

The mass-equivalent of a photon $_{p}m$ is defined as

$$_{p}m = 2 \times \pi \times \varepsilon_{0} \times \mu_{0} \times \hbar \times_{R} f \ . \tag{197}$$

Proof.
Due to Axiom I it is

$$+1 = +1. \tag{198}$$

Due to quantum theory it is $+1 = \frac{h}{2 \times \pi \times \hbar}$ and equally $+1 = c^{2} \times \varepsilon_{0} \times \mu_{0}$. We obtain

$$\frac{h}{2\times\pi\times\hbar}=c^{2}\times\varepsilon_{0}\times\mu_{0}\,. \tag{199}$$

Rearranging equation it follows that

$$\frac{h}{c^{2}}=2\times\pi\times\varepsilon_{0}\times\mu_{0}\times\hbar\,. \tag{200}$$

Multiplying the equation before by $_{R}f$, the frequency (from the standpoint of a stationary observer R), we obtain

$$\frac{h}{c^{2}}\times_{R}f=2\times\pi\times\varepsilon_{0}\times\mu_{0}\times\hbar\times_{R}f \tag{201}$$

which completes our proof. In general, the "mass-equivalent" of a photon from the standpoint of a stationary observer R) follows as

$$_{R}m=\frac{h}{c^{2}}\times_{R}f=2\times\pi\times\varepsilon_{0}\times\mu_{0}\times\hbar\times_{R}f \tag{202}$$

Quod erat demonstrandum.

3.21. Theorem. The "mass-equivalent" of a graviton $_{G}$m

Two photons ($_{p}m_{1}$ and $_{p}m_{2}$) with the frequency f = 1 and at a constant distance are moving uniformly and in straight line (inertial frame of reference) somewhere in deep space without being disturbed anyhow. Thus far, Newton's laws of motion are valid. Under these conditions, the "mass-equivalent" of a graviton can be calculated approximately as

$$_{G}m=1.34605337\times10^{-136} \tag{203}$$

Proof.
Due to Axiom I it is

$$+1=+1 \tag{204}$$

or

$$_{0}F={}_{0}F \tag{205}$$

According to Newton's law of gravitation, we obtain

$$_{0}F=\frac{_{0}G\times_{p}m_{1}\times_{p}m_{2}}{_{0}d\times_{0}d} \tag{206}$$

where ${}_{p}m_{1}$ denotes the "mass equivalent" of the one photon 1 and ${}_{p}m_{2}$ denotes the "mass equivalent" of the other photon 2, G is the Newtonian "constant" of gravitation and ${}_{0}d$ is the distance between the two photons. Rearranging this equation, we obtain

$$ {}_{0}F = \frac{{}_{0}G \times \frac{h}{c^2} \times \left({}_{0}f = 1\right)_1 \times \frac{h}{c^2} \times \left({}_{0}f = 1\right)_2}{c \times {}_{0}\left(t = 1\right) \times {}_{0}d} \tag{207} $$

since $c \times {}_{0}(t = 1) = {}_{0}d$ and the "mass equivalent" of the one photon is ${}_{p}m = \frac{h}{c^2} \times (f = 1)$. Multiplying this equation by ${}_{0}d$ we obtain

$$ {}_{0}F \times {}_{0}d = \frac{{}_{0}G \times \frac{h}{c^2} \times \frac{h}{c^2}}{c} \tag{208} $$

On the left part of the equation there is something like energy ${}_{0}E = {}_{0}F \times {}_{0}d = {}_{G}m \times c^2$ where ${}_{G}m$ denotes the "mass equivalent" of the graviton. Following from the above, one obtains according to Newtonian axioms and special relativity theory the equation

$$ {}_{G}m \times c^2 = \frac{{}_{0}G \times h \times h}{c \times c^2 \times c^2} \tag{209} $$

Dividing by c^2 it is

$$ {}_{G}m = \frac{{}_{0}G \times h \times h}{c \times c^2 \times c^2 \times c^2} \tag{210} $$

However, in SI Units, we obtain

$$ {}_{G}m = \frac{6.67259 \times 10^{-11} \times 6.6260755 \times 10^{-34} \times 6.6260755 \times 10^{-34}}{\left(299792458\right)^{+7}} \tag{211} $$

A convincing formulation of the "mass equivalent" of the graviton follows as

$$ {}_{G}m = 1.34605337 \times 10^{-136} \tag{212} $$

Quod erat demonstrandum.

Scholium.

The "mass equivalent" of the graviton $_{G}m$ does not imply that a graviton as such does possess any kind of a mass. The graviton (as a hypothetical elementary particle which mediates the force of gravitation in the framework of quantum field theory) is expected to be massless. This result suggests that, if it should be possible to convert the massless elementary particle graviton into energy completely, the "mass equivalent" of the graviton would be equivalent to $_{G}m = 1.346053370 \times 10^{-136}$. Thus far, there is no experimental evidence that a graviton exists. We are just assuming that a graviton exists.

4. Discussion

The theory quantum mechanics, perhaps the most revolutionary theory in the history of science, has raised innumerable questions to physicists, chemists and philosophers of science. Strictly speaking, the wave function is still one of the pillars of quantum mechanics. Thus far, there is nothing mysterious with the wave function. The wave function is existing independently of human mind and consciousness and something objective. The wave function is the quantum mechanical equivalent of the notion time $_{R}t$ of the special theory of relativity. In particular, another crucial aspect of quantum mechanics is the reduction of the state vector (i.e. collapse of the wavefunction).

The collapse of the wave function and the correct understanding of the collapse of the wave function addresses several distinct, important and far reaching issues of the foundations of today's physics and science as such. Originally, the concept of wavefunction collapse was introduced by Werner Heisenberg in his 1927 paper "Über den anschaulichen Inhalt der quantentheoretischen Kinematic und Mechanik" and later incorporated into the mathematical formalism of quantum mechanics by John von Neumann in his 1932 publication "Mathematische Grundlagen der Quantenmechanik". In his 1927 paper, Heisenberg writes

"durch die experimentelle Feststellung: "Zustand m" wählen wir aus der Fülle der verschiedenen Möglichkeiten (c_{nm}) eine bestimmte: m aus, zerstören aber gleichzeitig, wie nachher erläutert wird, alles, was an Phasenbeziehungen noch in den Größen c_{nm} enthalten war." [15]

It is easy understand the core of this problem. A (relativistic) system evolves in time by the continuous evolution via the Schrödinger equation or some relativistic equivalent. Under appropriate circumstances, the wave function, initially in a superposition of several eigenstates, collapses or reduces to a single eigenstate, that what is measured by a moving observer O. However, after the collapse of the wave function, a physical system is determined or described again by a wave function.

Thus far, the continuous evolution via the Schrödinger equation and the collapse of the wave function are the two basic processes by which quantum systems evolve in time. However, let us focus on the appropriate notion of the collapse of the wave function. Is the collapse of the wavefunction a fundamental and objective physical phenomenon of its own, rather than a non-real theoretical mathematical construct? Does the collapse of the wave function takes any time, the collapse time?

In contrast to the Copenhagen dominated interpretation of quantum mechanics, i. e. Penrose has put forward an approach that the phenomenon of wavefunction collapse is a real physical process. Penrose approach to the problem of the collapse of the wave function [16] may not be correct in detail, but the same goes into the right direction. The collapse of the wave function is something objective and happens everywhere around us. It appears to be possible that within the collapse of the wave function, the cause of the beginning of our world can be found.
It is clear, that if we divide i.e. $\mathbf{{}_{R}t/c^2}$ we will obtain something physical real and not only a new mathematical construct. But the question allowed is of course, is ${}_{R}t/c^2$ really identical with the notion gravitational field (under conditions of special theory of relativity). Thus far, the prediction of gravitational waves even under conditions of special theory of relativity is of course highly speculative but the same has to potential to help us to decide about the correctness of **the equivalence of time and gravitational field in general**.

5. Conclusion

The problem of the physical meaning of the wave function is solved. The wave function is quantum mechanical analogue of the notion time ${}_{R}t$ of special relativity.

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Appendix

None.

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