Quantum Gravitational Relativity
Part I

The theory presented in this paper is the first part of the quantum gravitational formulation of Einstein's special theory of relativity. The formulation is based on two postulates which take into account the discrete nature of space and time. Because the Fitzgerald-Lorentz length contraction formula violates the space quantization postulate, this formula is modified to avoid the violation. However Einstein's time dilation formula does not violate the time quantization postulate. This seems to indicate that we cannot treat time the same way we treat space.

by Rodolfo A. Frino

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1. Introduction

In this section I shall briefly describe a relativistic effect known as length contraction. Length contraction is the phenomenon whereby the length of an object with respect to an observer located in a moving reference frame will appear contracted in the direction of motion. The relation between the length of the object l_0 (proper length) with respect to a frame in which the object is at rest to the length of the object l ("contracted" length) measured by an observer moving with velocity v with respect to the stationary observer, is given by:

$$l = l_0 \sqrt{1-\frac{v^2}{c^2}} \tag{1.1}$$

This equation is sometimes written as

$$l = l_0 \sqrt{1-\beta^2} \tag{1.2}$$

where β is the ratio between the velocity of the body, v, to the speed of light, c:

$$\beta \equiv \frac{v}{c} \tag{1.3}$$

Relation (1.1/1.2) is the famous Fitzgerald-Lorentz length contraction formula (also known as Lorentz-Fitzgerald length contraction, Lorentz length contraction, relativity length contraction, length contraction). The formula can be easily derived from the Lorentz transformations. Note that, according to this formula, the length of the body is maximum in the reference frame in which the body is at rest. Thus moving bodies appear contracted or shrink in the direction of movement. However they do not appear contracted in the perpendicular directions to it. An interesting point to observe is that both Lorentz's ether theory and Einstein's special relativity produce exactly the same formula for the length contraction of a body. However, in contrast to Einstein's special theory of relativity, Lorentz's theory assumes the existence of an undetectable immobile ether. Einstein's theory, on the other hand, assumes that the speed of light in vacuum is independent of the motion of the light source (a postulate known as the *invariance of c*). Therefore the speed of light, *c*, turned out to be a universal constant. In Einstein's own words: *"Any ray of light moves in the 'stationary' system of co-ordinates with the determined velocity c, whether the ray be emitted by a stationary or by a moving body"* [1]. Because no experiment was able to confirm the existence of an immobile ether, Einstein's special theory of relativity is preferred over Lorentz's ether theory.

The purpose of this paper is to modify formula (1.1) to incorporate the discrete nature of space. This is done through the Planck length, which is defined as:

$$L_P \equiv \sqrt{\frac{h\,G}{2\,\pi\,c^3}} \tag{1.4}$$

Because the Planck length contains both the Planck's constant, *h*, and Newton's gravitational constant, *G*, it is reasonable to call this theory: the Quantum Gravitational Special Theory of Relativity or simply: Quantum Gravitational Relativity (QGR) (see **Notes**). The next section outlines the two postulates on which QGR is based upon. This does not require any sophisticated mathematical tools. **Appendix 1** contains the nomenclature used in this paper.

2. Postulates

This formulation assumes that the nature of spacetime is discrete or quantized. This quantization is implemented through the following two postulates:

(1) **Time quantization postulate**

Time is discrete. This means that there is a time, T_{MIN}, which is the minimum time with physical meaning. In other words there is no time or time interval smaller than T_{MIN}. It is likely that T_{MIN} to be equal to the Planck time, T_P.

(2) **Space quantization postulate**

Space is discrete. This means that there is a length, L_{MIN}, which is the minimum length with physical meaning. In other words there is no length or distance smaller than L_{MIN}. It is likely that L_{MIN} to be equal to the Planck length, L_P.

3. The Problem of Fitzgerald-Lorentz Length Contraction

Let us consider the problem with the Fitzgerald-Lorentz length contraction formula. This formula is:

$$l = l_0 \sqrt{1-\beta^2} \tag{3.1}$$

If we make l_0 equal to L_{MIN} then the value of l will be

$$l = L_{MIN} \sqrt{1-\beta^2} \tag{3.2}$$

Because $\sqrt{1-\beta^2}$ is less than 1 for all values of *v* in the range: $0 < v < c$, we deduce that the length, l , of the body in the direction of the body's movement will appear to be less than L_{MIN} (except for *v* = 0, in which case there is no movement). But this violates the second postulate (space quantization postulate) of this formulation which says that there is no length smaller than L_{MIN} . Thus we draw the conclusion that if postulate 2 is correct, and if length contraction is real, then the Fitzgerald-Lorentz length contraction formula is incorrect. Now one can ask: What should be the value of l when $l_0 = L_{MIN}$? The value of l for L_{MIN} should be equal to L_{MIN} (not equal to $L_{MIN} \sqrt{1-\beta^2}$ as predicted by the Lorentz formula). This means that we need to modify the Fitzgerald-Lorentz length contraction formula to correct this problem. The corrected formula will be called: The quantum gravitational length contraction (QGLC) formula. This new formula is introduced in the following section.

4. The Quantum Gravitational Length Contraction

The formula for the quantum gravitational length contraction is

Quantum Gravitational Length Contraction

$$l = l_0 \sqrt{1-\beta^2} + L_{MIN} \left(1 - \sqrt{1-\beta^2}\right) \tag{4.1}$$

This formula guaranties that the length, *l*, of a body will never appear to be smaller than the minimum length, L_{MIN} , regardless of the proper length, l_0 , of the body and its speed, *v*, with respect to the observer (of course l_0 must be greater or equal than L_{MIN}).

L_{MIN} is the minimum length with physical meaning. In other words nature does not have any length or distance smaller than L_{MIN} . So far no experiment could determine the exact value of L_{MIN} . However it is likely that this length to be equal to the Planck length, L_P . The quantum gravitational length contraction formula shown on **Table 7.1** (Summary section) assumes, precisely, that $L_{MIN} = L_P$. Having said that we have to keep in mind that L_{MIN} could be smaller than the Planck length (including a nil value in which case SR will hold). We just don't know.

Formula (4.1) guaranties that the length of a body will never appear to be smaller than the minimum length, L_{MIN} . This is reasonable since it doesn't make any sense than a body appears to be smaller than the minimum length imposed by nature (if there is one). **Appendix 2** illustrates the quantitative difference between the Fitzgerald-Lorentz contraction of Special Relativity and its counterpart of Quantum Gravitational Relativity.

5. Graphics

Table 5.1 shows the values of the function l/L_P for three different values of l_0 . The independent variable β varies between 0 and 1 in steps of 0.1. **Figure 5.1** shows the graphics corresponding to the three cases.

$$\frac{l}{L_P}=\frac{l_0}{L_P}\sqrt{1-\beta^2}+\left(1-\sqrt{1-\beta^2}\right) \qquad (5.1)$$

β	$\frac{l_0}{L_P}=1$	$\frac{l_0}{L_P}=5$	$\frac{l_0}{L_P}=10$
0.0	1	5	10
0.1	1	4.979 950	9.954 887
0.2	1	4.919 184	9.818 163
0.3	1	4.815 757	9.585 453
0.4	1	4.666 061	9.248 636
0.5	1	4.464 102	8.794 229
0.6	1	4.2	8.2
0.7	1	3.856 571	7.427 286
0.8	1	3.4	6.4
0.9	1	2.743 560	4.923 009
1.0	1	1	1

Table 5.1: The table shows the values of l/L_P for $l_0/L_P=1$, $l_0/L_P=5$ and $l_0/L_P=10$

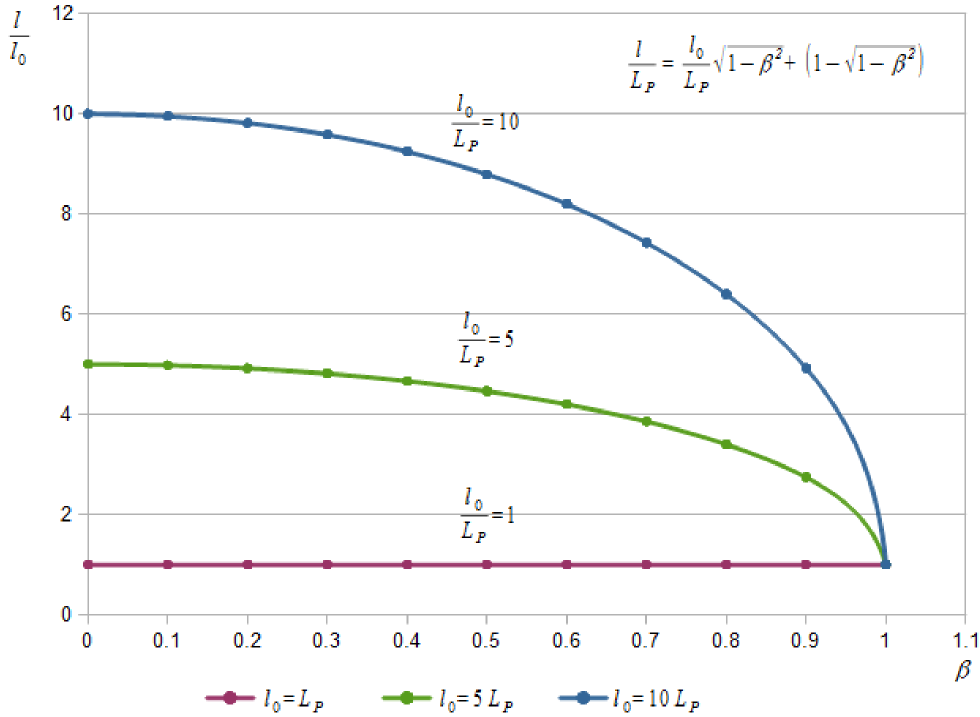

Figure 5.1: Graphical representation of function (5.1) for the three cases shown on Table 5.1

6. Analysis

In this section we shall analyse the data of section 5. According to Figure 5.1 we draw the following conclusions:

(1) When the proper length of a hypothetical particle is equal to the Planck length, L_P, then length contraction will never take place. This means that both an inertial observer in relative motion with respect to the particle and a stationary observer will measure the same length, L_P.

(2) When the proper length of a hypothetical particle is greater than the Planck length, L_P, then length contraction will always take place. This means that all inertial observers in relative motion with respect to the particle will measure a smaller length than that measured by the stationary observer with respect to the particle.

However, contrary to the predictions of Einstein's Special Relativity, the length of this particle, in both cases, will never appear to be shorter than the Planck length, regardless of the speed, *v,* of the particle with respect to the inertial observer.

Appendix 2 illustrates the difference between the results of the length contraction formulas from special relativity and quantum gravitational relativity.

7. Is There a Problem with Einstein's Time Dilation?

Einstein's special theory of relativity predicts that time does not flow at the same rate for observers in relative motion. Moving clocks appear to tick more slowly relative to stationary clocks. Einstein's found a quantitative description for the flow of time through the time dilation formula:

$$t = \frac{t_0}{\sqrt{1-\frac{v^2}{c^2}}} \tag{7.1}$$

In this formula *t* is the time measured by a moving clock ("dilated" time) and t_0 is the time measured by a stationary clock (proper time). However after inspecting formula (7.1) we find that is not necessary to modify this formula as we did in section 4 because the formula does not violate the time quantization principle. If the proper time is equal to the minimum time $t_0 = T_{MIN}$, then the "dilated" time *t* is

$$t_{MIN} = \frac{T_{MIN}}{\sqrt{1-\beta^2}} \tag{7.2}$$

Because the factor $1/\sqrt{1-\beta^2}$ is greater than 1 for all values of the speed *v* in the range: $0 < v < c$, the time t_{MIN} turns out to be greater than T_{MIN}. On the other hand, for *v* = 0, the time t_{MIN} turns out to be identical to T_{MIN}. In other words the "dilated" time *t* never gets smaller than T_{MIN}. This means that the time dilation formula does not violate postulate 1 (time quantization postulate). This in turn implies that we have an asymmetry between space and time: we cannot treat time the same way we treat space! Why do we have this asymmetry? I believe that nobody knows. To answer this question we need to deal further with this topic and this might not be an easy task. If we assume that the minimum time is equal to the Planck time:

$$t_{MIN} = T_P \tag{7.3}$$

then equation (7.2) becomes

$$t_P = \frac{T_P}{\sqrt{1-\beta^2}} \tag{7.4}$$

Which says that the "dilated" time t_P turns out to be greater than the Planck time, T_P.

8. Summary

Table 8.1 shows both the Lorentz length contraction formula and its quantum gravitational counterpart. For the quantum gravitational formula I have assumed that:

$$L_{MIN} = L_P \tag{8.1}$$

Name of the Formula	Original Relativistic Formula (SR)	Quantum Gravitational Formula (QGR)
Length contraction	$l = l_0\sqrt{1-\beta^2}$	$l = l_0\sqrt{1-\beta^2}+\left(1-\sqrt{1-\beta^2}\right)L_P$
Time dilation	$t = \dfrac{t_0}{\sqrt{1-\dfrac{v^2}{c^2}}}$	——

Table 8.1: The formulas for the Lorentz length contraction. Note that L_{MIN} has been replaced by L_P (the Planck length). Einstein's time dilation formula doesn't have a quantum gravitational counterpart.

9. Conclusions

The goal of this paper is to present a generalization of the Fitzgerald-Lorentz formula of length contraction. When do we have to use the quantum gravitational formula? We should used the QGLC formula instead of the original Lorentz formula whenever the length, *l*, of the body is of the same order of magnitude as L_{MIN} (close enough to this length). The final question one can ask is: is space really discrete? It may be years before we know the answer to this question, but everything seems to indicate that the space "fibres" of our universe are indeed quantized. The quantum gravitational treatment of black holes I carried out [2] seems to indicate that space is indeed quantised and that $L_{MIN} = L_P$. In summary, this paper shows that if the relativistic length contraction is a real effect and if space is quantized, then the Fitzgerald-Lorentz length contraction formula would not hold at the Planck scale. This in turn would imply that the Lorentz transformations would not hold at the Planck scale. This is so because they are classical transformations and therefore they do not take into account the space quantization postulate. If this were the case we would need to abandon the Lorentz transformations to make room for new and more general quantum gravitational transformations of space and time.

Notes

The first version of this article, which was published on 22/05/2015, was entitled: *Quantum Special Relativity*. Since then I changed the article's title to reflect the quantum gravitational nature of the formulation.

Appendix 1
Nomenclature

The following are the symbols used in this paper

$h =$ Planck's constant

$c =$ speed of light in vacuum
$G =$ Newton's gravitational constant
$l_0 =$ proper length
$l =$ “contracted” length
$t_0 =$ proper time
$t =$ “dilated” time
$t_P =$ “dilated” time corresponding to a proper time equal to the Planck time.
$T_{MIN} =$ Minimum time with physical meaning
$L_{MIN} =$ Minimum length with physical meaning
$T_P =$ Planck time
$L_P =$ Planck length
$v =$ speed of a massive body with respect to certain inertial observer
$\beta =$ ratio between the speed, *v*, of a massive body and the speed of light, *c*.
SR = Special relativity
QGR = Quantum gravitational relativity
QGLC = Quantum gravitational length contraction

Appendix 2
Calculation of the Fitzgerald-Lorentz Contraction for a Particle

Let us calculate the Fitzgerald-Lorentz contraction
(a) using Special Relativity, and
(b) using Quantum Gravitational Relativity
for a particle whose proper length, l_0 , is equal to 10 times the Planck length: $l_0 = 10\, L_P$. For the numeric calculations we shall assume that the velocity of the particle with respect to an inertial frame of reference is (i) 0.1*c*; and then (ii) 0.99*c*.

(a) Special Relativity

The formula in this case is

$$l = l_0 \sqrt{1-\beta^2}$$

(Case a-i) $v = 0.1\text{c}$

$$l(0.1\, c) = 10\, L_P \sqrt{1-0.1^2}$$
$$l(0.1\, c) \approx 1.608\ 098 \times 10^{-34}\, m$$

The ratio between this length and the Planck length is

$$\frac{l(0.1\, c)}{L_P} \approx 9.94988$$

(Case a-ii) $v = 0.99\, c$

$$l(0.99\, c) = 10\, L_P \sqrt{1-0.99^2}$$

$$l(0.99\,c) \approx 2.279\,930 \times 10^{-34}\,m$$

The ratio between this length and the Planck length is

$$\frac{l(0.99\,c)}{L_P} \approx 14.1067$$

(b) Quantum Gravitational Relativity

The formula in this case is

$$l = l_0\sqrt{1-\beta^2} + L_P\left(1-\sqrt{1-\beta^2}\right)$$

(Case b-i) $v = 0.1$c

$$l(0.1\,c) = 10\,L_P\sqrt{1-0.1^2} + L_P\left(1-\sqrt{1-0.1^2}\right)$$

$$l(0.1\,c) \approx 1.608\,908 \times 10^{-34}$$

The ratio between this length and the Planck length is

$$\frac{l(0.1\,c)}{L_P} \approx 9.9549$$

(Case b-ii) $v = 0.99$c

$$l(0.99\,c) = 10\,L_P\sqrt{1-0.99^2} + L_P\left(1-\sqrt{1-0.99^2}\right)$$

$$l(0.99\,c) \approx 3.668\,136 \times 10^{-34}\,m$$

The ratio between this length and the Planck length is

$$\frac{l(0.99\,c)}{L_P} \approx 22.6961$$

Comparing these results we draw the following conclusion: the faster a body moves with respect to an observer located in an inertial frame of reference, the greater the difference between the results given by the length contraction formulas of Special Relativity and Quantum Gravitational Relativity.

REFERENCES

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