Geometrical Universe Hypothesis

Essay written for the Gravity Research Foundation 2015 Awards for Essays on Gravitation

Author: Jacek Safuta

27/03/2015

Abstract

Max Tegmark, exploring implications of the External Reality Hypothesis (ERH) that there exists an external physical reality completely independent of us humans, claims that physics is so successfully described by mathematics because the physical world is completely mathematical, isomorphic to a mathematical structure, and that we are simply uncovering this bit by bit.[1] In this essay we have tried to discover that mathematical structure.

We start from the correspondence rule as it has a potential to look at the existing empirical domain from a different angle, outside the mainstream physics.

Geometrical Universe Hypothesis (GUH) is the theory of mathematical physics compatible with ERH. If GUH is correct, then the empirical domain and the correspondence rule are redundant in the sense that they can be inferred from Thurston geometries.
In this essay we present a sketch of the theory of mathematical physics compatible with ERH. We call it Geometrical Universe Hypothesis.

A theory of mathematical physics shall consist of:

- **correspondence rule** which links a mathematical structure with an empirical domain (e.g. in General Relativity, gravitational force that can be measured is only a manifestation of spacetime geometry that can be calculated)

- **empirical domain** (following the example above, we possess a lot of observational and experimental tests of GR)

- **mathematical structure** (consistently following, 3+1 dimensional, pseudo-Riemannian manifold)

**Correspondence rule**

We have started from the correspondence rule as it has a potential to look at the vast existing empirical domain from a different angle and lead us to the corresponding mathematical structures. We do not need new data nor new mathematical structures. We need this different angle - a **paradigm shift**.

The best example of a real paradigm shift is General Relativity that we have taken as an example of theory of mathematical physics. So let us start with GR which describes the relation between the geometry of 3+1 dimensional, pseudo-Riemannian manifold representing spacetime, and an energy–momentum contained in that spacetime. To continue that revolution we do not need Einstein's equations and even pseudo-Riemannian manifold. What we need, at the moment, is the paradigm that the **force** is simply a **manifestation of spacetime geometry**. This is the bond between the reality (geometry, mathematical structure) and our perception of reality (physics, experiments and measurements). We perceive and measure a force (physics), we know and calculate a deformation of spacetime (geometry).

By definition, a **force** is any **interaction** transferring **energy**. A **force field** is a vector field that describes a **non-contact force**. A non-contact force, in turn, is the force applied to an object by another object that does not stay in direct contact with it. Nowadays, the concept of direct contact forces is valid only in a colloquial language. Possibly, we could imagine a scientific meaning of that notion, in the case where an interaction/superposition of waves we called a contact (e.g. solitons can interact with other solitons and emerge from the “collision” unchanged, except for a phase shift). Regardless of the fact that the general term of force is commonly used and convenient, what we have presented above implies that any force (as being non-contact) is a manifestation of spacetime geometry.

Summing up: all **fundamental forces** are non-contact, manifestations of spacetime geometries.

So far we are aware of no other compelling explanation for the phenomenon of force/field. Therefore the concept deserves at least serious consideration.
Empirical domain

Today we recognize **four fundamental interactions** (forces/fields). There are substantial, measureable differences between them: a **scale/distance** they act over, an **attractive** or **repulsive** character, their **strength** and others like spin.

Gravitation and electromagnetism act over potentially infinite **distance**. The other two, act over subatomic distances. If we assumed, following previous considerations, the geometric character of them (an elastic spacetime distortion or curvature), we would understand that in reality the distances are also infinite. This is the attribute of any elastic medium. We find their limits as the strength of every field diminishes with distance to the point of being undetectable.

**Matter** does not have an universal definition. It is not even a fundamental concept in physics.

So called “ordinary matter” is composed of quarks and leptons.

With this notion we have completed our empirical domain and we are about to look for the proper mathematical structures.

Mathematical structures corresponding to our universe

On the observational and experimental basis (empirical domain) we can assume that spacetime is a **differentiable manifold**, can be given a differential structure locally by using the **homeomorphisms** in its atlas, is **continuous**, has **elastic** property that is **isotropic**.

The simplest mathematical structures, corresponding to the 3+1 dimensional spacetime of modern physics (including relativity), involves the set of **Thurston geometries** (the geometrization conjecture, proved by Perelman[2]). We can treat them as space-like, totally geodesic submanifolds of 3+1 dimensional spacetime.

In three dimensions, it is not always possible to assign a single geometry to a whole space. Every closed 3-manifold can be **decomposed** into pieces that each have one of eight types of geometric structure, resulting in an emergence of some attributes that we can observe. Thurston geometries include: $S^3$ geometry of constant positive scalar curvature, $E^3$ flat geometry, $H^3$ geometry of constant negative scalar curvature. The three are homogeneous and isotropic and five more exotic Riemannian manifolds are homogeneous but not isotropic.

In the tables below we assign interactions and matter to the proper Thurston geometries.
<table>
<thead>
<tr>
<th>Interaction</th>
<th>Gravitation</th>
<th>Electromagnetic</th>
<th>Weak</th>
<th>Strong</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum scale</td>
<td>infinite</td>
<td>infinite</td>
<td>subatomic</td>
<td>subatomic</td>
</tr>
<tr>
<td>(to become undetectable)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Local geometry</td>
<td>$S^3$</td>
<td>$E^3$</td>
<td>$S^3$</td>
<td>$E^3$</td>
</tr>
<tr>
<td>(Bosons - “particles mediating”)</td>
<td>(Gravitons?)</td>
<td>(Photons)</td>
<td>(W')</td>
<td>(Gluons)</td>
</tr>
<tr>
<td>notice1</td>
<td>notice1</td>
<td></td>
<td>$E^3$</td>
<td></td>
</tr>
<tr>
<td>notice2,3</td>
<td></td>
<td></td>
<td></td>
<td>$S^3$</td>
</tr>
<tr>
<td>notice2</td>
<td></td>
<td></td>
<td></td>
<td>$H^3$</td>
</tr>
<tr>
<td>notice3</td>
<td></td>
<td></td>
<td></td>
<td>$E^3$ (Z')</td>
</tr>
<tr>
<td>Acts on</td>
<td>All</td>
<td>Electrically charged</td>
<td>Quarks, Leptons</td>
<td>Quarks, Gluons, Hadrons</td>
</tr>
<tr>
<td>Strength (in the scale of quarks)</td>
<td>$10^{-41}$</td>
<td>$1$</td>
<td>$10^{-4}$</td>
<td>$60$</td>
</tr>
<tr>
<td>Spin (geometric structure on a Riemannian manifold?)</td>
<td>not applicable</td>
<td>$1$</td>
<td>$1$</td>
<td>$0, 1$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Matter</th>
<th>Leptons</th>
<th>Quarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Local geometry</td>
<td>$E^3$</td>
<td>$S^3$</td>
</tr>
<tr>
<td>(Fermions)</td>
<td>(Neutrinos)</td>
<td>(u, c, t)</td>
</tr>
<tr>
<td>notice2,3</td>
<td>$H^3$</td>
<td>$H^3$</td>
</tr>
<tr>
<td>notice2,3</td>
<td>(Electron)</td>
<td>(d, s, b)</td>
</tr>
<tr>
<td>notice2,3</td>
<td>(Muon)</td>
<td></td>
</tr>
<tr>
<td>notice2,3</td>
<td>(Tau)</td>
<td></td>
</tr>
<tr>
<td>Spins (geometric structure on a Riemannian manifold?)</td>
<td>$1/2$</td>
<td>$1/2$</td>
</tr>
</tbody>
</table>

Notice 1: gravity possibly can be an emerging interaction - a superposition of other geometries with $S^3$ being the outcome? Then it could be decomposed into the other geometries.

Notice 2: There is a metric associated with each geometry [6]

Notice 3: The geometries/metrics evolve by a wave equation. The constant curvature geometries arise as steady states of the Ricci flow, the other five homogeneous geometries arise naturally where the dynamics of the Ricci flow is more complicated and where topological changes (neck pinching or surgery: physicists might call these “wormholes”) happen. This picture is not yet completely clear [14][C][15]
The structures presented in the tables are static, space-like submanifolds (see Notice 3). The time is not included yet.

Let us recall our previous considerations that a force is any interaction transferring energy.

The classical descriptions of energy transfer methods are:

- thermal radiation that is electromagnetic radiation - this is the wave transfer of energy
- thermal conduction is the transfer of internal energy by microscopic “collisions” of particles. As the matter exhibits wave-like behavior and forces are non-contact, this is also the wave transfer of energy
- mass transfer - the matter\(^{(A)}\) exhibits wave-like behavior so this is also the wave transfer of energy.

As we can infer, wave is the only method of energy transfer (interaction). The wave is a periodic deformation of elastic medium (space or spacetime). The elasticity and energy are the preconditions for periodic motion.

Unfortunately since this point we have started to compose a wave theory. The weakness of wave theories was that waves would need an elastic medium for transmission. The existence of the hypothetical substance light-bearing aether was cast into doubt by Michelson–Morley experiment. These gentlemen attempted to detect the absolute motion of Earth through the stationary aether. They could not take into account that matter (the Earth) also could be a wave packet made of the same, elastic medium like the light. The wave is a disturbance/deformation, that travels through that medium, transferring the energy or matter. The wave motion transfers energy from one place to another, with no permanent displacement of the points of the medium. Conclusion: there is no motion of Earth through the aether. The nail in the coffin of the aether was the fact that Einstein's Special Relativity could generate the same mathematics without referring to the aether. That time it led most physicists to the conclusion that the notion of an aether was not a useful concept. We present a modern, Laughlin’s, Nobel Laureate in Physics, opinion:

The word “aether” has extremely negative connotations in theoretical physics because of its past association with opposition to relativity. This is unfortunate because, stripped of these connotations, it rather nicely captures the way most physicists actually think about the vacuum [...] The modern concept of the vacuum of space, confirmed every day by experiment, is a relativistic aether. But we do not call it this because it is taboo.\(^{[11]}\) This is the reason that we use the notion of elastic spacetime or space instead.

Summing up: the wave is a disturbance/deformation, that travels through the elastic medium of spacetime, transferring an energy and matter. The wave motion transfers energy and matter from one place to another, with no permanent displacement of the points of spacetime.

The description of geometrical structures here delivers the initial conditions. An approach to details we can find e.g. in\(^{[13]}\).

**Conclusion**

Finally GUH can be broken down into:

- correspondence rule that all interactions and matter are manifestations of spacetime geometry
- empirical domain - gravitational, electromagnetic, strong nuclear and weak nuclear measurements and cosmological observations
- geometric structure being the set of Thurston geometries with metrics and the wave transfer

GUH makes the testable prediction that five more Thurston geometrical structures remain to be uncovered in nature: \(S_2 \times \mathbb{R}, \mathbb{H}_2 \times \mathbb{R}, \text{SL}(2, \mathbb{R}), \text{Nil and Solv geometry.}\)
References


[2] Huai-Dong Cao, Xi-Ping Zhu, Hamilton-Perelman’s Proof of the Poincaré Conjecture and the Geometrization Conjecture

[6] Grady N., The eight geometries of the geometrization conjecture,
https://ir.library.oregonstate.edu/xmlui/bitstream/handle/1957/25249/The%20Eight%20Geometries%20of%20the%20Geometrization%20Conjecture.pdf?sequence=1


[13] Asselmeyer-Maluga T., Brans C.H., How to include fermions into General relativity by exotic smoothness

[14] Kusner R. The Poincare and Geometrization Conjectures: A Bit of (Cosmic) Background,
http://www.gang.umass.edu/~kusner/other/3mfd.html

[15] Bryant R., Ricci flow solitons in dimension three with SO(3)- symmetries
http://www.math.duke.edu/~bryant/3DRotSymRicciSolitons.pdf

[16] Nakad R., Spin\(^{c}\) Structures on Manifolds and Geometric Applications, Max Planck Institute for Mathematics 2012,
http://www.iecn.u-nancy.fr/~nakad/mini.pdf

Technical endnotes

[A] The elementary particles are certainly not eternal and indestructible units of matter, they can actually be transformed into each other. As a matter of fact, if two such particles, moving through space with a very high kinetic energy, collide, then many new elementary particles may be created from the available energy and the old particles may have disappeared in the collision. Such events have been frequently observed and offer the best proof that all particles are made of the same substance: energy.[7] Eddington remarked, when observing the ocean we perceive the moving waves as objects because they display a certain permanence, even though the water itself is only bobbing up and down.[8]

[C] We have to normalize the Ricci flow to obtain a flow which preserves volume

[D] 3-manifolds can have more than one type of geometric structure